

Whether Being Advantageous Matters Is Decided by the Size of the Population

— A mutation with a selection coefficient of **0.01** fixes **19.8013** times as often as a neutral one at $N = 500$, but only **1.2012** times as often at $N = 10$ — what decides is not s but Ns — and at $s = 0$ the probability is exactly the neutral value — [Paper 478]

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— *From thinking to syncing.* —

Abstract

An advantageous mutation is said to spread. This paper shows that what decides whether the advantage matters is not its size but its product with the population size—where Ns is small, being advantageous does no good. No new theorem or law is claimed. First, a mutation with $s = 0.01$ saturates at a fixation probability of **0.019801** even at $N = 10000$ (Section 2). Second, a neutral one is $1/(2N)$, falling without bound as N grows (Section 2). Third, this is the core of the paper. The ratio to neutral is **1.2012** at $N = 10$ and **19.8013** at $N = 500$ (Section 3). Fourth, the ratio rises monotonically with N , approaching neutral as Ns shrinks (Section 3). Fifth, at $s = 0$ the fixation probability is exactly $1/(2N)$ (Section 3). Sixth, the separator is whether the advantage acts or Ns acts (Section 4).
Keywords: fixation probability, genetic drift, population size, natural selection, nearly neutral theory.

Scope of this paper (scope note): No new theorem or law is claimed—Kimura’s fixation probability, genetic drift and the nearly neutral theory are all standard. No evolutionary history is argued—particular traits, lineages and dates are not the subject. The diffusion approximation is used—non-overlapping generations, constant population size and weak selection are assumed; real populations often violate these. Effective size is not distinguished— N is treated as the effective population size, which can differ greatly from the census count. Linkage and recombination are excluded—hitchhiking, background selection and linkage disequilibrium are not the subject. Relation to earlier papers: Paper 477 showed that adding to a non-minimal element does nothing—there too, it was a pairing and not a quantity alone that acted. Paper 300 showed that common root and distinct root can be decided—“selection is acting” and “ Ns is large” are distinct roots by that criterion. Paper 446 showed that a floor moves with the demand—there too, a quantity alone was undetermined without its condition. What is added is giving the fixation probability at four population sizes, giving the ratio to neutral as 1.2012 and 19.8013, confirming that the ratio rises monotonically with N , showing that $Ns = 0.02$ lies within five per cent of neutral, confirming the exact neutral value at $s = 0$, and placing the separator at s versus Ns .

1. Notation and the Question

Setting: In a diploid population of effective size N , let a mutation of selection coefficient s arise once in one individual, at initial frequency $p_0 = 1/(2N)$. Under the diffusion approximation, the probability that it spreads through the whole population is

$$u = \frac{1 - e^{-4Ns p_0}}{1 - e^{-4Ns}}$$

which reduces to $u = p_0 = 1/(2N)$ at $s = 0$. Below, $s = 0.01$.

Observation 1.1

An advantageous mutation is said to spread. Advantage means being selected—apparently the very definition of selection.

Observation 1.2

But at first only one individual carries it. Advantage or not, chance can remove it—and how much chance matters is set by how small the population is.

Question: Which of the two wins.

2. Advantage Acts Only in Large Populations

Computation 2.1: Take the fixation probability at four population sizes.

Population N	Fixation probability	Neutral $1/(2N)$
10	0.060062	0.050000
100	0.020171	0.005000
1000	0.019801	0.000500
10000	0.019801	0.000050

Observation 2.2

The probability saturates at **0.019801** however large N becomes. The same at $N = 1000$ and $N = 10000$ —once the population is large enough, the fate of an advantageous mutation no longer depends on its size.

Observation 2.3

That value is close to $2s$. 0.019801 for $s = 0.01$ —even a one per cent advantage spreads only once in about fifty tries.

Observation 2.4

The neutral case, meanwhile, falls without bound. From 0.050000 to 0.000050—a thousandfold.

Observation 2.5

So the ratio is what must be looked at. In absolute terms the small population fixes more often—not because of advantage, but because chance spreads things there.

3. The Ratio Runs from 1.2012 to 19.8013

Computation 3.1: Take the ratio to neutral at five population sizes.

N	Ns	Ratio to neutral
10	0.10	1.2012
25	0.25	1.5663
50	0.50	2.2901
100	1.00	4.0342
500	5.00	19.8013

Observation 3.2

At $N = 10$ it is only **1.2012**. Barely distinguishable from neutral—being advantageous is doing almost no work.

Observation 3.3

At $N = 500$ it is **19.8013**. With the same $s = 0.01$ —the advantage unchanged, only the population differing.

Observation 3.4

The ratio rises monotonically with N . $1.2012 < 4.0342 < 19.8013$, with the order holding at the intermediate points too—the smaller Ns , the closer to neutral.

Observation 3.5

At $Ns = 0.02$ the ratio is within five per cent of neutral. 1.0302—and in the limit it coincides.

Observation 3.6

And at $s = 0$ the probability is exactly $1/(2N)$. A boundary, and a check that the expression is set up correctly—not approximately but exactly.

Observation 3.7

A deleterious mutation can also fix in a small population. 0.8215 times neutral at $s = -0.01$ and $N = 10$ —against 7.738×10^{-172} at $N = 10000$, effectively zero. The same disadvantage, a different fate by size.

4. The Separator

The separator: does the advantage act, or does Ns .

Observation 4.1

If the advantage acts, s tells all. One per cent advantageous is one per cent advantageous anywhere—look only at the trait.

Observation 4.2

If Ns acts, nothing follows without the population. The same s giving 1.2 or 19.8—the trait alone does not settle it.

Observation 4.3

These are distinct roots by the criterion of Paper 300. Both are called “selection is acting”—one a property of a trait, one of a trait paired with a population.

Observation 4.4

The same shape as Paper 477. There, what acted was not how much was added but where—here it is not how large the advantage but which population. In both, a quantity alone does not settle the answer.

Observation 4.5

It overlaps Paper 446 as well. There, “how many times the floor” was undetermined without the recovery—here “an advantageous mutation” is undetermined without the population size.

Observation 4.6

So harmful mutations can accumulate in small populations. Selection cannot sort them out—this is the content of the nearly neutral theory, and it matters for populations near extinction.

Observation 4.7

To be honest—this rests on the diffusion approximation. Non-overlapping generations, constant size and weak selection are assumed, and real populations often violate them—19.8013 belongs to the approximation; that what acts is Ns and not s holds broadly across this family of models.

5. Roll Call and Closing

1. 0.019801 at $N = 10000$ (Computation 2.1): it saturates.
2. Close to $2s$ (Observation 2.3): once in about fifty.
3. Neutral from 0.050000 to 0.000050 (Computation 2.1): a thousandfold.
4. Small populations fix more in absolute terms (Observation 2.5): by chance.
5. 1.2012 at $N = 10$ (Computation 3.1): the core.
6. 19.8013 at $N = 500$ (Computation 3.1): the same s .
7. The ratio rises monotonically (Observation 3.4): Ns acts.
8. Exactly $1/(2N)$ at $s = 0$ (Observation 3.6): a boundary confirmed.
9. Deleterious can fix when small (Observation 3.7): 0.8215 times.

10. The separator is s or Ns (Observation 4.3): distinct roots by Paper 300.

Closing: An advantageous mutation is said to spread—advantage means being selected, apparently the very definition. But at first only one individual carries it: chance can remove it whatever its advantage, and how much chance matters is set by how small the population is. Advantage acts only in large populations—a mutation with $s = 0.01$ fixes with probability **0.019801** at $N = 1000$ and the same at $N = 10000$: once the population is large enough, its fate no longer depends on the size. That value is close to $2s$ —even a one per cent advantage spreads only once in about fifty tries. The neutral case meanwhile falls from 0.050000 to 0.000050, a thousandfold. So in absolute terms the small population fixes more often, not because of advantage but because chance spreads things there. The ratio runs from 1.2012 to 19.8013—at $N = 10$ ($Ns = 0.10$) it is only **1.2012**, barely distinguishable from neutral: being advantageous is doing almost no work. At $N = 500$ ($Ns = 5.00$) it is **19.8013**, with the same $s = 0.01$ —the advantage unchanged, only the population differing. The ratio rises monotonically with N , with $1.2012 < 4.0342 < 19.8013$ holding at the intermediate points too: the smaller Ns , the closer to neutral. At $Ns = 0.02$ it is 1.0302, within five per cent of neutral, and in the limit it coincides. And at $s = 0$ the probability is exactly $1/(2N)$ —a boundary and a check on the expression. A deleterious mutation can fix in a small population too: 0.8215 times neutral at $s = -0.01$ and $N = 10$, against 7.738×10^{-172} at $N = 10000$. The same disadvantage, a different fate by size. One thing separates them—does the advantage act, or does Ns . If the advantage acts, s tells all and one need look only at the trait. If Ns acts, nothing follows without the population, the same s giving 1.2 or 19.8. They are distinct roots by the criterion of Paper 300, one a property of a trait and one of a trait paired with a population. It is the same shape as Paper 477, where what acted was not how much was added but where: in both, a quantity alone does not settle the answer. It overlaps Paper 446 too, where “how many times the floor” was undetermined without the recovery: here “an advantageous mutation” is undetermined without the population size. So harmful mutations can accumulate in small populations, because selection cannot sort them out—this is the content of the nearly neutral theory, and it matters for populations near extinction. To be honest—this rests on the diffusion approximation, assuming non-overlapping generations, constant size and weak selection, which real populations often violate. 19.8013 belongs to the approximation; that what acts is Ns and not s holds broadly across this family of models.

References

1. Fisher, R. A. (1930). *The Genetical Theory of Natural Selection*. Clarendon Press.
2. Wright, S. (1931). Evolution in Mendelian Populations. *Genetics*.
3. Kimura, M. (1962). On the Probability of Fixation of Mutant Genes in a Population. *Genetics*.
4. Ohta, T. (1973). Slightly Deleterious Mutant Substitutions in Evolution. *Nature*.
5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

6. Yamagishi, Y. (2026). *The Theoretical Floor Rises When You Ask for More* [Paper 446]. Zenodo. 10.5281/zenodo.22732460
7. Yamagishi, Y. (2026). *While the Wrong Input Is Being Added, the Limit Is Somewhere Else* [Paper 477]. Zenodo. 10.5281/zenodo.22733686

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