

The Same Number Appears Seven Times and Splits into Two Groups and One

— $1/e = 0.367879441$ appears in seven places — three come from the stationarity condition $\ln x = -1$, three from the binomial-to-Poisson limit, and one from how a function was written down — at finite n two of them read **0.300000** against **0.398690** and agree exactly only in the limit — [Paper 479]

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— *From thinking to syncing.* —

Abstract

The same number is taken to mean the same reason. This paper shows that the seven appearances of $1/e$ in the corpus split **3 + 3 + 1**—the two groups are distinct roots of each other, and the last is a coincidence of convention. No new theorem or law is claimed. First, for the limiting functional $P(x) = x \ln(1/x)$ the maximiser and the maximum coincide exactly (Section 2). Second, but at finite n they differ: **0.300000** against **0.398690** at $n = 10$ (Section 2). Third, this is the core of the paper. The seven split into three from stationarity, three from a Poisson limit, and one from convention (Section 3). Fourth, the minimiser of x^x is $1/e$ but its minimum is $e^{-1/e} = 0.692200628$, a different number (Section 3). Fifth, the $\varphi(0) = 1/e$ of Paper 178 becomes **0.135335283** if the numerator in the exponent is changed to -2 (Section 3). Sixth, the separator is whether it comes from stationarity, from a limit, or from how it was written (Section 4). **Keywords:** $1/e$, secretary problem, derangement, Poisson limit, optimal stopping.

Scope of this paper (scope note): No new theorem or law is claimed—the secretary problem, derangements, the Poisson limit and the minimum of x^x are all standard. e is not mystified—it is not claimed that the seven unify into one; the two groups are distinct roots. No earlier paper is called wrong—Paper 388 judged by the criterion of varying the variable, and under that criterion its verdict reads as distinct. This paper treats it as an instance of criteria disagreeing. Numerical precision is limited—a stationary point is flat, so numerical search resolves it only to about 1.5×10^{-8} . History is not discussed—who found what first is not the subject. Relation to earlier papers: Paper 300 showed that common root and distinct root can be decided—this paper applies its five criteria to seven cases. Paper 405 showed that the evidence of a common root is a bijection—not a count of agreeing digits. Paper 388 judged the two secretary quantities to be distinct roots—this paper shows that the verdict depends on the first criterion. Paper 178 introduced a test function—its $\varphi(0)$ is the seventh. What is added is tabulating all seven, giving the finite- n discrepancy as 0.098690, showing the exact coincidence in the limit, giving the $3 + 3 + 1$ classification, showing the Paper 178 value to be a convention by changing the numerator, and placing the separator in three parts.

1. Notation and the Question

Setting: Count the places where $1/e = 0.367879441171442$ appears in the corpus. In the secretary problem one passes over the first k of n candidates and then takes the first who exceeds all seen so far. The probability of taking the best is

$$P_n(k) = \frac{k}{n} \sum_{j=k+1}^n \frac{1}{j-1}$$

and as $n \rightarrow \infty$ with $x = k/n$ this becomes

$$P(x) = x \ln \frac{1}{x}$$

Below, this P is called the limiting functional.

Observation 1.1

The same number is taken to mean the same reason. 0.367879 appears in four papers—suggesting a single root.

Observation 1.2

But Paper 300 settled that numerical agreement alone is not a common root. Five criteria decide it—and applying them should split the set.

Question: Into how many groups do the seven fall.

2. They Coincide Only in the Limit

Computation 2.1: Take the optimal pass rate and the probability of success at four values of n .

n	Pass rate k^*/n	Probability P	Difference
10	0.300000	0.398690	0.098690
100	0.370000	0.371043	0.001043
1000	0.368000	0.368196	0.000196
10000	0.367900	0.367911	0.000011

Observation 2.2

At $n = 10$ they differ by **0.098690**. 0.300000 against 0.398690—not the same number at all, but thirty per cent apart.

Observation 2.3

The discrepancy shrinks monotonically with n . $0.098690 > 0.001043 > 0.000196$ —reaching 0.000011 at $n = 10^4$.

Computation 2.4: Take the maximiser and the maximum of the limiting functional $P(x) = x \ln(1/x)$.

Maximiser $x^* = 0.367879441$, maximum $P(x^*) = 0.367879441$. They coincide exactly.

Observation 2.5

The coincidence is not accidental but a consequence of stationarity. $P'(x) = \ln(1/x) - 1 = 0$ gives $\ln(1/x^*) = 1$ —whence $P(x^*) = x^* \cdot 1 = x^*$. The expression forces the maximum to equal the maximiser.

Observation 2.6

So the verdict of Paper 388 depends on which criterion was used. Under “vary the variable” (Paper 31) the two move separately at finite n , hence distinct roots—under “the path” (Paper 279) they come from one stationarity condition, hence a common root. Paper 388 is not called wrong; this is an instance of criteria disagreeing.

Observation 2.7

Numerically the maximiser resolves no finer than 1.5×10^{-8} . Because the peak is flat—moving x by 3×10^{-9} changes P only below double precision. That is a limit of the arithmetic, not of the expression.

3. The Seven Split Three, Three and One

Computation 3.1: List the seven places where $1/e$ appears.

Source	Value	Root
Secretary pass rate k^*/n	0.367900 ($n = 10^4$)	stationarity
Secretary success P	0.367911 ($n = 10^4$)	stationarity
Minimiser of x^x	0.367879	stationarity
Derangements $D_n/n!$	0.367879441 ($n = 20$)	Poisson limit
$(1 - 1/n)^n$	0.367695425 ($n = 10^3$)	Poisson limit
Poisson $P(0)$	0.367879441171442	Poisson limit
$\varphi(0)$ of Paper 178	0.367879441171442	convention

Observation 3.2

The first group all come from $\ln x = -1$. The minimiser of x^x also satisfies $\ln x^* = -1$ —the secretary problem and x^x look entirely unrelated and solve the same stationarity condition.

Observation 3.3

But the minimum of x^x is not $1/e \cdot e^{-1/e} = \mathbf{0.692200628}$ —maximiser and maximum coincide only for $P(x) = x \ln(1/x)$, not for x^x . The same stationarity condition does not make the values agree.

Observation 3.4

The second group are all the binomial-to-Poisson limit. $(1 - 1/n)^n \rightarrow e^{-1}$ is the body of it, and Poisson $P(0)$ is that limit itself—derangements follow because the number of fixed points converges to Poisson (1).

Observation 3.5

The two groups are distinct roots of each other. One is a derivative set to zero, the other the value of a limit—both return $1/e$, but e enters by different routes.

Observation 3.6

The seventh is neither. In the test function $\varphi(x) = e^{-1/(1-x^2)}(1 + 0.9x)$ of Paper 178, $\varphi(0) = 1/e$ because the numerator in the exponent was written as -1 —write -2 and it becomes **0.135335283**.

Observation 3.7

So it is a convention, not a theorem. Changing the radius of support leaves $\varphi(0)$ untouched, but changing the numerator moves it—the value is set by the writer, not by nature.

Observation 3.8

So the census is $3 + 3 + 1$. Two groups sharing a root, and one pure coincidence—“the seven unify into one” cannot be written.

4. The Separator

The separator: does it come from stationarity, from a limit, or from how it was written.

Observation 4.1

From stationarity, and e appears in a derivative set to zero. $\ln x = -1$ — e enters as the base of the logarithm.

Observation 4.2

From a limit, and e appears in the operation $n \rightarrow \infty$. $(1 - 1/n)^n$ — e enters as the value of a limit.

Observation 4.3

From how it was written, and changing a coefficient removes it. A numerator of -2 and there is no $1/e$ — e was not there. The writer put it in.

Observation 4.4

This is a threefold split by the criteria of Paper 300. All are called “ $1/e$ appeared”—one a quantity of stationarity, one of a limit, one of convention.

Observation 4.5

In the language of Paper 405, the evidence of a common root is a bijection. Within the stationarity group a correspondence through $\ln x = -1$ can be written—across groups it cannot. Counting agreeing digits is no evidence.

Observation 4.6

And the one convention can be deleted with the other six surviving. Replace the φ of Paper 178 with any other bump function and nothing else in the corpus changes—a constant whose removal changes no conclusion is not a mechanism.

Observation 4.7

The same shape as Paper 478. There, what acted was Ns and not s —here $1/e$ appears not because of e but because of stationarity or a limit. In both, the reason does not lie with the conspicuous quantity.

Observation 4.8

To be honest—seven may not be all of them. The sweep used the literal string “0.367879” and the word for derangements—a passage writing only e^{-1} without digits would not be caught. An eighth is not excluded.

5. Roll Call and Closing

1. 0.300000 against 0.398690 at $n = 10$ (Computation 2.1): a gap of 0.098690.
2. A gap of 0.000011 at $n = 10^4$ (Computation 2.1): shrinking monotonically.
3. Maximiser equals maximum in the limit (Computation 2.4): 0.367879441.
4. A consequence of stationarity (Observation 2.5): $\ln(1/x^*) = 1$.
5. The criteria disagree (Observation 2.6): Paper 388 is not called wrong.
6. The seven split $3 + 3 + 1$ (Computation 3.1): the core.
7. The minimum of x^x is 0.692200628 (Observation 3.3): not the minimiser.
8. A numerator of -2 gives 0.135335283 (Observation 3.6): a convention.
9. Three separators (Observation 4.3): stationarity, limit, convention.
10. An eighth is not excluded (Observation 4.8): a limit of the sweep.

Closing: The same number is taken to mean the same reason—0.367879 in four papers suggests a single root. But Paper 300 settled that numerical agreement alone is not a common root, and applying its five criteria should split the set. They coincide only in the limit—the secretary pass rate and the probability of success differ by 0.098690 at $n = 10$, reading 0.300000 against 0.398690: not the same number at all, but thirty per cent apart. The

discrepancy shrinks monotonically to 0.000011 at $n = 10^4$. For the limiting functional $P(x) = x \ln(1/x)$ the maximiser 0.367879441 and the maximum 0.367879441 coincide exactly, and not by accident: $\ln(1/x^*) = 1$ forces $P(x^*) = x^*$. So the verdict of Paper 388 depends on the criterion—“vary the variable” gives distinct roots at finite n , “the path” gives a common root. Paper 388 is not called wrong; this is an instance of criteria disagreeing. Numerically the maximiser resolves no finer than 1.5×10^{-8} because the peak is flat, which is a limit of the arithmetic and not of the expression. The seven split three, three and one—the first group, the secretary pass rate, the probability of success and the minimiser of x^x , all come from $\ln x = -1$: entirely unrelated problems solving the same stationarity condition. But the minimum of x^x is $e^{-1/e} = \mathbf{0.692200628}$, not $1/e$: the same condition does not make the values agree. The second group, derangements, $(1 - 1/n)^n$ and Poisson $P(0)$, are all the binomial-to-Poisson limit. The two groups are distinct roots of each other: one a derivative set to zero, the other the value of a limit, with e entering by different routes. The seventh is neither. In Paper 178 the value $\varphi(0) = 1/e$ follows from writing -1 in the exponent; write -2 and it becomes **0.135335283**. It is a convention, not a theorem—the value is set by the writer, not by nature. So the census is **3 + 3 + 1**, and “the seven unify into one” cannot be written. Three things separate them—stationarity, a limit, or how it was written. From stationarity, e enters as the base of a logarithm in a derivative set to zero. From a limit, e enters as the value of $n \rightarrow \infty$. From how it was written, changing a coefficient removes it— e was not there, and the writer put it in. This is a threefold split by the criteria of Paper 300, one a quantity of stationarity, one of a limit and one of convention. In the language of Paper 405 the evidence is a bijection: within the stationarity group a correspondence through $\ln x = -1$ can be written, across groups it cannot, and counting agreeing digits is no evidence. And the one convention can be deleted with the other six surviving: replace the φ of Paper 178 with any other bump function and nothing else changes. A constant whose removal changes no conclusion is not a mechanism. To be honest—seven may not be all of them. The sweep used the literal string “0.367879” and the word for derangements, so a passage writing only e^{-1} without digits would not be caught. An eighth is not excluded.

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