

# Putting the Criteria Themselves to the Test

— Applying the five criteria for a common root to six test cases and building the  $5 \times 5$  agreement table, **9** of the **10** pairs disagree — the worst is “vary the variable” against “dimension” at **3** of **6** — and the dimension criterion answers “common root” in **5** of **6**, the bluntest of the five — [Paper 480]

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— *From thinking to syncing.* —

## Abstract

**Paper 300 showed that a common root can be decided by criteria. This paper shows that those five criteria do not share a root with each other—9 of the 10 pairs disagree on at least one test case. No new theorem or law is claimed. First, the five criteria were applied to six test cases and a  $5 \times 5$  agreement table built (Section 2). Second, the diagonal is 6 throughout and the table is symmetric (Section 2). Third, this is the core of the paper. 9 of the 10 pairs disagree; only one agrees on every case (Section 3). Fourth, the worst pair is “vary the variable” against “dimension” at 3 of 6 (Section 3). Fifth, the dimension criterion answers “common root” in 5 of 6, the bluntest of the five (Section 3). Sixth, the separator is which group of transformations the identity is asked under (Section 4). Keywords: common root and distinct root, criteria of judgement, agreement table, self-application of criteria, structural interpretation.**

Scope of this paper (scope note): No new theorem or law is claimed—the five criteria of Paper 300 already stand; this paper only applies them to one another.No claim that the five collapse into one—the table is built and the disagreements counted, nothing more. The restraint of Paper 300 is kept.The verdicts were placed by reading—the answers of the five criteria on the six cases were not produced by a machine but placed by me. The table itself is open to refutation.Six cases are not claimed to exhaust—more cases could change the count of disagreeing pairs, though only upwards.No earlier paper is called wrong—each criterion works correctly within its own terms.Relation to earlier papers: Paper 300 noted in observations 6.3–6.4 that asking whether the five criteria share a root itself needs a criterion, and stopped there—this paper carries it out.Paper 405 showed that the evidence of a common root is a bijection—a bijection is the strongest form of the correspondence criterion.Paper 479 split the seven appearances of  $1/e$  into  $3 + 3 + 1$ —those six splittings are the test cases here.Paper 411 showed that only the cross ratio survives a projective transformation—the purest example in the corpus of asking what is invariant under which group.What is added is applying the five criteria to one another, building the  $5 \times 5$  agreement table, counting the disagreeing pairs at 9, naming the worst pair and the bluntest criterion, and placing the separator at which group the identity is asked under.

## 1. Notation and the Question

Setting: Paper 300 laid down five criteria for deciding whether two quantities share a root.

| Symbol | Criterion                                            | Source    |
|--------|------------------------------------------------------|-----------|
| $C_1$  | Does the agreement survive varying the variable      | Paper 31  |
| $C_2$  | Can an identity-preserving correspondence be written | Paper 56  |
| $C_3$  | Do they travel the same path                         | Paper 279 |
| $C_4$  | Do they carry the same dimension                     | Paper 116 |
| $C_5$  | Do they have the same number of maps                 | Paper 153 |

Each criterion returns “common root” or “distinct root” for a given pair.To ask whether the criteria share a root, make them the objects of judgement: apply them to the same test cases and count how far the answers agree.

### Observation 1.1

Paper 300 noted this nesting and stopped. “Asking whether the five criteria share a root itself needs a criterion”—with the risk of an infinite regress.

### Observation 1.2

But one step can be taken without regress. Apply the five to one another and count the agreements—that counting needs no further criterion.

Question: How far do the five agree.

## 2. Applied to Six Test Cases

Computation 2.1: Take six pairs from the seven appearances of  $1/e$  treated in Paper 479, and record each criterion’s answer as 1 = common root, 0 = distinct root.

| Test case                                    | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|----------------------------------------------|-------|-------|-------|-------|-------|
| Secretary $k^*/n$ and $P$ (finite $n$ )      | 0     | 1     | 1     | 1     | 1     |
| Secretary $k^*/n$ and $P$ (limit)            | 1     | 1     | 1     | 1     | 1     |
| Derangements and $(1 - 1/n)^n$               | 1     | 1     | 1     | 1     | 1     |
| The stationarity group and the Poisson group | 0     | 0     | 0     | 1     | 0     |
| The bump of Paper 178 and the other six      | 0     | 0     | 0     | 1     | 0     |
| Minimiser and minimum of $x^x$               | 0     | 0     | 1     | 0     | 0     |

### Observation 2.2

This table was not produced by a machine. I read the six cases and placed the answers—so the table itself is open to refutation. Read them differently and the numbers change.

Computation 2.3: Count, for each pair of criteria, how many of the six cases they answer alike.

|                         | $C_1$ | $C_2$ | $C_3$ | $C_4$ | $C_5$ |
|-------------------------|-------|-------|-------|-------|-------|
| $C_1$ vary the variable | 6     | 5     | 4     | 3     | 5     |
| $C_2$ correspondence    | 5     | 6     | 5     | 4     | 6     |
| $C_3$ path              | 4     | 5     | 6     | 3     | 5     |
| $C_4$ dimension         | 3     | 4     | 3     | 6     | 4     |
| $C_5$ number of maps    | 5     | 6     | 5     | 4     | 6     |

**Observation 2.4**

The diagonal is 6 throughout. Every criterion agrees with itself on every case—obvious, and a check that the table is built correctly.

**Observation 2.5**

The table is symmetric. The agreement of  $C_i$  with  $C_j$  does not depend on the order—agreement is a relation and carries no direction.

### 3. Nine of the Ten Pairs Disagree

**Computation 3.1:** Count the pairs whose agreement falls short of 6.

| Pair           | Cases in disagreement |
|----------------|-----------------------|
| $C_1$ vs $C_4$ | <b>3</b>              |
| $C_3$ vs $C_4$ | <b>3</b>              |
| $C_1$ vs $C_3$ | <b>2</b>              |
| $C_2$ vs $C_4$ | <b>2</b>              |
| $C_4$ vs $C_5$ | <b>2</b>              |
| $C_1$ vs $C_2$ | <b>1</b>              |
| $C_1$ vs $C_5$ | <b>1</b>              |
| $C_2$ vs $C_3$ | <b>1</b>              |
| $C_3$ vs $C_5$ | <b>1</b>              |
| $C_2$ vs $C_5$ | <b>0</b>              |

### Observation 3.2

**9** of the 10 pairs disagree. Only  $C_2$  and  $C_5$  agree throughout—the five do not share a root with one another.

### Observation 3.3

The worst is “vary the variable” against “dimension”. Three of six—split on half the cases. Two criteria returning opposite answers to one question.

### Observation 3.4

“Path” against “dimension” splits on the same three. Both have dimension as the other party—dimension alone is the one that disagrees readily.

### Observation 3.5

The dimension criterion answers “common root” in 5 of 6. The most of the five—the bluntest. While dimensionless quantities are being compared, dimension separates nothing.

### Observation 3.6

$C_2$  (correspondence) and  $C_5$  (number of maps) agreed on all six. Every case—on these six, writing an identity-preserving correspondence and having the same number of maps cannot be told apart.

### Observation 3.7

That is what Paper 405 said. The evidence of a common root is a bijection—a bijection satisfies correspondence and count at once. In their strong form the two criteria overlap.

### Observation 3.8

But agreement on six cases is not evidence that the two share a root. That is the first criterion of Paper 300 itself—more cases might separate them. Numerical agreement is not enough.

## 4. The Separator

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The separator: which group of transformations the identity is asked under.

### Observation 4.1

All five criteria have the shape “move something and see what stays”. Vary the variable, map through a correspondence, follow the path, change the units, count the maps—only the thing being moved differs.

#### Observation 4.2

So the disagreements reduce to the choice of group.  $C_1$  moves a group of parameters,  $C_4$  a group of units—a different group leaves different things invariant.

#### Observation 4.3

This has the shape of classifying geometry by groups. As a geometry is fixed by what it leaves invariant, a common root is fixed by the group under which identity is asked—a common root is not an absolute relation but one relative to a group.

#### Observation 4.4

Paper 411 is its purest example. Under the projective group neither length nor ratio survives, and only the cross ratio remains—the wider the group, the fewer the invariants.

#### Observation 4.5

So attach the group when asking whether two things share a root. The two secretary quantities of Paper 479 are distinct roots over the family of finite  $n$  and a common root at the single point of the limit—both are right. The questions differ.

#### Observation 4.6

It is not claimed that the five collapse into one. The table was built and the disagreements counted—one step past where Paper 300 stopped.

#### Observation 4.7

To be honest—the verdicts on the test cases were placed by reading. No machine produced them—read them differently and the number 9 changes. But adding cases cannot reduce the count of disagreeing pairs: once a pair splits anywhere, it belongs to that count.

## 5. Roll Call and Closing

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1. Five criteria on six cases (Computation 2.1): verdicts placed by reading.
2. The diagonal is 6 (Observation 2.4): a check on the table.
3. The table is symmetric (Observation 2.5): agreement has no direction.
4. 9 of 10 pairs disagree (Computation 3.1): the core.
5. Worst is  $C_1$  vs  $C_4$  at 3 (Observation 3.3): split on half.
6.  $C_3$  vs  $C_4$  also 3 (Observation 3.4): dimension disagrees readily.
7. Dimension says “common” 5/6 (Observation 3.5): the bluntest.
8.  $C_2$  and  $C_5$  agree throughout (Observation 3.6): but that is no evidence.
9. The separator is which group (Observation 4.3): relative, not absolute.
10. No claim that five collapse (Observation 4.6): the restraint of Paper 300.

Closing: Paper 300 showed that a common root can be decided by criteria—and then noted in observations 6.3–6.4 that asking whether the five criteria share a root itself needs a criterion, stopping there for fear of an infinite regress. But one step can be taken without regress: apply the five to one another and count the agreements, which needs no further criterion. The table can be built—six pairs were taken from the seven appearances of  $1/e$  in Paper 479 and each criterion’s answer recorded as 1 or 0. The table was not produced by a machine but placed by me, so it is itself open to refutation. Counting agreements gives a diagonal of 6 throughout and a symmetric table: every criterion agrees with itself, and agreement is a relation without direction. And nine of the ten pairs disagree—only  $C_2$  (correspondence) and  $C_5$  (number of maps) agree throughout. The five do not share a root with one another. The worst is “vary the variable” against “dimension” at 3 of 6: split on half the cases, two criteria returning opposite answers to one question. “Path” against “dimension” splits on the same three, both having dimension as the other party—dimension alone disagrees readily. The dimension criterion answers “common root” in 5 of 6, the bluntest of the five: while dimensionless quantities are being compared, dimension separates nothing.  $C_2$  and  $C_5$  agreed on all six, which is what Paper 405 said—a bijection satisfies correspondence and count at once, so in their strong form the two overlap. But agreement on six cases is not evidence that they share a root. That is the first criterion of Paper 300 itself. One thing separates them—which group of transformations the identity is asked under. All five have the shape “move something and see what stays”: vary the variable, map through a correspondence, follow the path, change the units, count the maps. Only the thing being moved differs. So the disagreements reduce to the choice of group— $C_1$  moves parameters,  $C_4$  moves units, and a different group leaves different things invariant. This has the shape of classifying geometry by groups: as a geometry is fixed by what it leaves invariant, a common root is fixed by the group under which identity is asked. It is not an absolute relation but one relative to a group. Paper 411 is its purest example, where under the projective group neither length nor ratio survives and only the cross ratio remains—the wider the group, the fewer the invariants. So attach the group when asking. The two secretary quantities of Paper

479 are distinct roots over finite  $n$  and a common root at the limit; both are right, and the questions differ. It is not claimed that the five collapse into one: the table was built and the disagreements counted, one step past where Paper 300 stopped. To be honest—the verdicts were placed by reading, not by a machine, so reading them differently changes the number 9. But adding cases cannot reduce the count of disagreeing pairs: once a pair splits anywhere, it belongs to that count.

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