

The Bet That Maximises Expectation Goes Bust with Probability One

— At a win rate of **0.6** and even money, expectation peaks at **1.200000** when the whole bankroll is staked, yet fifty rounds of that bet ruin the player with probability **1.000000000** — growth peaks at **$f^* = 0.200000$** , and growth returns to zero not at **$2f^*$** but at **0.389391**, a factor of **1.946953** — “twice Kelly is zero” holds exactly only inside the quadratic approximation — [Paper 484]

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— *From thinking to syncing.* —

Abstract

A stake is thought to belong where expectation is highest. This paper shows that such a stake goes bust with probability **1**—expectation peaks at $f = 1$ and growth at $f^* = 0.200000$, a gap of **0.8000**. No new theorem or law is claimed. First, expectation peaks at **1.200000** with the whole bankroll staked—against **1.040000** at $f = 0.2$ (Section 2). Second, fifty rounds of that bet give a probability of ruin of **1.000000000** (Section 2). Third, growth peaks at **$f^* = 0.200000$** , where the closed form and numerical maximisation agree (Section 3). Fourth, this is the core of the paper. Growth returns to zero not at **$2f^*$** but at **0.389391**, a factor of **1.946953** (Section 3). Fifth, inside the quadratic approximation it is exactly **0.400000**—“twice Kelly is zero” is a property of that approximation (Section 3). Sixth, the separator is whether the average runs across parallel worlds or along time (Section 4). **Keywords:** Kelly criterion, expected value, probability of ruin, logarithmic utility, ergodicity.

Scope of this paper (scope note): No new theorem or law is claimed—the Kelly criterion, logarithmic utility and the calculation of ruin probabilities are all standard. No strategy is recommended—which stake to choose is not the subject; only that two objective functions peak in different places, shown with numbers. Real markets are excluded—transaction costs, correlation, estimation error in the win rate and the indivisibility of capital are not included. The win rate and the payout are taken as known. Utility theory is not entered—whether logarithmic utility is the right preference is not discussed. The logarithm here is not a preference but the tool that turns products into sums. Ruin is defined once—it means only that staking the whole bankroll and losing once leaves zero. Relation to earlier papers: Paper 292 showed that only the logarithm adds—this paper places the maximum of that sum. Paper 298 wrote in its scope note that it would not enter the Kelly criterion, utility functions or the minimisation of ruin—this paper takes the seat that line left open. Paper 383 showed that a premium is not the expected loss—there too, expectation alone did not settle it. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is setting the expectation maximum of 1.200000 beside the growth maximum at $f^* = 0.200000$, giving the fifty-round ruin probability as 1.000000000, confirming that the closed form and numerical maximisation agree, measuring the return to zero at 0.389391, a factor of 1.946953, and showing that the quadratic approximation puts it at exactly twice.

1. Notation and the Question

Setting: Take a win rate $p = 0.60$, a loss probability $q = 1 - p = 0.40$ and even money ($b = 1.0$). Staking a fraction f of the bankroll each round multiplies it by

$$1 + bf \text{ (with probability } p), \quad 1 - f \text{ (with probability } q)$$

Set the per-round expectation beside the growth rate under repetition. The growth rate is

$$g(f) = p \ln(1 + bf) + q \ln(1 - f)$$

which writes Paper 292's "only the logarithm adds" as a function of the stake.

Observation 1.1

A stake is thought to belong where expectation is highest. If the expectation is positive, more is better—the picture that falls straight out of the per-round arithmetic.

Observation 1.2

But a bankroll moves by multiplication. Once it reaches zero it does not return—an additive expectation does not see that.

Question: Where, and by how much, do the two objectives part.

2. Expectation Peaks at the Whole Bankroll

Computation 2.1: Evaluate the per-round expected multiplier.

Stake fraction f	Expected multiplier
0.2	1.040000
1.0	1.200000

Observation 2.2

Expectation peaks at $f = 1$. $1.200000 > 1.040000$ —the expected multiplier is linear in f , so a positive slope puts the maximum at the endpoint.

Observation 2.3

Fifty rounds of that bet ruin the player with probability $1 - p^{50}$ —one loss takes the bankroll to zero, and nothing staked afterwards brings it back.

Observation 2.4

A positive expectation and a growing bankroll are different things. An expected multiplier of 1.2—what grows is an average taken across parallel worlds, not one traced along a single world.

Observation 2.5

The cross-sectional average is positive because a rare large win carries it. Winning all fifty has probability p^{50} —that single path holds up the average, and the rest are all zero.

3. The Growth Maximum and the Return to Zero

Computation 3.1: Tabulate $g(f)$ by stake.

Stake fraction f	Growth rate $g(f)$
0.0500	+0.008757
0.1000	+0.015042
0.2000	+0.020136
0.3000	+0.014749
0.4000	−0.002447
0.5000	−0.033980

Observation 3.2

The maximum is $f^* = 0.200000$. The closed form $f^* = p - q/b$ agrees with numerical maximisation—0.200000 against 0.200000, an independent check.

Observation 3.3

At $2f^* = 0.4$ the growth rate is negative. −0.002447—overbet, and the bankroll shrinks even while the expectation stays positive.

Computation 3.4: Solving numerically for the return to zero gives $f = 0.389391$, a factor of **1.946953** times f^* . Recomputing g there returns −0.000000000.

Observation 3.5

It is not exactly twice. $1.946953 < 2$ —growth has already crossed zero 5.3 per cent short of it.

Observation 3.6

Inside the quadratic approximation it is exactly twice. The approximate solution is 0.400000—truncating the logarithms at second order makes g a parabola with vertex at f^* and roots at 0 and $2f^*$.

Observation 3.7

So “twice Kelly is zero” is a property of the parabola, not of the logarithm. The logarithm gives 1.946953—the third and higher terms pull the zero crossing nearer.

4. The Separator

The separator: does the average run across parallel worlds, or along time in one.

Observation 4.1

Across, and the answer is $f = 1$. An expected multiplier of 1.200000—held up by a rare single path, with no guarantee of drawing it.

Observation 4.2

Along, and the answer is $f^* = 0.200000$. A growth rate of 0.020136—what accrues while tracing one world.

Observation 4.3

The two answers differ by 0.8000. 1.0 against 0.2000—distinct roots by the criterion of Paper 300, differing in objective and in the kind of average.

Observation 4.4

The gap comes from turning products into sums. As Paper 292 showed, only the logarithm adds—the expectation of a multiplier is a sum, the growth of a bankroll a product, and the logarithm is what joins them.

Observation 4.5

Paper 298 stood on the same edge and stopped. Its scope note reads that it enters neither the Kelly criterion, nor utility functions, nor the minimisation of ruin—this paper sits in the seat that line kept open.

Observation 4.6

This is one of the separators of Paper 481. When “the expectation is positive so the bet is sound” turns out wrong—whether the win rate was misjudged or the average was the wrong kind sends the repair elsewhere.

Observation 4.7

To be honest—the ruin here is ruin by definition. Stake everything and one loss leaves zero—allow an indivisible minimum unit, or capital that survives a loss, and the probability is no longer 1. This is a property forced by the endpoint $f = 1$, and $f < 1$ needs a different argument.

5. Roll Call and Closing

1. Expectation peaks at 1.200000 (Computation 2.1): at $f = 1$.
2. Ruin probability 1.000000000 in fifty (Observation 2.3): one loss ends it.
3. One path of probability p^{50} holds the average (Observation 2.5): the rest are zero.
4. Growth peaks at $f^* = 0.200000$ (Observation 3.2): matching the closed form.
5. Negative growth at $2f^*$ (Observation 3.3): -0.002447 .
6. Zero returns at 0.389391 (Computation 3.4): the core.
7. A factor of 1.946953, not 2 (Observation 3.5): 5.3 per cent short.
8. The quadratic gives 0.400000 (Observation 3.6): a property of the parabola.

9. The separator is across or along (Observation 4.3): distinct roots by Paper 300.

10. Ruin by definition (Observation 4.7): forced by the endpoint.

Closing: A stake is thought to belong where expectation is highest—if the expectation is positive then more is better, the picture that falls straight out of the per-round arithmetic. But a bankroll moves by multiplication and once it reaches zero it does not return, and an additive expectation does not see that. Expectation peaks at the whole bankroll—at a win rate of 0.60 and even money, the per-round expected multiplier is 1.040000 at $f = 0.2$ and **1.200000** at $f = 1.0$; being linear in f , it peaks at the endpoint. Fifty rounds of that bet give $1 - p^{50} = \mathbf{1.000000000}$ for ruin, since one loss takes the bankroll to zero and nothing staked afterwards brings it back. A positive expectation and a growing bankroll are different things: what grows is an average across parallel worlds, not one traced along a single world. The cross-sectional average is positive because the rare path that wins all fifty, of probability p^{50} , holds it up while the rest are all zero. The growth maximum and the return to zero— $g(f) = p \ln(1 + bf) + q \ln(1 - f)$ peaks at $f^* = \mathbf{0.200000}$ with $g = 0.020136$, and the closed form $f^* = p - q/b$ agrees with numerical maximisation, 0.200000 against 0.200000. At $2f^* = 0.4$ growth is already -0.002447 : overbet, and the bankroll shrinks even while the expectation stays positive. Solving for the return to zero gives **0.389391**, a factor of **1.946953**, and recomputing g there returns -0.000000000 . It is not exactly twice—growth crosses zero 5.3 per cent short of it. Inside the quadratic approximation it is exactly 0.400000, since truncating the logarithms at second order makes g a parabola with vertex at f^* and roots at 0 and $2f^*$. So “twice Kelly is zero” is a property of the parabola, not of the logarithm; the third and higher terms pull the crossing nearer. One thing separates them—does the average run across parallel worlds, or along time in one. Across, and the answer is $f = 1$ with an expected multiplier of 1.200000, held up by a rare single path with no guarantee of drawing it. Along, and the answer is $f^* = 0.200000$ with a growth rate of 0.020136, what accrues while tracing one world. The two differ by 0.8000, and are distinct roots by the criterion of Paper 300, differing in objective and in the kind of average. The gap comes from turning products into sums: as Paper 292 showed, only the logarithm adds, the expectation of a multiplier being a sum and the growth of a bankroll a product. Paper 298 stood on the same edge and stopped, its scope note reading that it enters neither the Kelly criterion, nor utility functions, nor the minimisation of ruin; this paper sits in the seat that line kept open. This is one of the separators of Paper 481: when “the expectation is positive so the bet is sound” turns out wrong, whether the win rate was misjudged or the average was the wrong kind sends the repair elsewhere. To be honest—the ruin here is ruin by definition: stake everything and one loss leaves zero. Allow an indivisible minimum unit, or capital that survives a loss, and the probability is no longer 1. It is a property forced by the endpoint $f = 1$, and $f < 1$ needs a different argument.

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