

The “Exactly Half” in Three Fields Is the Vertex of a Parabola, Not Physics

— An efficiency of **0.500000** at the matched load, half the jam density at **75.000000**, half the demand intercept at **50.000000** — nondimensionalised, all three fall on **$1/2$** , namely [**0.499999994, 0.5, 0.5**] — put Betz beside them as a control and the optimum sits at **0.333333330** instead, because its objective is cubic — what fixed the half was not the field but the degree — [Paper 486]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

Three fields whose optima land on exactly half are taken to do so for reasons of their own. This paper shows that the three are one vertex of one parabola—make the objective cubic and the half becomes **$1/3$** . No new theorem or law is claimed. First, at the matched load $R^* = 1.000000$, equal to the internal resistance, the efficiency is exactly **0.500000** (Section 2). Second, in traffic flow $k^* = 75.000000$ is half the jam density and $v^* = 50.000000$ half the free-flow speed (Section 2). Third, under monopoly $p^* = 50.000000$ is half the demand intercept (Section 2). Fourth, this is the core of the paper. Nondimensionalised, the three give [**0.499999994, 0.5, 0.5**] and fall on the same **$1/2$** (Section 3). Fifth, the Betz control gives an optimum of **0.333333330** and a maximum of **0.592592593**, that is **$16/27$** —because its objective is cubic (Section 3). Sixth, the separator is the degree of the objective, not the field (Section 4). **Keywords:** vertex of a parabola, matched load, Betz limit, monopoly pricing, traffic capacity.

Scope of this paper (scope note): No new theorem or law is claimed—the matched load, the Greenshields traffic model, monopoly pricing and the Betz limit are all standard. The models are not assessed—whether traffic obeys a linear speed-density relation, or demand is a straight line, is not the subject. The models are taken as given and only the location of the optimum is examined. No general theory of optimisation—convexity, first-order conditions and the multivariate case are not treated; only the vertex in one variable. The three are not claimed to be one phenomenon—no shared mechanism between circuits, traffic and markets is asserted. What is shared is the shape of the objective, not the object. Betz is not re-derived—the theory of wind turbines is not the subject; it appears only as a control of different degree. Relation to earlier papers: Paper 357 showed that maximum power and maximum efficiency are not the same point—this paper takes only that maximum-power point and sets it beside other fields. Paper 479 found three roots beneath one number—this paper runs the other way, finding one root beneath three settings. Paper 300 showed that a shared root can be decided—this is a case on the shared side. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is placing the three optima in one table, nondimensionalising them to $[0.499999994, 0.5, 0.5]$, giving the Betz control as 0.333333330 and 16/27, attributing the gap of 0.166667 to degree, and placing the separator at degree rather than field.

1. Notation and the Question

Setting: Write the optimum of three settings in one variable each.

$$P(R) = \frac{E^2 R}{(r + R)^2}, \quad q(k) = v_f k \left(1 - \frac{k}{k_j}\right), \quad \Pi(Q) = Q(a - bQ) - cQ$$

From the left: the power a source of internal resistance r delivers to a load R ; the traffic flow at density k ; the monopolist's profit at output Q . Each has a “half” at its maximum.

Observation 1.1

The three halves are taken to follow from the physics of each. A circuit, a road, a market—each arriving at a half for its own reason, runs the picture.

Observation 1.2

But in all three the maximised quantity is quadratic in the variable. $q(k)$ is quadratic in k , $\Pi(Q)$ in Q —and the vertex of a parabola sits midway between its roots.

Question: Is the half fixed by the field, or by the degree.

2. Three Optima

Computation 2.1: Solve the three numerically.

Setting	Optimum	Value there
Matched load ($r = 1$)	$R^* = 1.000000$	efficiency 0.500000
Traffic flow	$k^* = 75.000000$	$q_{\max} = 3750.000000$
Monopoly	$Q^* = 25.000000$	$p^* = 50.000000$

Observation 2.2

At the matched load the optimum equals the internal resistance. $R^* = 1.000000$ with $r = 1$ —the voltage then divides half at the source and half at the load, and the efficiency is exactly 0.500000.

Observation 2.3

In traffic the optimum is half the jam density. 75.000000 against $k_j/2 = 75.0$ —and the speed there is half the free-flow value at 50.000000 , two halves standing at once.

Observation 2.4

Under monopoly the price is half the demand intercept. 50.000000 against 50.0 —with zero marginal cost, and a profit of 1250.000000 .

Observation 2.5

Each half is half of something different. A resistance, a density, a price—different units and different objects, not comparable as they stand.

3. Nondimensionalised, They Become One

Computation 3.1: Divide each variable by its own upper bound.

Setting	Nondimensionalised optimum
Matched load	0.499999994
Traffic flow	0.500000000
Monopoly	0.500000000

Observation 3.2

All three fall on $1/2.[0.499999994, 0.5, 0.5]$ —only the matched load differs in the last digits, from the step size of the numerical search; the closed form is $1/2$.

Computation 3.3: As a control, solve the Betz problem the same way. For an extraction fraction a the power coefficient is $C_p = 4a(1 - a)^2$, cubic in a .

Quantity	Value
Optimal a	0.333333330
C_p there	0.592592593 (16/27)

Observation 3.4

Betz does not land on $1/2.0.333333330$ —that is $1/3$, a different place from the three halves.

Observation 3.5

The maximum matches the known value. 0.592592593 against $16/27 = 0.592592593$ —an independent check that the control itself is computed correctly.

Observation 3.6

The only difference is degree. Quadratic gives $1/2$, cubic gives $1/3$, a gap of 0.166667 —because the vertex of $x(1 - x)^{n-1}$ sits at $1/n$, with the field playing no part.

Observation 3.7

So the half is a shape, not a physics. One shape placed in three fields—what agreed was not the object but the degree of the objective.

4. The Separator

The separator: is the optimum fixed by the field, or by the degree of the objective.

Observation 4.1

By the field, and the values should differ between fields. A circuit, a road, a market—with no reason for the same $1/2$ three times.

Observation 4.2

By the degree, and changing the degree should move it. Cubic gives $1/3$ —and it moved.

Observation 4.3

Since it moved, the degree was what fixed it. Three at 0.5, one at 0.333333330—sorting by field does not cut; sorting by degree does.

Observation 4.4

Paper 479 runs the other way. There, three roots beneath one number—here, one root beneath three settings. The same test, used in reverse.

Observation 4.5

Paper 357 showed that maximum power and maximum efficiency are not the same point. Only the maximum-power point is taken here—and the 0.500000 appearing there is the efficiency, not the quantity being maximised.

Observation 4.6

This is one of the separators of Paper 481. When “the optimum is at half” turns out wrong—whether the model failed or the objective was not quadratic sends the repair elsewhere.

Observation 4.7

To be honest—why the three are quadratic is not explained. What is shown reaches as far as: quadratic puts it at $1/2$, cubic at $1/3$ —why traffic flow and monopoly can both be written quadratically is a question about how those models were chosen, and lies outside this paper.

5. Roll Call and Closing

1. Efficiency 0.500000 at the matched load (Computation 2.1): at $R^* = 1.000000$.
2. $k^* = 75.000000$ in traffic (Observation 2.3): speed half too, at 50.000000.
3. $p^* = 50.000000$ under monopoly (Observation 2.4): half the intercept.
4. Each half is half of something different (Observation 2.5): not comparable as they stand.
5. $[0.499999994, 0.5, 0.5]$ (Computation 3.1): the core.
6. Betz at 0.333333330 (Observation 3.4): not at $1/2$.
7. C_p matches $16/27$ (Observation 3.5): an independent check.
8. The gap 0.166667 is degree (Observation 3.6): $x(1-x)^{n-1}$ peaks at $1/n$.
9. The separator is degree, not field (Observation 4.3): sorting by degree cuts.
10. Why quadratic is not explained (Observation 4.7): a question about model choice.

Closing: The three halves are taken to follow from the physics of each—a circuit, a road and a market each arriving at a half for its own reason. But in all three the maximised quantity is quadratic in the variable, and the vertex of a parabola sits midway between its roots. Three optima—at the matched load the optimum equals the internal

resistance, $R^* = 1.000000$, where the voltage divides half at the source and half at the load and the efficiency is exactly **0.500000**. In traffic the optimum is half the jam density at **75.000000**, with the speed there half the free-flow value at 50.000000. Under monopoly the price is half the demand intercept at **50.000000**, with zero marginal cost and a profit of 1250.000000. Each half is half of something different—a resistance, a density, a price—so they are not comparable as they stand. Nondimensionalised, they become one—dividing each variable by its own upper bound gives **[0.499999994, 0.5, 0.5]**, all three on $1/2$, with only the matched load differing in the last digits from the step size of the search. As a control, Betz gives $C_p = 4a(1 - a)^2$, cubic in a , with an optimum at **0.333333330** and $C_p = \mathbf{0.592592593}$, that is $16/27$. Betz lands on $1/3$, not $1/2$. That the maximum matches the known $16/27$ is an independent check on the control itself. The only difference is degree: quadratic gives $1/2$, cubic gives $1/3$, a gap of 0.166667, because the vertex of $x(1 - x)^{n-1}$ sits at $1/n$ and the field plays no part. So the half is a shape, not a physics; what agreed was not the object but the degree of the objective. One thing separates them—is the optimum fixed by the field, or by the degree of the objective. By the field, the values should differ between fields, with no reason for the same $1/2$ three times. By the degree, changing the degree should move it—and cubic gives $1/3$, so it moved. Since it moved, the degree was what fixed it: sorting by field does not cut, sorting by degree does. Paper 479 runs the other way, finding three roots beneath one number where this finds one root beneath three settings—the same test, used in reverse. Paper 357 showed that maximum power and maximum efficiency are not the same point; only that maximum-power point is taken here, and the 0.500000 appearing there is the efficiency, not the quantity being maximised. This is one of the separators of Paper 481: when “the optimum is at half” turns out wrong, whether the model failed or the objective was not quadratic sends the repair elsewhere. To be honest—why the three are quadratic is not explained. What is shown reaches as far as: quadratic puts it at $1/2$, cubic at $1/3$. Why traffic flow and monopoly can both be written quadratically is a question about how those models were chosen, and lies outside this paper.

References

1. Greenshields, B. D. (1935). A Study of Traffic Capacity. *Proceedings of the Highway Research Board*.
2. Betz, A. (1920). Das Maximum der theoretisch möglichen Ausnützung des Windes durch Windmotoren. *Zeitschrift für das gesamte Turbinenwesen*.
3. Varian, H. R. (1992). *Microeconomic Analysis* (3rd ed.). W. W. Norton.
4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
5. Yamagishi, Y. (2026). *Maximum Power and Maximum Efficiency Are Not the Same Point* [Paper 357]. Zenodo. 10.5281/zenodo.22697180
6. Yamagishi, Y. (2026). *The Same Number Appears Seven Times and Splits into Two Groups and One* [Paper 479]. Zenodo. 10.5281/zenodo.22733844

7. Yamagishi, Y. (2026). *A Broken Law Can Be Blamed on Any of the Assumptions* [Paper 481]. Zenodo. 10.5281/zenodo.22733927

— *From thinking to syncing.* —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)