

2.5PN Is the First Dissipative Order Because Mass Conservation Kills $n = 1$

— 3rd ed.: Where the Even-Order Rule of the Yamagishi Higher-Order Delay Interpolation Breaks — [Paper 49]

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— *From thinking to syncing.* —

ABSTRACT

This paper set out that the c^{-2} hierarchy of the post-Newtonian (PN) expansion descends from the Taylor expansion in the retarded time $t_{\text{ret}} = t - r/c$, and showed that **the PN expansion branches into a conservative series (integer orders) and a dissipative one (half-integer orders). The second edition recovered that branching correctly and retracted the first edition's error that “odd orders vanish by parity” — an important correction as physics, and it is not altered here. The two other retractions of the second edition — the correspondence of c^{-5} with Paper 34's $\frac{c^5}{5}$ (“5D bulk capacity”), and the identification of 5PN accuracy with a sub-pixel limit of 10^{-10} — also stand and are not reinstated. All numbers were re-verified and all four agree: Mercury's perihelion advance $42.990''$ per century (observed $42.98'' \pm 0.04''$), the chirp mass of a $1.4 + 1.4M_{\odot}$ binary $1.2188M_{\odot}$, the cycle count from 10 Hz $N_{\text{cyc}} = 1.603 \times 10^4$, the required accuracy $\delta_{\text{req}} = 1/(2\pi N_{\text{cyc}}) = 9.93 \times 10^{-6}$, and the 5PN relative term $(0.3)^{10} = 5.905 \times 10^{-6}$. What the third edition adds is an answer to one question the second edition left open: why does the dissipative series start at 2.5PN? First, the second edition's reason is correct but nearly definitional. It said “in a conservative system the Lagrangian is invariant under $t \rightarrow -t$, so terms with odd time derivatives cannot contribute”. True — but that amounts to saying a conservative system has no dissipative term, and it does not explain why the first dissipative term is 2.5PN rather than 0.5PN or 1.5PN. Second, the answer lies in the $n = 1$ term of the near-zone expansion. Expanding the retarded potential,**

$$V = G \int \frac{\rho(t - r'/c)}{r'} d^3x' = G \sum_n \frac{(-1)^n}{n! c^n} \frac{\partial^n}{\partial t^n} \int \rho r'^{n-1} d^3x',$$

the first odd term, $n = 1$, is $-\frac{G}{c} \dot{M}$ and vanishes by conservation of mass. The $n = 2$ term is even and contributes at 1PN. Hence the first surviving odd term is $n = 3$, the third time derivative of the quadrupole moment — the Burke--Thorne radiation-reaction potential. A c^{-3} in the potential becomes c^{-5} in the energy loss rate: that is 2.5PN. Third, this gives a reason for the agreement with the quadrupole formula. The second edition correctly said that “the c^{-5} of the quadrupole formula and the entry of radiation reaction at 2.5PN necessarily agree as a matter of energy balance”. The third edition names the source of that agreement: both come from the same $n = 3$ term, the third derivative of the quadrupole. The two c^{-5} share a root. Fourth, the Layer-3 vocabulary is removed from the body. The second edition itself limited “latency correction”, “frame interpolation”, “discharge” and the like as “vocabulary for a reading, not the content of a theorem”. That limitation was right, and this edition carries it out by removing the vocabulary from the body. Scope: structural interpretation and self-audit of established standard facts (Layer 2). No prediction of general relativity is altered and nothing is new to physics — what is new here is internal to this series.

Standing of this paper (discipline): the physics used here is **entirely standard** — the near-zone expansion of the retarded potential, Burke--Thorne (1970) radiation reaction, the PN theory of Blanchet (2014) and Maggiore (2007), Einstein's (1915) perihelion result, and the phase condition of matched filtering. **Not one new theorem is proved.** The third edition's contribution is **answering “why 2.5”**: (i) noting that the second edition's time-reversal reason is **nearly definitional**, (ii) showing that **$n = 1$ vanishes by mass conservation**, (iii) identifying the first surviving odd term as **the third derivative of the quadrupole**, (iv) stating that the two **c^{-5} share a root**, and (v) removing the Layer-3 vocabulary from the body. **The most important restraints: no prediction of general relativity is altered. The Mercury agreement is not treated as support for this series' reading** (the second edition's note is kept — the same number is obtained without that reading). **c^{-5} is not matched to a “5D bulk capacity”** ($\frac{c^5}{5}$ is retracted; Papers 34, 42). **The destination of the outflowing energy is not a higher dimension** (it is a gravitational wave propagating in ordinary three-dimensional space). **5PN accuracy is not identified with a sub-pixel limit** (retracted in the second edition). **The phase-coherence condition is not read as a sharp threshold** (it is an order-of-magnitude estimate). Layer-3 lenses are quarantined under a non-scientific label.

1. Introduction: The Second Edition Recovered the Branching; What Remained Is “Why 2.5”

The first edition stated that “**odd-order terms vanish by parity**” and thereby **dropped radiation reaction — the very cause of a binary's inspiral — out of the series**. The second edition retracted this and recovered the branching.

Series	Orders	Powers of $1/c$	Character
Conservative	1PN, 2PN, 3PN, 4PN	$c^{-2}, c^{-4}, c^{-6}, c^{-8}$	time-reversal symmetric
Dissipative	2.5PN, 3.5PN, 4.5PN	c^{-5}, c^{-7}, c^{-9}	asymmetric; radiation reaction

That recovery is correct and is not altered here. The second edition's explicit statement that “**this is not new to physics but an internal correction**” is also kept.

But one question was left unanswered.

Why does the dissipative series start at 2.5PN — and not at 0.5PN or 1.5PN?

2. The Second Edition's Reason Is Correct, but Nearly Definitional

Established Fact 2.1 (The second edition's explanation)

“In a conservative system the Lagrangian is invariant under time reversal $t \rightarrow -t$, so terms containing odd time derivatives cannot contribute to the equations of motion.”

Observation 2.1 (Correct, but it does not give the position)

The explanation is correct. A time-reversal-symmetric system does not admit a term that breaks time reversal. But that amounts to saying that a conservative system has no dissipative term, which is roughly what “conservative” means. And it leaves out the important part — where the dissipation begins. Since the expansion advances in units of $\frac{1}{c} \frac{\partial}{\partial t}$, an odd term could in principle appear at $n = 1$, that is at 0.5PN. It does not. Why?

3. The Answer Is at $n = 1$: Mass Conservation Kills It

Theorem 3.1 (The near-zone expansion and the first survivor)

Expanding the retarded potential in the near zone,

$$V = G \int \frac{\rho(t - r'/c)}{r'} d^3x' = G \sum_{n \geq 0} \frac{(-1)^n}{n! c^n} \frac{\partial^n}{\partial t^n} \int \rho r'^{n-1} d^3x',$$

the terms fare as follows.

n	Term	Fate
0	Newtonian potential	survives (even)
1	$-\frac{G}{c} \dot{M}$	zero (mass conservation)
2	$+\frac{G}{2c^2} \frac{\partial^2}{\partial t^2} \int \rho r' d^3x'$	survives (even; 1PN)
3	$-\frac{G}{6c^3} \frac{\partial^3}{\partial t^3} \int \rho r'^2 d^3x'$	first surviving odd term

Observation 3.1 ($n = 3$ is the quadrupole's third derivative)

$\int \rho r'^2 d^3x'$ is nothing other than the quadrupole moment. So the $n = 3$ term is the third time derivative of the quadrupole, which is the Burke--Thorne radiation-reaction potential (Burke--Thorne 1970). The c^{-3} in the potential becomes c^{-5} in the energy loss rate: the reaction force is $F \sim c^{-3} \ddot{Q}$, its power is $P = F \cdot v$, and balancing against the quadrupole formula fixes c^{-5} . 2.5PN is the first dissipative order because $n = 1$ is killed by mass conservation. Had mass not been conserved, dissipation would have begun at 0.5PN. Time-reversal symmetry says only that odd terms are absent from a conservative system; where the prohibition lifts is decided by a conservation law.

4. The Two c^{-5} Share a Root

The second edition said, correctly, that “the c^{-5} of the quadrupole formula and the entry of radiation reaction at 2.5 PN necessarily agree as a matter of energy balance”.

Observation 4.1 (The source of the agreement)

The third edition names its source: both come from the same $n = 3$ term.

Quadrupole formula $\frac{G}{5c^5} \langle \ddot{Q}\ddot{Q} \rangle$	far field: the square of the quadrupole's third derivative
2.5PN reaction force	near zone, $n = 3$: the quadrupole's third derivative

The two are not “coincidentally the same order”; they are two appearances of one quantity. One counts, from outside, the energy the system loses; the other counts, from inside, the force the system feels. This is the reverse of a judgement this series has often passed — Paper 86 warned that “a common ancestor is not a common root” and Paper 100 that “a resonance is a signature, not a proof”. But here the common source can actually be pointed at: one place, the $n = 3$ term. The discipline of distinguishing distinct roots from a common root does not forbid a common root. It only demands that they be distinguished.

5. The Numbers, Re-verified (All as in the Second Edition)

Established Fact 5.1 (All four agree)		
Quantity	Second edition	Re-verified here
Mercury's perihelion advance	42.99"/century	42.990" (obs. 42.98" ± 0.04")
Chirp mass (1.4 + 1.4M _⊙)	—	1.2188 M _⊙
N _{cyc} (from 10 Hz)	1.6 × 10 ⁴	1.603 × 10 ⁴
δ _{req} = 1/(2πN _{cyc})	1.0 × 10 ⁻⁵	9.93 × 10 ⁻⁶
5PN relative term (0.3) ¹⁰	5.9 × 10 ⁻⁶	5.905 × 10 ⁻⁶

Every number in the second edition is correct. Mercury's $\Delta\phi = \frac{6\pi GM}{c^2 a(1-e^2)}$ and the phase-coherence condition $\delta_{\text{req}} \sim 1/(2\pi N_{\text{cyc}})$ are both unchanged.

Observation 5.1 (But “exactly at the threshold” is an order-of-magnitude statement)

The second edition wrote that “the two are of the same order, and the 5PN correction sits exactly at the threshold of the required accuracy”. The agreement of orders is confirmed here (9.93×10^{-6} against 5.905×10^{-6}). But “exactly at the threshold” overstates it. δ_{req} is an order-of-magnitude estimate from the rule of thumb that the accumulated phase error should stay under one radian, and $(v/c)^{10}$ is the size of the relative term at the worst point, just before merger. Since the phase accumulates mostly at low frequency, the actual phase contribution of a high PN order is smaller than this estimate. This edition therefore keeps “the same order of magnitude” and withdraws “exactly at the threshold”. The second edition's observation that this agrees with the actual methodology — analytic waveforms to 3.5PN--4PN, matched to numerical relativity near merger — remains correct.

6. On the Mercury Agreement (The Second Edition's Note Is Kept)

Established Fact 6.1 (What the second edition wrote correctly)

“This agreement is a standard result of general relativity and not support for this paper’s picture.” The formula of Proposition 6.1 was derived by Einstein in 1915, and this paper merely recomputes it. **The same number is obtained without adopting that reading.**

This note is the most disciplined sentence in the paper. It is kept unchanged in the third edition, together with its limitation of the result to **the role of an anchor** — confirming that the first step of the c^{-2} hierarchy corresponds to an established observation.

7. Removing the Layer-3 Vocabulary from the Body

The second edition stated at the outset that **“latency correction”, “frame interpolation”, “discharge” and the like are vocabulary this series supplies for a reading, and not the content of a theorem. That limitation is right.**

But the second edition left the vocabulary standing throughout the body — in Section 5, **“the required accuracy is the tolerance of each pipeline stage so that the accumulated frame drift does not reach one radian”**; in Section 7, **“a standard computational algorithm interpolating the positions of dynamic objects by Taylor expansion in a rendering system of finite propagation speed”**; in Corollary 4.2, **“local frame interpolation” and “discharge out of the system”**.

The third edition carries the limitation out: the vocabulary is removed from the body.

Nothing is lost by removing it. That integer-order PN is conserved within the system while half-integer PN radiates out of it is **physics**, resting on **time-reversal symmetry and mass conservation**, and does not depend on the vocabulary.

The “discharge into a higher dimension” retracted in the second edition is likewise not reinstated. The energy flows out as a gravitational wave propagating in ordinary three-dimensional space.

8. The Scope, Restated

Item	What it is
The c^{-2} hierarchy from the retarded expansion	Standard fact
The conservative / dissipative branching	Standard fact (Blanchet 2014 etc.)
That $n = 1$ vanishes by mass conservation	Standard fact (made explicit here)
Why 2.5 PN is the first dissipative order	$n = 3$ is the first survivor
That the two c^{-5} share a root	The same $n = 3$ term
Mercury 42.990'' /century	Fact (re-verified; Einstein 1915)
$\delta_{\text{req}} \approx 9.93 \times 10^{-6}$; $(0.3)^{10} = 5.905 \times 10^{-6}$	Fact (same order)
“Exactly at the threshold”	Withdrawn (an order-of-magnitude estimate)
That Mercury supports this series' reading	Not shown (the second edition's note kept)
c^{-5} matched to a 5D bulk capacity	Retracted (second edition; not reinstated)
5PN accuracy as a sub-pixel limit of 10^{-10}	Retracted (second edition; not reinstated)
“Odd orders vanish by parity”	Retracted (second edition; not reinstated)
Any change to the predictions of general relativity	None

Of the thirteen rows, not one carries a claim proper to this paper. They are standard facts, re-verifications, withdrawals, or retractions. **What the third edition added is only that it pointed explicitly at one standard fact — it is 2.5 because $n = 1$ vanishes.**

9. Conclusion

The second edition recovered the branching. **It put back into the series the radiation reaction the first edition had dropped — the very cause of a binary spiralling inward. That was a large correction.**

But “why 2.5” stood open. Time-reversal symmetry says only that a conservative system has no odd terms; it does not say where dissipation begins.

The answer was on the second line of the expansion. The $n = 1$ term is proportional to $\dot{M}$ and vanishes because mass is conserved. So the first surviving odd term is $n = 3$, the third derivative of the quadrupole.

Time reversal forbids the odd terms.

Mass conservation decides where the prohibition lifts.

What is not claimed (restraint): no prediction of general relativity is altered / the Mercury agreement is not treated as support for this series' reading (the second edition's note is kept) / c^{-5} is not matched to a 5D bulk capacity (retracted; Papers 34, 42) / the destination of the outflowing energy is not a higher dimension (a gravitational wave in ordinary three-dimensional space) / 5PN accuracy is not identified with a sub-pixel limit (retracted) / the phase-coherence condition is not read as a sharp threshold (an order-of-magnitude estimate) / the content of Section 4 is not claimed as new to physics (established in Blanchet 2014, Burke--Thorne 1970 and elsewhere) / Layer-3 vocabulary is not placed in the body (quarantined under a non-scientific label) / no new mathematics and no prediction (Layer 2).

Revision Record

Third edition (August 2026): all three corrections of the second edition were right and none is reinstated — (a) the retraction of the first edition's “odd-order terms vanish by parity” (half-integer terms exist and describe radiation reaction), (b) the retraction of the correspondence between c^{-5} and Paper 34's $\frac{c^5}{5}$ (“5D bulk capacity”), which came from $\int E^2 dc$ and is itself retracted, and (c) the retraction of “5PN accuracy = sub-pixel limit 10^{-10} ” (correctly $\sim 10^{-5}$ by the phase-coherence condition). All four numbers were re-verified and agree — Mercury $42.990''/\text{century}$, chirp mass $1.2188M_\odot$, $N_{\text{cyc}} = 1.603 \times 10^4$, $\delta_{\text{req}} = 9.93 \times 10^{-6}$, $(0.3)^{10} = 5.905 \times 10^{-6}$. Three things are added. (i) “Why 2.5PN?” is answered: the second edition's reason (a conservative system is time-reversal symmetric, so it has no odd terms) is correct but nearly definitional, and does not explain where dissipation begins. In the near-zone expansion $V = G \sum_n \frac{(-1)^n}{n!c^n} \partial_t^n \int \rho r^{n-1}$, the first odd term $n = 1$ is $-\frac{G}{c} \dot{M}$ and vanishes by conservation of mass. Hence the first surviving odd term is $n = 3$, the third time derivative of the quadrupole moment (the Burke--Thorne radiation-reaction potential), which is 2.5PN. Time reversal forbids the odd terms; a conservation law decides where the prohibition lifts. (ii) The two c^{-5} were shown to share a root — both the c^{-5} of the quadrupole formula and the c^{-5} of the 2.5PN reaction come from the same $n = 3$ term (the quadrupole's third derivative). This supplies the source of the second edition's “they necessarily agree as a matter of energy balance”. (iii) The Layer-3 vocabulary was removed from the body — the second edition had correctly limited it at the outset as “vocabulary for a reading, not the content of a theorem”, but left the vocabulary itself in Sections 5 and 7 and in Corollary 4.2. This edition carries the limitation out. In addition, “the 5PN correction sits exactly at the threshold” was weakened to “the same order of magnitude” — δ_{req} is an order-of-magnitude estimate, and since the phase accumulates mostly at low frequency the actual contribution of a high PN order is smaller.

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