

Something Is Resting on the Summit of a Hill

— L_4 is a maximum of the effective potential, lower in all eight surrounding directions (-1.499998500005 against a surrounding maximum of -1.499998503754) — and it is stable all the same, fenced by a mass ratio of **24.9599357944** — the Sun-Earth ratio of **333029.7043** lies on the stable side and a ratio of **20** on the unstable one — what holds it is not the potential but the Coriolis force, and one sense of “stable” has been swapped for another — [Paper 490]

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— *From thinking to syncing.* —

Abstract

A stable point is taken to be the bottom of a valley. This paper shows that L_4 is a maximum of the effective potential and stable nonetheless—the fence is not a shape but a mass ratio of **24.9599357944**. No new theorem or law is claimed. First, L_4 is a stationary point—its gradient is $(0.000 \times 10^0, 1.110 \times 10^{-10})$, becoming -0.040634 when displaced by 0.05 (Section 2). Second, this is the core of the paper. That stationary point is a maximum—the effective potential is -1.499998500005 while the largest of the eight surrounding values is -1.499998503754 , all lower (Section 2). Third, it is stable all the same, the condition being $27\mu(1 - \mu) < 1$ —a threshold mass ratio of **24.9599357944** (Section 3). Fourth, the Sun-Earth ratio of **333029.7043** lies on the stable side and a ratio of **20** on the unstable one (Section 3). Fifth, L_1 solves numerically to **1491426526** m against the approximation's **1496433206** m, a relative difference of **0.003357** (Section 4). Sixth, the separator is whether what holds it is a potential or a velocity-dependent force (Section 5). **Keywords:** Lagrange points, restricted three-body problem, L_4 and L_5 , linear stability, celestial mechanics.

Scope of this paper (scope note): No new theorem or law is claimed—the restricted three-body problem, the Lagrange points and the linear stability condition for L_4 and L_5 are all standard. No celestial mechanics is built—perturbations, eccentricity, radiation pressure and the actual distribution of Trojans are not the subject; only the circular restricted problem. Nonlinear stability is excluded—what is treated is linear stability; long-term behaviour and Arnold diffusion are not entered. Not all five points are located—numbers are given for L_1 and L_4 only; L_2 , L_3 and L_5 are named and left. The Hill radius is not derived—Paper 337’s scope note declines to enter the three-body problem or the derivation of the Hill radius, and this paper is placed beside it. Relation to earlier papers: Paper 195 showed that “stable” is six separate words, separated by what is perturbed—this paper gives a case where two of those six part company. Paper 487 showed that a stationary point comes in three kinds— L_4 is the maximum, and stable besides. Paper 337 showed that the edge of the solar system is five edges—in the same system, “a stable place” is likewise not one. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is confirming the maximality of L_4 in eight directions, giving its gradient as 1.110×10^{-10} , giving the threshold as 24.9599357944 and checking it by inversion, bracketing the boundary with 20 and 25, and comparing the numerical L_1 with the approximation at 0.003357.

1. Notation and the Question

Setting: Two bodies of masses m_1 and m_2 orbit their common centre in a circle, with mass fraction $\mu = m_2/(m_1 + m_2)$. In the co-rotating frame a third body of negligible mass feels the effective potential

$$\Omega(x, y) = - \left[\frac{1}{2}(x^2 + y^2) + \frac{1 - \mu}{r_1} + \frac{\mu}{r_2} \right]$$

in units where $G(m_1 + m_2) = 1$, the separation is 1 and the angular velocity is 1. It has five stationary points, of which L_4 and L_5 form equilateral triangles with the two bodies.

Observation 1.1

A stable point is taken to be the bottom of a valley. Displace it and it returns—the picture of falling towards lower potential.

Observation 1.2

But a rotating frame carries a velocity-dependent force. The Coriolis force—which cannot be written as a potential.

Question: Is L_4 a valley or a hill.

2. L_4 Is the Summit of a Hill

Computation 2.1: At $\mu = 3.0 \times 10^{-6}$, close to the Sun-Earth value, evaluate the gradient at L_4 .

$$\left(\frac{\partial \Omega}{\partial x}, \frac{\partial \Omega}{\partial y}\right) = (0.000 \times 10^0, 1.110 \times 10^{-10})$$

Observation 2.2

The gradient vanishes. 1.110×10^{-10} —the size of the finite-difference step; a stationary point.

Observation 2.3

Displaced by 0.05, it ceases to be stationary. -0.040634 —eight orders larger, confirming that the test responds to position.

Computation 2.4: Compare Ω at L_4 with the eight surrounding directions.

Position	Effective potential Ω
L_4	-1.499998500005
Largest of the eight around it	-1.499998503754
Smallest of the eight around it	-1.499998513721

Observation 2.5

Everything around it is lower.Even the largest, $-1.499998503754 < -1.499998500005$ —the potential falls in every direction: L_4 is a maximum.

Observation 2.6

It is not a minimum.The smallest around it is -1.499998513721 —the directions differ, so it is not flat either.

Observation 2.7

In the three kinds of Paper 487, this is the maximum. A stationary point that is a hill—there the optical path was longest; here an object rests.

3. Stable All the Same, Fenced by a Mass Ratio

Computation 3.1: Solve the linear stability condition $27\mu(1 - \mu) < 1$ for L_4 and L_5 .

Quantity	Value
Critical mass fraction μ_c	0.038520896505
$27\mu_c(1 - \mu_c)$ (inverted)	0.99999999999999
Threshold mass ratio $(1 - \mu_c)/\mu_c$	24.9599357944

Observation 3.2

The inversion returns 1.0.99999999999999—an independent check on both the algebra and the arithmetic.

Observation 3.3

The fence is 24.9599357944. Stable if the heavier exceeds the lighter by more than 24.96 times—a question of ratio, not of shape.

Computation 3.4: Check both sides of the boundary.

System	Mass ratio	Verdict
Sun and Earth	333029.7043	stable
A binary at 25	25	stable (just above)
A binary at 20	20	unstable

Observation 3.5

Sun and Earth stand four orders from the fence.333029.7043 against 24.9599357944—comfortably on the stable side.

Observation 3.6

25 and 20 bracket the boundary.One stable, one not—confirming the threshold lies between them.

Observation 3.7

What holds it is the Coriolis force.Displaced, the body is turned sideways and enters a circulation before it can descend—being velocity-dependent, it never appears in the picture of the potential.

4. L_1 Is No Valley, but a Saddle

Computation 4.1: Solve for L_1 in the Sun-Earth system and compare with the approximation.

Route	Distance from Earth
Numerical	1491426526 m (1,491,427 km)
Approximation $R(m_2/3m_1)^{1/3}$	1496433206 m
Difference	5006679 m (relative 0.003357)

Observation 4.2

It matches the familiar 1.5 million km.1,491,427 km—where observing spacecraft are placed, an independent check.

Observation 4.3

The approximation is 0.34 per cent out. A difference of 5006679 m—not agreement, but the right order: the range where the approximation works can be read.

Observation 4.4

L_1 is a saddle and is not stable. It falls in one direction—a spacecraft placed there must keep station.

Observation 4.5

So the five stationary points are not of one kind. Three saddles, two maxima—and it is the maxima that are stable.

5. The Separator

The separator: is what holds it a potential, or a velocity-dependent force.

Observation 5.1

A potential, and nothing but a minimum is stable. The floor of a valley—which L_4 is not.

Observation 5.2

A velocity-dependent force, and a maximum may be stable. The Coriolis force—and L_4 is exactly that.

Observation 5.3

Two of Paper 195's six senses of “stable” part company here. “A minimum of energy” and “perturbations do not grow”— L_4 satisfies the second and not the first: distinct roots by the criterion of Paper 300.

Observation 5.4

So “stable, therefore in a valley” cannot be said. A counterexample exists—the Trojan asteroids rest on that summit.

Observation 5.5

That the fence is a ratio rather than a shape follows from the same place. 24.9599357944—the balance between the Coriolis force and the slope of the hill is fixed by the mass ratio alone.

Observation 5.6

This is one of the separators of Paper 481. When something drifts off a point that was supposed to be stable—which sense of stability failed sends the repair elsewhere.

Observation 5.7

To be honest—only linear stability is treated. That perturbations do not grow to first order—what happens over long times, and how nonlinear effects enter, is another question this paper does not touch. Not all of why the Trojans have lasted billions of years is written here.

6. Roll Call and Closing

1. Gradient 1.110×10^{-10} at L_4 (Computation 2.1): a stationary point.
2. -0.040634 when displaced by 0.05 (Observation 2.3): the test responds.
3. $\Omega(L_4) = -1.499998500005$ (Computation 2.4): the core.
4. Surrounding maximum -1.499998503754 (Observation 2.5): all lower; a maximum.
5. Threshold 24.9599357944 (Computation 3.1): inverting gives 0.9999999999999999.
6. Sun and Earth at 333029.7043 (Computation 3.4): four orders of margin.
7. Bracketed by 25 and 20 (Observation 3.6): both sides of the boundary.
8. L_1 at 1491426526 m (Computation 4.1): 0.003357 from the approximation.
9. The separator is potential or velocity (Observation 5.3): two of Paper 195’s six part.

10. Linear stability only (Observation 5.7): not the long-term question.

Closing: A stable point is taken to be the bottom of a valley—displace it and it returns, falling towards lower potential. But a rotating frame carries a velocity-dependent force, the Coriolis force, which cannot be written as a potential. L_4 is the summit of a hill—at $\mu = 3.0 \times 10^{-6}$ the gradient at L_4 is $(0.000 \times 10^0, 1.110 \times 10^{-10})$, the size of the finite-difference step, and displacing by 0.05 gives -0.040634 , eight orders larger. The effective potential there is -1.499998500005 while the largest of the eight surrounding values is -1.499998503754 and the smallest -1.499998513721 . The potential falls in every direction, so L_4 is a maximum, not a minimum, and the directions differ so it is not flat either. In the three kinds of Paper 487 this is the maximum: there the optical path was longest, here an object rests. Stable all the same, fenced by a mass ratio—solving $27\mu(1 - \mu) < 1$ gives $\mu_c = 0.038520896505$, whose inversion returns 0.9999999999999999 , and a threshold mass ratio of **24.9599357944**. Stability requires the heavier to exceed the lighter by more than 24.96 times: a question of ratio, not of shape. The Sun-Earth ratio **333029.7043** stands four orders from the fence, while 25 is stable and 20 is not, bracketing the boundary. What holds it is the Coriolis force: displaced, the body is turned sideways and enters a circulation before it can descend, and being velocity-dependent it never appears in the picture of the potential. L_1 is no valley, but a saddle—solving for the Sun-Earth system gives **1491426526** m from Earth, or 1,491,427 km, matching where observing spacecraft are placed. The approximation $R(m_2/3m_1)^{1/3}$ gives 1496433206 m, a difference of 5006679 m or 0.003357: not agreement, but the right order. L_1 is a saddle and is not stable, so a spacecraft there must keep station. The five stationary points are not of one kind—three saddles, two maxima—and it is the maxima that are stable. One thing separates them—is what holds it a potential, or a velocity-dependent force. A potential, and nothing but a minimum is stable, which L_4 is not. A velocity-dependent force, and a maximum may be stable, which L_4 is. Two of Paper 195's six senses of “stable” part company here—“a minimum of energy” and “perturbations do not grow”—and L_4 satisfies the second and not the first, distinct roots by the criterion of Paper 300. So “stable, therefore in a valley” cannot be said: the Trojan asteroids rest on that summit. That the fence is a ratio rather than a shape follows from the same place, the balance between the Coriolis force and the slope being fixed by the mass ratio alone. This is one of the separators of Paper 481: when something drifts off a point supposed to be stable, which sense of stability failed sends the repair elsewhere. To be honest—only linear stability is treated, that perturbations do not grow to first order. What happens over long times, and how nonlinear effects enter, is another question this paper does not touch, and not all of why the Trojans have lasted billions of years is written here.

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