

The Classical Doppler Effect Is Two Formulas, and Relativity Is Their Geometric Mean

— At $\beta = 0.1$ the classical answer is **1.111111111** if the source moves and **1.100000000** if the observer does —
relativity's **1.105541597** agrees with their **geometric** mean to **15** digits (**1.105541596785133** against
1.105541596785133) and differs from their arithmetic mean by **0.000013959** — and transversely the classical
value is **1.000000000** while relativity gives **0.994987437**: only one of the two has an effect there — [Paper
491]

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— *From thinking to syncing.* —

Abstract

The Doppler effect is taken to be one phenomenon. This paper shows that the classical account has two answers and relativity's agrees with their geometric mean to 15 digits—and that transversely only one of the two has any effect at all. No new theorem or law is claimed. First, at $\beta = 0.1$ the two classical formulas give **1.111111111** and **1.100000000**—a difference of **0.011111111** (Section 2). Second, this is the core of the paper. Relativity's **1.105541597** agrees exactly with their geometric mean—**1.105541596785133** against **1.105541596785133** (Section 3). Third, it does not agree with their arithmetic mean—a difference of **0.000013959** (Section 3). Fourth, as $\beta \rightarrow 0$ all three agree—to 5.000×10^{-13} at $\beta = 10^{-6}$ (Section 3). Fifth, transversely the classical value is **1.000000000** and the relativistic one **0.994987437**, so only the latter has an effect (Section 4). Sixth, the separator is whether the effect asks who is moving (Section 5). **Keywords:** Doppler effect, relativistic Doppler effect, geometric mean, transverse Doppler effect, special relativity.

Scope of this paper (scope note): No new theorem or law is claimed—the two classical Doppler formulas, the relativistic formula and the transverse effect are all standard. The physics of the medium is excluded—dispersion of sound, wind and inhomogeneous media are not the subject; only a medium at rest, and light in vacuum. No experiment is designed—the construction and error budget of the Ives-Stilwell experiment are not entered. Acceleration is excluded—only uniform motion; nothing is said about accelerating sources. No mechanism is offered for the geometric mean—the agreement is an algebraic identity, and this paper goes as far as confirming it. Relation to earlier papers: Paper 24 showed that the one-way speed of light cannot be measured, using the fact that the Doppler ratio carries no synchronisation—this paper splits that very ratio in two. Paper 292 showed that only the logarithm adds—a geometric mean appears because the averaging is done in a multiplicative world. Paper 300 showed that a shared root can be decided—the two classical formulas are distinct roots, and the relativistic one restores a single root. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is setting the two classical values 1.11111111 and 1.100000000 side by side, confirming the geometric mean to 15 digits, giving the gap to the arithmetic mean as 0.000013959, checking the $\beta \rightarrow 0$ side at 5.000×10^{-13} , and showing the transverse shift to be second order by a factor of 4.030693.

1. Notation and the Question

Setting: A source of frequency f_0 and an observer approach at speed v , and write $\beta = v/c$. In the rest frame of the medium, the classical answer depends on which of them is moving.

$$\frac{f}{f_0} = \frac{1}{1 - \beta} \quad (\text{source moving}), \quad \frac{f}{f_0} = 1 + \beta \quad (\text{observer moving})$$

Relativity has only one.

$$\frac{f}{f_0} = \sqrt{\frac{1 + \beta}{1 - \beta}}$$

Numbers below are at $\beta = 0.1$.

Observation 1.1

The Doppler effect is taken to be one phenomenon. Higher approaching, lower receding—one formula, runs the picture.

Observation 1.2

But classically there are two. Source moving, or observer moving—with a medium as a third party, the same relative velocity gives different answers.

Question: How far apart are the two, and where among them does relativity fall.

2. The Classical Account Has Two Answers

Computation 2.1: Evaluate all three at $\beta = 0.10$.

Case	f/f_0
Classical (source moving)	1.111111111
Classical (observer moving)	1.100000000
Relativistic	1.105541597

Observation 2.2

The two classical formulas disagree. A difference of 0.011111111—the same relative velocity, answers 1.1 per cent apart.

Observation 2.3

They part at second order. $1/(1 - \beta) = 1 + \beta + \beta^2 + \dots$ against $1 + \beta$ —identical to first order, separating at β^2 .

Observation 2.4

So the presence of a medium is in principle measurable. See which formula holds—for sound this can be done, and the two are distinguished.

Observation 2.5

For light the distinction cannot be drawn. Relativity has one formula—with nowhere to ask who is moving.

3. Relativity Is Their Geometric Mean

Computation 3.1: Average the two classical values two ways and compare with relativity.

Quantity	Value	Difference from relativity
Geometric mean $\sqrt{(1/(1-\beta))(1+\beta)}$	1.105541596785133	0
Relativistic $\sqrt{(1+\beta)/(1-\beta)}$	1.105541596785133	—
Arithmetic mean	1.105555556	0.000013959

Observation 3.2

The geometric mean agrees to 15 digits. 1.105541596785133 against 1.105541596785133—not close, but algebraically the same.

Observation 3.3

Multiplying the formulas shows it. $\frac{1}{1-\beta} \cdot (1+\beta) = \frac{1+\beta}{1-\beta}$ —take the square root and the relativistic formula appears.

Observation 3.4

The arithmetic mean does not agree. A difference of 0.000013959—“between the two” is not enough; which mean matters.

Observation 3.5

A geometric mean appears because this is a world of ratios. Frequency ratios compose by multiplication—and as Paper 292 showed, only the logarithm adds.

Observation 3.6

As $\beta \rightarrow 0$ all three agree. A difference of 5.000×10^{-13} at $\beta = 10^{-6}$ —the other side of the boundary: at low speed the theories cannot be told apart.

4. Transversely, Only One of Them Has an Effect

Computation 4.1: Evaluate motion perpendicular to the line of sight.

Theory	f/f_0	Shift
Classical	1.000000000	0
Relativistic	0.994987437	0.005013

Observation 4.2

Classically nothing happens. 1.000000000—neither approaching nor receding, so of course.

Observation 4.3

Relativistically the frequency drops. 0.994987437—matching $1/\gamma$, and returning to 0.994987437107 on inversion.

Observation 4.4

So this is time dilation itself. The distance does not change, yet the frequency falls—nothing but the slowing of a moving clock is at work.

Observation 4.5

The shift is second order. 0.005012563 at $\beta = 0.10$ and 0.020204103 at 0.20, a ratio of 4.030693—doubling β roughly quadruples it, so it is not a first-order effect.

Observation 4.6

So the transverse effect is no residue of the longitudinal one. It survives where the longitudinal effect vanishes—a quantity of separate origin.

5. The Separator

The separator: does the effect ask who is moving, or not.

Observation 5.1

Asking, and there are two answers. 1.11111111 and 1.10000000—because a medium provides a reference, as with sound.

Observation 5.2

Not asking, and there is one. 1.105541597—fixed by relative velocity alone, as with light.

Observation 5.3

The two are distinct roots by the criterion of Paper 300. Both are called the Doppler effect—one an effect of velocity relative to a medium, the other of relative velocity: different independent variables.

Observation 5.4

And the transverse effect is the test of that separator. If it is about a medium, nothing happens transversely—something happening puts it on the relativistic side.

Observation 5.5

Paper 24 used this ratio for something else. The Doppler ratio is built from two events on one clock, so it carries no synchronisation—this paper watches that same ratio split in two on the classical side.

Observation 5.6

This is one of the separators of Paper 481. When “the Doppler formula does not fit” is reported—whether a medium was overlooked or a second-order term dropped sends the repair elsewhere.

Observation 5.7

To be honest—no mechanism is offered for the geometric mean. It is the algebraic identity

$\frac{1}{1-\beta} \cdot (1 + \beta) = \frac{1+\beta}{1-\beta}$ —why relativity should land exactly between the two classical answers is a question this paper does not answer. What was confirmed is only that it does.

6. Roll Call and Closing

1. Classical 1.111111111 and 1.100000000 (Computation 2.1): differing by 0.011111111.
2. They part at second order (Observation 2.3): identical to first.
3. Geometric mean to 15 digits (Computation 3.1): the core.
4. Arithmetic mean off by 0.000013959 (Observation 3.4): which mean matters.
5. 5.000×10^{-13} as $\beta \rightarrow 0$ (Observation 3.6): the other side.
6. Transverse classical 1.000000000 (Computation 4.1): nothing happens.
7. Transverse relativistic 0.994987437 (Observation 4.3): matching $1/\gamma$.
8. The transverse shift is second order (Observation 4.5): a factor of 4.030693.
9. The separator is who is moving (Observation 5.3): distinct roots by Paper 300.
10. No mechanism for the mean (Observation 5.7): only the fact was confirmed.

Closing: The Doppler effect is taken to be one phenomenon—higher approaching, lower receding, one formula. But classically there are two: source moving or observer moving, and with a medium as a third party the same relative velocity gives different answers. The classical account has two answers—at $\beta = 0.10$, 1.111111111 if the source moves and 1.100000000 if the observer does, a difference of 0.011111111: the same relative velocity, answers 1.1

per cent apart. They part at second order, since $1/(1 - \beta) = 1 + \beta + \beta^2 + \dots$ and $1 + \beta$ agree to first. So the presence of a medium is in principle measurable, and for sound the two are distinguished. For light they are not: relativity has one formula, with nowhere to ask who is moving. Relativity is their geometric mean—the geometric mean $\sqrt{(1/(1 - \beta))(1 + \beta)}$ is **1.105541596785133** and the relativistic $\sqrt{(1 + \beta)/(1 - \beta)}$ is **1.105541596785133**, agreeing to 15 digits. They are algebraically the same: $\frac{1}{1-\beta} \cdot (1 + \beta) = \frac{1+\beta}{1-\beta}$, and the square root gives the relativistic formula. The arithmetic mean 1.105555556 differs by 0.000013959, so “between the two” is not enough—which mean matters. A geometric mean appears because this is a world of ratios, frequency ratios composing by multiplication, and as Paper 292 showed, only the logarithm adds. As $\beta \rightarrow 0$ all three agree to 5.000×10^{-13} at $\beta = 10^{-6}$: at low speed the theories cannot be told apart. Transversely, only one of them has an effect—for motion perpendicular to the line of sight the classical value is **1.000000000** while relativity gives **0.994987437**, matching $1/\gamma$ and returning to 0.994987437107 on inversion. So this is time dilation itself: the distance does not change, yet the frequency falls, and nothing but the slowing of a moving clock is at work. The shift is second order—0.005012563 at $\beta = 0.10$ and 0.020204103 at 0.20, a ratio of 4.030693. So it is no residue of the longitudinal effect: it survives where the longitudinal one vanishes, a quantity of separate origin. One thing separates them—does the effect ask who is moving, or not. Asking, and there are two answers, because a medium provides a reference, as with sound. Not asking, and there is one, fixed by relative velocity alone, as with light. The two are distinct roots by the criterion of Paper 300: both are called the Doppler effect, but one is an effect of velocity relative to a medium and the other of relative velocity, with different independent variables. And the transverse effect is the test of that separator—if it is about a medium, nothing happens transversely, so something happening puts it on the relativistic side. Paper 24 used this ratio for something else, that it is built from two events on one clock and so carries no synchronisation; this paper watches the same ratio split in two on the classical side. This is one of the separators of Paper 481: when “the Doppler formula does not fit” is reported, whether a medium was overlooked or a second-order term dropped sends the repair elsewhere. To be honest—no mechanism is offered for the geometric mean. It is the algebraic identity $\frac{1}{1-\beta} \cdot (1 + \beta) = \frac{1+\beta}{1-\beta}$, and why relativity should land exactly between the two classical answers is a question this paper does not answer. What was confirmed is only that it does.

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