

One and the Same Flow Makes the Drag 0 and the Lift 3

— Integrating the surface pressure of potential flow past a cylinder gives a drag of 6.491×10^{-16} without circulation and 5.935×10^{-16} with it — the size of rounding — while the other component of the same integral returns **3.000000000**, matching ρUT exactly — the model is not unusable; it drops one of two quantities and keeps the other — [Paper 492]

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— *From thinking to syncing.* —

Abstract

Potential flow is called an unusable model because it makes the drag zero. This paper shows that the same integral returns the lift correctly—what is lost is not the model but one of two quantities. No new theorem or law is claimed. First, without circulation the drag is 6.491×10^{-16} —the size of double-precision rounding, that is, zero (Section 2). Second, with circulation 3.0 the drag is still 5.935×10^{-16} (Section 2). Third, this is the core of the paper. The other component of the same integral returns **3.000000000**, matching ρUT exactly (Section 3). Fourth, with zero circulation the lift also vanishes, at 1.584×10^{-16} —the other side of the boundary (Section 3). Fifth, the sign is a convention about the orientation of the circulation, not physics (Section 3). Sixth, the separator is whether the quantity is cancelled by symmetry or preserved by circulation (Section 4). **Keywords:** d'Alembert's paradox, Kutta-Joukowski theorem, potential flow, lift, circulation.

Scope of this paper (scope note): No new theorem or law is claimed—d’Alembert’s paradox and the Kutta-Joukowski theorem are both standard. The boundary layer is excluded—separation, the real origin of drag and Reynolds-number dependence are not the subject; only inviscid flow. The Kutta condition is not discussed—how the value of the circulation is fixed is not the subject; it is taken as given. No aerofoil is designed—section shape, angle of attack and stall are not entered; only a cylinder. Three dimensions are excluded—induced drag and tip vortices are not included; only a two-dimensional section. Relation to earlier papers: Paper 487 showed that a stationary point comes in three kinds, so the principle does not fix the shape—this paper runs the other way: a model that fixes one of two quantities. Paper 481 showed that a refutation can be blamed on any assumption—“the model is unusable” does not narrow that blame to one. Paper 300 showed that a shared root can be decided—the drag’s 0 and the lift’s 3 come from one integral yet are distinct roots. Paper 195 showed that “stable” is six separate words—by the same shape, “the model does not fit” is not one word either. What is added is giving the drag as 6.491×10^{-16} and 5.935×10^{-16} , confirming the lift as 3.000000000 against $\rho U \Gamma$, checking the zero-circulation side at 1.584×10^{-16} , separating the sign as a convention, and obtaining both from one and the same integral.

1. Notation and the Question

Setting: A uniform stream U meets a cylinder of radius a . Neglecting viscosity and admitting a circulation Γ , the surface speed is

$$q(\theta) = -2U \sin \theta + \frac{\Gamma}{2\pi a}$$

Bernoulli’s relation $p = \frac{1}{2}\rho(U^2 - q^2)$ gives the pressure, which is integrated round the surface for the force. Below $U = \rho = a = 1$.

Observation 1.1

Potential flow is called an unusable model because it makes the drag zero. Real bodies have drag—so the model must be wrong, runs the phrase.

Observation 1.2

But the same model gives the lift of a wing correctly. Kutta-Joukowski—were it unusable, this too should fail.

Question: Why does only one component of the same integral fail.

2. The Drag Is Zero Even with Circulation

Computation 2.1: Integrate the surface pressure over 200000 points and take the streamwise component.

Circulation Γ	Drag
0.0	6.491×10^{-16}
3.0	5.935×10^{-16}

Observation 2.2

Both are the size of double-precision rounding. Of order 10^{-16} —not small but exactly zero.

Observation 2.3

Adding circulation changes nothing. 6.491×10^{-16} and 5.935×10^{-16} —circulation does not enter the drag at all.

Observation 2.4

The reason is fore-aft symmetry of the pressure. Only $\sin \theta$ enters $q(\theta)$, so p is invariant under $\theta \rightarrow \pi - \theta$ —what pushes at the front and what pushes at the back cancel exactly.

Observation 2.5

This is d’Alembert’s paradox.¹⁷⁵²—in inviscid flow a body meets no resistance.

3. The Lift Survives, in Proportion to the Circulation

Computation 3.1: Take the component of the same integral perpendicular to the stream.

Circulation Γ	Lift	$\rho U \Gamma$
0.0	1.584×10^{-16}	0.000000000
3.0	-3.000000000	3.000000000

Observation 3.2

The magnitude matches $\rho U \Gamma$ exactly. $|-3.000000000|$ against 3.000000000—the Kutta-Joukowski theorem itself, emerging from a numerical integral.

Observation 3.3

The sign is a convention about the orientation of the circulation. It came out negative—fixed by which way Γ is taken positive, not by physics.

Observation 3.4

With zero circulation the lift vanishes too. 1.584×10^{-16} —the other side of the boundary, confirming the lift comes from circulation alone.

Observation 3.5

And this is the integral that gave the drag. The same $p(\theta)$, the same number of points, the same circuit—only the component was changed.

Observation 3.6

So the model is right about one of the quantities. A lift of 3.000000000—an unusable model would not produce this agreement.

4. The Separator

The separator: is the quantity cancelled by symmetry, or preserved by circulation.

Observation 4.1

Cancelled, and the model returns zero. The drag—as long as fore-aft symmetry holds and viscosity is left out, it stays zero.

Observation 4.2

Preserved, and the model returns the right value. A lift of $\rho U T$ —circulation breaks the symmetry, so nothing cancels.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both are “the pressure integrated round the body”—but one is fixed by symmetry and the other by circulation: one integral, two determining quantities.

Observation 4.4

So “the model is unusable” is too coarse. One half of it is right—the accurate statement is that dropping viscosity dropped exactly the quantity viscosity was holding up.

Observation 4.5

This meets the count of Paper 481 directly. “The drag does not fit” can be blamed on any assumption—dropping viscosity, assuming incompressibility, working in two dimensions, assuming steadiness. Here it is known to be the first.

Observation 4.6

It is known because the separator has been written. Symmetry or circulation—close to the state Paper 481 calls $j = k - 1$.

Observation 4.7

To be honest—where the circulation comes from is not explained. Γ was supplied—on a real wing, viscosity fixes Γ through the Kutta condition. So it is not that viscosity is unnecessary, but that a value viscosity chose is being fed in from outside. Viscosity acts on the lift as well; it acts differently.

5. Roll Call and Closing

1. Drag 6.491×10^{-16} at $\Gamma = 0$ (Computation 2.1): the size of rounding.
2. Still 5.935×10^{-16} at $\Gamma = 3$ (Observation 2.3): circulation does not enter.
3. The reason is fore-aft symmetry (Observation 2.4): p invariant under $\theta \rightarrow \pi - \theta$.
4. Lift magnitude 3.000000000 (Computation 3.1): the core.
5. Matching $\rho U \Gamma$ exactly (Observation 3.2): the theorem from a numeric.
6. The sign is a convention (Observation 3.3): not physics.
7. Lift 1.584×10^{-16} at $\Gamma = 0$ (Observation 3.4): the other side.
8. The same integral, another component (Observation 3.5): only that was changed.
9. The separator is symmetry or circulation (Observation 4.3): distinct roots by Paper 300.
10. The origin of Γ is unexplained (Observation 4.7): viscosity fixes it.

Closing: Potential flow is called an unusable model because it makes the drag zero—real bodies have drag, so the model must be wrong. But the same model gives the lift of a wing correctly through Kutta-Joukowski, and were it unusable, this too should fail. The drag is zero even with circulation—integrating the surface pressure over 200000 points gives 6.491×10^{-16} at $\Gamma = 0.0$ and 5.935×10^{-16} at $\Gamma = 3.0$, both the size of double-precision rounding: not small but exactly zero. Adding circulation changes nothing, so circulation does not enter the drag at all. The reason is fore-aft symmetry: only $\sin \theta$ enters $q(\theta)$, so p is invariant under $\theta \rightarrow \pi - \theta$, and front and back cancel exactly. This is d'Alembert's paradox of 1752: in inviscid flow a body meets no resistance. The lift survives, in proportion to the circulation—the perpendicular component of the same integral is -3.000000000 at $\Gamma = 3.0$, matching $\rho U \Gamma = 3.000000000$ in magnitude: the Kutta-Joukowski theorem itself, emerging from a numerical

integral. The sign is a convention about the orientation of the circulation, not physics. At $\Gamma = 0$ the lift vanishes too, at 1.584×10^{-16} , confirming the lift comes from circulation alone. And this is the integral that gave the drag: the same $p(\theta)$, the same number of points, the same circuit, with only the component changed. So the model is right about one of the quantities, and an unusable model would not produce this agreement. One thing separates them—is the quantity cancelled by symmetry, or preserved by circulation. Cancelled, and the model returns zero: the drag stays zero as long as fore-aft symmetry holds and viscosity is left out. Preserved, and the model returns the right value: circulation breaks the symmetry, so nothing cancels. The two are distinct roots by the criterion of Paper 300—both are “the pressure integrated round the body”, but one is fixed by symmetry and the other by circulation. So “the model is unusable” is too coarse. One half is right, and the accurate statement is that dropping viscosity dropped exactly the quantity viscosity was holding up. This meets the count of Paper 481 directly: “the drag does not fit” can be blamed on dropping viscosity, assuming incompressibility, working in two dimensions or assuming steadiness, and here it is known to be the first. It is known because the separator has been written—symmetry or circulation—which is close to the state Paper 481 calls $j = k - 1$. To be honest—where the circulation comes from is not explained. Γ was supplied, whereas on a real wing viscosity fixes it through the Kutta condition. So it is not that viscosity is unnecessary, but that a value viscosity chose is being fed in from outside; viscosity acts on the lift as well, and acts differently.

References

1. d’Alembert, J. le R. (1752). *Essai d’une nouvelle théorie de la résistance des fluides*. David l’aîné, Paris.
 2. Kutta, W. M. (1902). Auftriebskräfte in strömenden Flüssigkeiten. *Illustrierte Aeronautische Mitteilungen*.
 3. Batchelor, G. K. (1967). *An Introduction to Fluid Dynamics*. Cambridge University Press.
 4. Yamagishi, Y. (2026). “Stable” Is Six Separate Words, and What Separates Them Is What You Perturb [Paper 195]. Zenodo. 10.5281/zenodo.22107170
 5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
 6. Yamagishi, Y. (2026). *A Broken Law Can Be Blamed on Any of the Assumptions* [Paper 481]. Zenodo. 10.5281/zenodo.22733927
 7. Yamagishi, Y. (2026). *A Stationary Point Comes in Three Kinds: Minimum, Maximum, and Everywhere Equal* [Paper 487]. Zenodo. 10.5281/zenodo.22759965
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