

A Lagrange Multiplier Is a Temperature Here and a Pressure Next Door

— The multiplier β from maximising entropy agrees with an independently computed dS/dE at **0.700000000** against **0.700000000**, and again at **1.900000000** for $\beta = 1.9$ — it does not agree with a wrong value of **0.9** — the same device appears in Paper 196 as the multiplier enforcing incompressibility, where it is the pressure:

what the multiplier is, the constraint decides — [Paper 494]

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— *From thinking to syncing.* —

Abstract

A Lagrange multiplier is taken to be an auxiliary quantity that appears mid-calculation. This paper shows that the multiplier agrees digit for digit with an independently defined physical quantity—and that which quantity it becomes is decided by the constraint imposed. No new theorem or law is claimed. First, maximising the entropy at $\beta = 0.700$ gives $E = 0.727444451$ and $S = 1.132814574$ (Section 2). Second, this is the core of the paper. An independently computed dS/dE returns **0.700000000**, agreeing with the multiplier (Section 2). Third, it agrees again at $\beta = 1.9$, giving **1.900000000**—not a fit at one point (Section 2). Fourth, it does not agree with a wrong value of **0.9**—the test responds to the value (Section 2). Fifth, as $\beta \rightarrow 0$, $S = 1.386294361 = \ln 4$, and at $\beta = 50$, $E = 1.929 \times 10^{-22}$ (Section 3). Sixth, the separator is whether the multiplier is a device or a quantity (Section 4).

Keywords: maximum entropy principle, Lagrange multiplier, temperature, statistical mechanics, pressure.

Scope of this paper (scope note): No new theorem or law is claimed—the maximum-entropy principle and the identification of the multiplier with inverse temperature are standard. Statistical mechanics is not founded here—ergodicity, the origin of thermal equilibrium and quantum statistics are not the subject; only a four-level finite system. The philosophy of inference is not entered—whether maximum entropy is a correct norm of inference is not discussed; only what the formulas return. Units are excluded— k_B is set to 1, and the constant relating β to $1/T$ is not the subject. The pressure side is not computed—the multiplier enforcing incompressibility, from Paper 196, is named as a control and no numbers are given for it. Relation to earlier papers: Paper 196 showed that pressure appears as the multiplier enforcing incompressibility—the multiplier here is the same device becoming a different physical quantity. Paper 112 counted six distinct roots sharing one rhyme—the multiplier has that shape: one procedure yielding several quantities. Paper 300 showed that a shared root can be decided—temperature and pressure share a root as procedures and differ as quantities. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is confirming the agreement of dS/dE with β at 0.700000000, confirming it again at $\beta = 1.9$, showing that a wrong value does not agree, checking both limits at $\ln 4$ and 1.929×10^{-22} , and setting it beside the pressure of Paper 196.

1. Notation and the Question

Setting: Take a four-level system with energies $E_i \in \{0, 1, 2, 3\}$. Maximising the entropy $S = -\sum p_i \ln p_i$ at fixed mean energy gives, through a multiplier β ,

$$p_i = \frac{e^{-\beta E_i}}{Z}, \quad Z = \sum_j e^{-\beta E_j}$$

where β was introduced only to impose the constraint. Thermodynamics, separately, defines temperature by $1/T = dS/dE$. The two are computed independently and compared.

Observation 1.1

A Lagrange multiplier is taken to be an auxiliary quantity that appears mid-calculation. Put in to impose a constraint, removed at the end—scaffolding, runs the picture.

Observation 1.2

But β does not go away. It sits inside the answer—the scaffolding has become part of the building.

Question: Does β agree with an independently defined quantity.

2. The Multiplier Agrees with an Independently Computed dS/dE

Computation 2.1: Build the distribution at $\beta = 0.700$ and evaluate E and S .

Quantity	Value
Multiplier β	0.700
Mean energy E	0.727444451
Entropy S	1.132814574
Numerical dS/dE	0.700000000

Observation 2.2

dS/dE agrees with the multiplier.0.700000000 against 0.700000000—both S and E were built from β , but the derivative looks only at the relation between S and E and never uses β directly.

Observation 2.3

It agrees at another β too.1.900000000 at $\beta = 1.9$ —not a fit at one point.

Observation 2.4

It does not agree with a wrong value.0.700000000 against 0.9—the test responds to the value, so it could have failed.

Observation 2.5

So β is not an auxiliary quantity.It is $1/T$ itself—what was taken for scaffolding was a temperature.

Observation 2.6

And the temperature does not come from the maximisation. It appeared when a constraint was imposed—the constraint is producing the quantity.

3. What Happens at Either End

Computation 3.1: Send β to both extremes.

β	E	S
$\rightarrow 0$	1.499999999	1.386294361
0.700	0.727444451	1.132814574
50	1.929×10^{-22}	9.644×10^{-21}

Observation 3.2

As $\beta \rightarrow 0$ the distribution becomes uniform. $S = 1.386294361$ against $\ln 4 = 1.386294361$ —matching an independently computed value, with E at the uniform mean of 1.5.

Observation 3.3

As β grows it falls to the ground state. $E = 1.929 \times 10^{-22}$, $S = 9.644 \times 10^{-21}$ —the other side of the boundary, with the formulas holding at both ends.

Observation 3.4

Larger β means smaller E . From 0.727444451 to 1.929×10^{-22} —consistent with reading β as $1/T$.

Observation 3.5

β can also be negative. Because the levels are finite in number—then E exceeds 1.5 and the temperature is negative, which cannot happen in a system with infinitely many levels.

4. The Separator

The separator: is the multiplier a device of the calculation, or an independently defined quantity.

Observation 4.1

A device should disappear at the end. Put in only to impose a constraint—but it did not disappear.

Observation 4.2

A quantity should agree with another definition. dS/dE —and it agreed, at 0.700000000.

Observation 4.3

Which quantity it becomes is decided by the constraint. Fix the energy and it is a temperature—in Paper 196, fixing $\nabla \cdot u = 0$ made the multiplier a pressure.

Observation 4.4

By the criterion of Paper 300 they share a root as procedures and differ as quantities. Both are Lagrange multipliers—one an inverse temperature, one a pressure, differing even in units.

Observation 4.5

The shape is the one Paper 112 counted. Six distinct roots sharing one rhyme—one procedure producing a different quantity for each constraint.

Observation 4.6

So “Lagrange multiplier” is not the name of a quantity. It names a position in a procedure—and what fills it is decided by the constraint placed there.

Observation 4.7

This is one of the separators of Paper 481. When “the multiplier has a wrong value” is reported—whether the arithmetic failed or a different constraint’s multiplier was being read sends the repair elsewhere.

Observation 4.8

To be honest—the pressure side was not computed. Paper 196’s result was borrowed—that the two come from one procedure can be read off the form of the equations, but what was confirmed numerically here is only the temperature side. Setting them side by side is a claim, not a check.

5. Roll Call and Closing

1. $E = 0.727444451$ at $\beta = 0.700$ (Computation 2.1): $S = 1.132814574$.
2. $dS/dE = 0.700000000$ (Observation 2.2): **the core**.
3. Agreement again at $\beta = 1.9$ (Observation 2.3): **not one point**.
4. No agreement with 0.9 (Observation 2.4): **the test responds**.
5. $S = \ln 4$ as $\beta \rightarrow 0$ (Computation 3.1): **uniform**.
6. $E = 1.929 \times 10^{-22}$ at $\beta = 50$ (Observation 3.3): **the other end**.
7. β can be negative (Observation 3.5): **because the levels are finite**.
8. The constraint fixes the quantity (Observation 4.3): **pressure in Paper 196**.
9. Same procedure, different quantity (Observation 4.4): **by the criterion of Paper 300**.
10. The pressure side is uncomputed (Observation 4.8): **a claim, not a check**.

Closing: A Lagrange multiplier is taken to be an auxiliary quantity that appears mid-calculation—scaffolding, put in to impose a constraint and removed at the end. But β does not go away: it sits inside the answer, and the scaffolding has become part of the building. The multiplier agrees with an independently computed dS/dE —building the distribution of a four-level system at $\beta = 0.700$ gives $E = 0.727444451$ and $S = 1.132814574$, and the numerical

dS/dE returns **0.700000000**. Both S and E were built from β , but the derivative looks only at the relation between them and never uses β directly. It agrees again at $\beta = 1.9$, giving **1.900000000**, so this is not a fit at one point. It does not agree with a wrong value of 0.9, so the test could have failed. So β is not auxiliary but $1/T$ itself: what was taken for scaffolding was a temperature. And the temperature does not come from the maximisation—it appeared when a constraint was imposed, so the constraint is producing the quantity. What happens at either end—as $\beta \rightarrow 0$ the distribution becomes uniform, with $S = 1.386294361$ matching $\ln 4 = 1.386294361$ and E at the uniform mean of 1.5. At $\beta = 50$ it falls to the ground state, $E = 1.929 \times 10^{-22}$ and $S = 9.644 \times 10^{-21}$, so the formulas hold at both ends. Larger β means smaller E , consistent with reading β as $1/T$. Because the levels are finite in number, β can also be negative, and then E exceeds 1.5 and the temperature is negative—which cannot happen with infinitely many levels. One thing separates them—is the multiplier a device of the calculation, or an independently defined quantity. A device should disappear at the end, and it did not. A quantity should agree with another definition, and it agreed at 0.700000000. Which quantity it becomes is decided by the constraint: fix the energy and it is a temperature, while in Paper 196 fixing $\nabla \cdot u = 0$ made it a pressure. By the criterion of Paper 300 they share a root as procedures and differ as quantities—both Lagrange multipliers, one an inverse temperature and one a pressure, differing even in units. The shape is the one Paper 112 counted: one procedure producing a different quantity for each constraint. So “Lagrange multiplier” is not the name of a quantity but of a position in a procedure, and what fills it is decided by the constraint placed there. This is one of the separators of Paper 481: when the multiplier is reported to have a wrong value, whether the arithmetic failed or a different constraint’s multiplier was being read sends the repair elsewhere. To be honest—the pressure side was not computed. Paper 196’s result was borrowed, and while the common procedure can be read off the form of the equations, what was confirmed numerically here is only the temperature side. Setting them side by side is a claim, not a check.

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