

# The $4\pi$ Appears Twice, Cancels, and Then the Sum Diverges

— The  $4\pi r^2 dr$  that counts the stars and the  $1/4\pi r^2$  that dims them cancel completely: a shell contributes **0.010000000** at  $r = 1$  and at  $r = 1000$  alike, a difference of **0** — what remains is  $nLdr$ , free of  $r$ , and the sum diverges linearly in the radius (doubling  $R$  doubles it, by **2.000000000**) — the night sky is dark not for want of stars but for what stops the sum — [Paper 496]

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— *From thinking to syncing.* —

## Abstract

The night sky is taken to be dark because distant stars are faint. This paper shows that the  $4\pi$  that dims and the  $4\pi$  that multiplies the stars cancel completely—what remains is free of  $r$ , and the sum diverges. No new theorem or law is claimed. First, a shell at radius  $r$  contributes **0.010000000** at  $r = 1$  and at  $r = 1000$  alike (Section 2). Second, this is the core of the paper. The difference between the two is **0**—not close, but cancelled symbolically, both the  $4\pi$  and the  $r^2$  (Section 2). Third, that value equals  $nLdr$ —and returns on inversion (Section 2). Fourth, the sum diverges linearly in the radius—doubling  $R$  multiplies it by **2.000000000** (Section 3). Fifth, cut off at a finite radius it stays finite—**10.000000** at  $R = 1000$  and **0** at  $R = 0$  (Section 3). Sixth, the separator is whether the stars are too faint or the sum is cut off (Section 4). **Keywords:** Olbers' paradox, dark night sky, age of the universe, inverse-square law.

Scope of this paper (scope note): No new theorem or law is claimed—Olbers’ paradox, and its resolution through finite age and finite stellar lifetime, are standard. No cosmology is written—expansion, redshift, the temperature of the microwave background and an evolving luminosity function are not the subject; only a uniform static idealisation. Obscuration is excluded—nearer stars hiding farther ones is not included. In the idealisation the obscuring star shines at the same surface brightness, so it does not help, but that calculation is not done here. No units—number density and luminosity are in arbitrary units, and no prediction is made about the actual brightness of the sky. No age is derived—where the cutoff radius falls is not the subject; only that a cutoff is required. Relation to earlier papers: Paper 56 treated the  $4\pi$  patch as a structure—this is a case where that  $4\pi$  appears twice and disappears. Paper 483 showed that a radius cancels when two forces share an exponent—here too the exponents match and  $r$  cancels, but what remains is a divergent sum rather than a finite ratio. Paper 493 showed that a refinement can give three answers—both have the shape “before summing, write what stops it”. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is confirming the shell contribution over four decades of radius, showing the difference to be 0, confirming agreement with  $nLdr$  by inversion, showing the divergence to be linear at 2.000000000, and bracketing with 10.000000 and 0.

## 1. Notation and the Question

Setting: Let stars of luminosity  $L$  fill space at uniform number density  $n$ . A shell of radius  $r$  and thickness  $dr$  holds  $n \cdot 4\pi r^2 dr$  of them, and each delivers  $L/(4\pi r^2)$ . Multiplying,

$$dF = n 4\pi r^2 dr \cdot \frac{L}{4\pi r^2} = n L dr$$

Below,  $n = 10^{-2}$  and  $L = 1$  in arbitrary units.

### Observation 1.1

The night sky is taken to be dark because distant stars are faint. Dimming as the inverse square—so the far contribution is negligible, runs the picture.

### Observation 1.2

But farther out there are also more stars at that distance. The shell volume goes as  $r^2$ —the multiplying side and the dimming side carry the same  $r^2$ .

Question: Do the two cancel, or does one win.

## 2. Both the $4\pi$ and the $r^2$ Cancel Completely

Computation 2.1: Evaluate the contribution of a shell of thickness 1 over four decades of radius.

| Radius $r$ | Contribution |
|------------|--------------|
| 1.0        | 0.010000000  |
| 10.0       | 0.010000000  |
| 100.0      | 0.010000000  |
| 1000.0     | 0.010000000  |

### Observation 2.2

A thousandfold change in radius leaves the contribution untouched. A difference of 0—not even within rounding: the cancellation is symbolic.

### Observation 2.3

The value equals  $nLdr$ . 0.010000000 against 0.010000000—matching an independently computed closed form, and returning on inversion.

### Observation 2.4

Both the  $4\pi$  and the  $r^2$  cancel.  $4\pi r^2 dr$  on the counting side,  $1/(4\pi r^2)$  on the dimming side—the  $4\pi$  Paper 56 treated as a structure appears twice here and vanishes.

### Observation 2.5

So “distant stars are faint and do not matter” does not hold. Every shell contributes equally—distance balances the penalty in brightness against the bonus in number, exactly.

### 3. The Sum Diverges Linearly in the Radius

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Computation 3.1: Add the shells.

| Cutoff radius $R$ | Total received |
|-------------------|----------------|
| 0                 | 0              |
| 1000              | 10.000000      |
| 2000              | 20.000000      |

**Observation 3.2**

Doubling  $R$  doubles the total. A factor of 2.000000000—diverging at first order, with no prospect of convergence.

**Observation 3.3**

Cut off at a finite radius, it stays finite. 10.000000 at  $R = 1000$ —the boundary: only the cutoff keeps the value finite.

**Observation 3.4**

At  $R = 0$  nothing is received. The other boundary—confirming that the total is fixed by the cutoff radius alone.

**Observation 3.5**

So the brightness of the sky is not fixed by the properties of stars.  $n$  and  $L$  are only constants of proportionality—what fixes it is  $R$ .

### 4. The Separator

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The separator: are the stars too faint, or is the sum cut off.

#### Observation 4.1

If brightness, raising  $L$  should brighten the sky. Proportionally—but raising  $R$  raises it just as much.

#### Observation 4.2

If the cutoff, the search is for why  $R$  is finite. The age of the universe and the lifetimes of stars—beyond some distance the light has not arrived, and cannot be added.

#### Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both may be called “why the night sky is dark”—one is the size of the integrand, the other the end of the interval: different places in the same expression.

#### Observation 4.4

Beside Paper 483, the same cancellation ends differently. There, matching exponents made the radius cancel and left a finite ratio—here they make  $r$  cancel and leave a divergent sum. The cancellation itself does not fix the ending.

#### Observation 4.5

It has the shape of Paper 493 too. Before summing or taking a limit, write what stops it—there a refinement ratio, here the end of an integral.

#### Observation 4.6

This is one of the separators of Paper 481. When “the sky is dark, so the universe is finite” turns out wrong—giving up uniformity, giving up staticity or giving up infinity sends the repair elsewhere.

### Observation 4.7

To be honest—obscuration was not computed. Nearer stars hiding farther ones—in the idealisation the obscuring star shines at the same surface brightness and so does not help, but that calculation is not done here. What was shown in numbers reaches as far as the cancellation and the divergence.

## 5. Roll Call and Closing

1. Shell contribution  $0.010000000$  (Computation 2.1): over four decades.
2. A difference of  $0$  (Observation 2.2): the core.
3. Equal to  $nLdr$  (Observation 2.3): returning on inversion.
4. The  $4\pi$  appears twice (Observation 2.4): the  $4\pi$  of Paper 56.
5. Distance balances both sides (Observation 2.5): every shell equal.
6.  $2.000000000$  for a doubled  $R$  (Observation 3.2): first-order divergence.
7.  $10.000000$  at  $R = 1000$  (Computation 3.1): the cutoff fixes it.
8.  $0$  at  $R = 0$  (Observation 3.4): the other boundary.
9. The separator is integrand or endpoint (Observation 4.3): distinct roots by Paper 300.
10. Obscuration uncomputed (Observation 4.7): cancellation and divergence only.

Closing: The night sky is taken to be dark because distant stars are faint—dimming as the inverse square, so the far contribution is negligible. But farther out there are also more stars at that distance: the shell volume goes as  $r^2$ , and the multiplying side and the dimming side carry the same  $r^2$ . Both the  $4\pi$  and the  $r^2$  cancel completely—a shell of thickness  $1$  contributes  $0.010000000$  at  $r = 1.0, 10.0, 100.0$  and  $1000.0$  alike, a thousandfold change in radius leaving a difference of  $0$ : not even within rounding, the cancellation is symbolic. The value equals  $nLdr$ , matching an independently computed closed form and returning on inversion. What cancels is  $4\pi r^2 dr$  on the counting side against  $1/(4\pi r^2)$  on the dimming side—the  $4\pi$  Paper 56 treated as a structure, appearing twice and vanishing. So “distant stars are faint and do not matter” does not hold: every shell contributes equally, and distance balances the penalty in brightness against the bonus in number, exactly. The sum diverges linearly in the radius—adding the shells gives  $10.000000$  at  $R = 1000$  and  $20.000000$  at  $R = 2000$ , so doubling  $R$  multiplies the total by  $2.000000000$ : first-order divergence, with no prospect of convergence. Cut off at a finite radius it stays finite, and only the cutoff keeps the value finite. At  $R = 0$  nothing is received, confirming that the total is fixed by the cutoff radius alone. So the brightness of the sky is not fixed by the properties of stars:  $n$  and  $L$  are only constants of proportionality, and what fixes it is  $R$ . One thing separates them—are the stars too faint, or is the sum cut off. If brightness, raising  $L$  should brighten the sky proportionally—but raising  $R$  raises it just as much. If the cutoff, the

search is for why  $R$  is finite: the age of the universe and the lifetimes of stars, beyond which the light has not arrived and cannot be added. The two are distinct roots by the criterion of Paper 300: one is the size of the integrand, the other the end of the interval, different places in the same expression. Beside Paper 483 the same cancellation ends differently—there matching exponents left a finite ratio, here they leave a divergent sum, so the cancellation itself does not fix the ending. It has the shape of Paper 493 too: before summing or taking a limit, write what stops it—there a refinement ratio, here the end of an integral. This is one of the separators of Paper 481: when “the sky is dark, so the universe is finite” turns out wrong, giving up uniformity, staticity or infinity sends the repair elsewhere. To be honest—obscuration was not computed. In the idealisation the obscuring star shines at the same surface brightness and so does not help, but that calculation is not done here. What was shown in numbers reaches as far as the cancellation and the divergence.

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