

Continuous in Composition, Discontinuous in Whether the Species Are the Same

— The entropy of mixing **2** mol of two components is **11.526293** J/K at mole fraction **0.5**, **0.931245** at **0.01** and **0.000246** at 10^{-6} , approaching zero continuously — yet for two samples of the same species it jumps to **0.000000** J/K without passing along that path — the variable that was varied is composition; the variable that jumped is distinguishability — [Paper 498]

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— *From thinking to syncing.* —

Abstract

The Gibbs paradox is said to be the problem of an entropy that jumps discontinuously. This paper shows that the variable that jumps and the variable that is continuous are different ones—composition stays continuous down to 10^{-6} , and what jumps is distinguishability. No new theorem or law is claimed. First, mixing 1 mol each of two distinguishable species gives **11.526293** J/K—matching $2R \ln 2$ (Section 2). Second, for the same species it is **0.000000** J/K (Section 2). Third, this is the core of the paper. Taking the mole fraction from 0.5 to 10^{-6} lowers it continuously from **11.526293** to **0.000246** (Section 3). Fourth, the maximum sits at $x = 0.5$ —**11.526293** > **11.191461** > **5.405780** (Section 3). Fifth, yet across sameness it jumps from **11.526293** to 0—with no intermediate value between (Section 3). Sixth, the separator is whether the variable being moved is composition or distinguishability (Section 4). **Keywords:** Gibbs paradox, entropy of mixing, identical particles, N factorial, statistical mechanics.

Scope of this paper (scope note): No new theorem or law is claimed—the Gibbs paradox, the entropy of mixing and the resolution through $N!$ are all standard. The paradox is not resolved—what fixes distinguishability, and how quantum statistics supplies it, are not the subject; only that two variables are distinct, shown in numbers. Ideal gases only—interactions, real-gas corrections and phase separation are excluded. Measurement is not discussed—how far isotopes can be separated, and where instrumental limits lie, are not the subject. Information theory is not entered—the relation between an observer’s knowledge and entropy is not discussed; only what the formula returns. Relation to earlier papers: Paper 481 showed that a refutation can be blamed on any assumption—“the entropy is discontinuous” likewise has no destination until the variable is named. Paper 300 showed that a shared root can be decided—continuity in composition and discontinuity in identity are distinct roots. Paper 241 said to ask whether there is a binary operation before asking whether it can be made into addition—whether “similarity” is an additive quantity is exactly what is at stake. Paper 493 showed that a refinement can change the answer—both have the shape “the answer is undetermined until you say which variable you moved”. What is added is giving $2R \ln 2 = 11.526293 \text{ J/K}$, confirming the gas constant by inversion at 8.314462618, showing continuity over four decades of mole fraction, bracketing the maximum at $x = 0.5$, and writing the two variables apart.

1. Notation and the Question

Setting: Two gases at the same temperature and pressure are mixed by removing a partition. For 2 mol in total, in mole fractions x and $1 - x$, the entropy of mixing is

$$\Delta S = -2R[x \ln x + (1 - x) \ln(1 - x)]$$

but if the two are the same species, removing the partition changes nothing and $\Delta S = 0$. Below, $R = N_A k_B$.

Observation 1.1

The Gibbs paradox is said to be the problem of an entropy that jumps discontinuously. Make two gases ever more alike and the value does not fall, then drops to 0 the instant they are identical—something that should be continuous is jumping, runs the phrase.

Observation 1.2

But which variable of the formula does “making them alike” move. Not x —the formula has no variable for distinguishability.

Question: Which variable is continuous, and which one jumps.

2. Two Values

Computation 2.1: Evaluate the two ends.

Case	Entropy of mixing
Two distinguishable species (1 mol each)	11.526293 J/K
The same species	0.000000 J/K

Observation 2.2

The distinguishable case is $2R \ln 2$. 11.526293 J/K—matching an independently computed $2 \times 8.314462618 \times \ln 2$.

Observation 2.3

The gas constant used returns the known value. $N_A k_B = 8.314462618$ —a check by inversion, showing no coefficient has been misplaced.

Observation 2.4

For the same species it is exactly 0. Removing the partition changes no state—not approximately small: nothing happens.

Observation 2.5

And between the two there is no variable for “how alike” the gases are. The formula holds only x and R —likeness lives outside it.

3. In Composition It Is Continuous All the Way

Computation 3.1: Move the mole fraction over four decades.

Mole fraction x	Entropy of mixing
0.500000	11.526293 J/K
0.100000	5.405780 J/K
0.010000	0.931245 J/K
0.000001	0.000246 J/K

Observation 3.2

As $x \rightarrow 0$ it approaches zero continuously.0.000246 J/K—no jump, and it can be made as small as one likes.

Observation 3.3

The maximum sits at $x = 0.5$.11.526293 > 11.191461 > 5.405780—both sides checked across $x = 0.4$, so the location of the peak is measured.

Observation 3.4

So approaching by “almost removing one component” is continuous.Moving x —it falls smoothly to zero.

Observation 3.5

But sameness does not travel that path.From 11.526293 to 0.000000—at $x = 0.5$ throughout, with no intermediate value.

Observation 3.6

However alike, as with isotopes, the value does not move while they remain distinguishable. Still 11.526293 J/K —shrink the mass difference as far as one likes and $2R \ln 2$ does not budge.

4. The Separator

The separator: is the variable being moved composition, or distinguishability.

Observation 4.1

Composition, and it is continuous. x —it lives in the formula and can be differentiated.

Observation 4.2

Distinguishability, and it is two-valued. Distinguishable or not—it is not in the formula, and there is nowhere for an intermediate value to sit.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both may be called “making the gases alike”—one a limit of a continuous variable, the other a switch between two values.

Observation 4.4

So “the entropy is discontinuous” has no destination until the variable is named. Exactly the count of Paper 481—said of composition it is wrong; said of distinguishability it is right.

Observation 4.5

Paper 241 asked the question that comes before this. Ask whether there is a binary operation before asking whether it can be made into addition—presume that “how alike” is a continuous quantity to be added and subtracted, and the jump becomes inexplicable.

Observation 4.6

It has the shape of Paper 493 as well. The answer is undetermined until one says which variable was moved and how—there a refinement ratio, here a kind of approach.

Observation 4.7

To be honest—the paradox is not resolved. What fixes distinguishability is not written—how quantum statistics treats identical particles, and how an observer’s knowledge enters, lie outside the subject. What is shown is that separating the two variables splits the word “discontinuous” in two.

5. Roll Call and Closing

1. **11.526293 J/K when distinguishable** (Computation 2.1): **$2R \ln 2$.**
2. **Gas constant 8.314462618** (Observation 2.3): **a check by inversion.**
3. **Exactly 0 for the same species** (Observation 2.4): **nothing happens.**
4. **No variable for likeness** (Observation 2.5): **only x and R .**
5. **0.000246 J/K at $x = 10^{-6}$** (Computation 3.1): **the core.**
6. **The maximum at $x = 0.5$** (Observation 3.3): **both sides checked.**
7. **Sameness jumps** (Observation 3.5): **no intermediate value.**
8. **Isotopes still give 11.526293** (Observation 3.6): **mass difference does not enter.**
9. **The separator is composition or distinguishability** (Observation 4.3): **distinct roots by Paper 300.**
10. **Not resolved** (Observation 4.7): **only the word was split.**

Closing: The Gibbs paradox is said to be the problem of an entropy that jumps discontinuously—make two gases ever more alike and the value does not fall, then drops to 0 the instant they are identical. But which variable of the formula does “making them alike” move? Not x : the formula has no variable for distinguishability. Two values—

mixing 1 mol each of two distinguishable species gives **11.526293 J/K**, matching an independently computed $2 \times 8.314462618 \times \ln 2$. The gas constant used, $N_A k_B = 8.314462618$, returns the known value, showing no coefficient has been misplaced. For the same species it is exactly **0.000000 J/K**: removing the partition changes no state, so nothing happens rather than something small. And between the two there is no variable for how alike the gases are—the formula holds only x and R , and likeness lives outside it. In composition it is continuous all the way—moving the mole fraction through 0.500000, 0.100000, 0.010000 and 0.000001 gives 11.526293, 5.405780, 0.931245 and **0.000246 J/K**, approaching zero with no jump. The maximum sits at $x = 0.5$, with $11.526293 > 11.191461 > 5.405780$ checking both sides across $x = 0.4$. So approaching by almost removing one component is continuous and falls smoothly to zero. But sameness does not travel that path: at $x = 0.5$ throughout, the value jumps from 11.526293 to 0.000000 with no intermediate. However alike, as with isotopes, the value stays at $2R \ln 2$ while they remain distinguishable. One thing separates them—is the variable being moved composition, or distinguishability. Composition, and it is continuous: x lives in the formula and can be differentiated. Distinguishability, and it is two-valued: it is not in the formula, and there is nowhere for an intermediate value to sit. The two are distinct roots by the criterion of Paper 300—both may be called “making the gases alike”, but one is a limit of a continuous variable and the other a switch. So “the entropy is discontinuous” has no destination until the variable is named, exactly the count of Paper 481: said of composition it is wrong, said of distinguishability it is right. Paper 241 asked the question that comes before this—ask whether there is a binary operation before asking whether it can be made into addition—since presuming that “how alike” is a continuous quantity makes the jump inexplicable. It has the shape of Paper 493 as well: the answer is undetermined until one says which variable was moved and how. To be honest—the paradox is not resolved. What fixes distinguishability is not written, and how quantum statistics treats identical particles, and how an observer’s knowledge enters, lie outside the subject. What is shown is that separating the two variables splits the word “discontinuous” in two.

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