

Every Paper in the Corpus Cites Papers That Are Cited 2.15 Times Better Than Average

— Measured on the citation graph, the mean degree is **11.799197** while the mean degree of a neighbour is **25.366236**, an excess of **13.567039** — which agrees to six digits with the variance divided by the mean — on a ring where every degree is equal the excess is exactly **0**, and on a star it opens to **5.5125** — it is the one identity of Paper 308's buses — [Paper 501]

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— *From thinking to syncing.* —

Abstract

The papers one happens to cite are taken to have been the conspicuous ones by chance. This paper shows that this is no bias but the consequence of an identity holding on any graph—the excess equals the variance divided by the mean. No new theorem or law is claimed. First, on the corpus's citation graph the mean degree is **11.799197** (Section 2). Second, the mean degree of a neighbour is **25.366236**, a factor of **2.149827** (Section 2). Third, this is the core of the paper. The excess of **13.567039** equals the variance over the mean, **13.567039** (Section 2). Fourth, on a ring where every degree is equal the excess is exactly **0** (Section 3). Fifth, on a star it opens to **5.5125**—the more the degrees vary, the larger the excess (Section 3). Sixth, the separator is whether one is counting vertices or edges (Section 4). **Keywords:** friendship paradox, citation network, size-biased sampling, network science, corpus of papers.

Scope of this paper (scope note): No new theorem or law is claimed—the friendship paradox and size-biased sampling are standard. No social claim—why people feel below average, and what follows socially, are not the subject. The corpus is not evaluated—a heavily cited paper is not called a good one; only edges are counted. Direction is discarded—citing and being cited are not distinguished; the graph is treated as undirected, and the values change if direction is kept. No causal claim—the identity says that variance produces the excess; why the degrees vary is not treated. Relation to earlier papers: Paper 308 showed that the gap between the buses you board is twice the average—this paper shows the same expression holding on a graph rather than in time. Paper 500 measured the citation graph over 498 papers—this paper measures the same graph by a different quantity. Paper 300 showed that a shared root can be decided—the buses and the graph differ in object but share a root. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is giving 11.799197 and 25.366236 for the corpus’s own graph, confirming the excess against the closed form to six digits, giving both extremes as exactly zero on a ring and 5.5125 on a star, confirming the same identity on the bus sequence, and placing the separator at vertices versus edges.

1. Notation and the Question

Setting: Write k_v for the degree of a vertex v . Picking a vertex and reading its degree, and picking an edge and reading the degree at its end, are different samplings. The first averages to $\langle k \rangle$, the second to

$$\frac{\sum_v k_v^2}{\sum_v k_v} = \frac{\langle k^2 \rangle}{\langle k \rangle} = \langle k \rangle + \frac{\sigma^2}{\langle k \rangle}$$

Below, the corpus’s internal citations are read as an undirected graph and both are measured.

Observation 1.1

The papers one happens to cite are taken to have been the conspicuous ones by chance. A bias in the choosing—correct it and it goes away, runs the picture.

Observation 1.2

But no “bias” appears in that expression. It is an identity—on any graph whose degrees vary, it must happen.

Question: By how much does the corpus’s graph exceed.

2. Measuring the Corpus's Graph

Computation 2.1: Read the citations of the 499 numbered papers as an undirected graph and average two ways.

Quantity	Value
Mean degree $\langle k \rangle$	11.799197
Mean neighbour degree $\langle k^2 \rangle / \langle k \rangle$	25.366236
Ratio	2.149827
Variance of the degrees σ^2	160.080160
$\sigma^2 / \langle k \rangle$	13.567039
Excess (neighbour — self)	13.567039

Observation 2.2

A neighbour is cited 2.149827 times better.25.366236 against 11.799197—from every paper, the papers it cites look above average.

Observation 2.3

The excess matches the closed form.13.567039 against 13.567039—agreeing to six digits: an identity, not a coincidence.

Observation 2.4

So the impression of being below average has no ground.Everyone observes the same thing—the comparison selects its own comparands.

Observation 2.5

The larger the variance, the larger the excess. $\sigma^2 = 160.080160$ —about 13.6 times the mean of 11.799197: the degrees vary widely.

3. Checking Both Extremes

Computation 3.1: Evaluate the same quantities on a graph with no variance and on an extreme one.

Graph	Mean degree	Mean neighbour	Ratio
Ring (12 vertices, all of degree 2)	2.000000	2.000000	1.0000
The corpus's citation graph	11.799197	25.366236	2.1498
Star (1 centre, 20 leaves)	1.904762	10.500000	5.5125

Observation 3.2

On the ring the excess is exactly zero. A difference of 0.000×10^0 —equal degrees mean zero variance, and the identity gives zero excess.

Observation 3.3

On the star it opens to 5.5125. 10.500000 against 1.904762—one centre neighbours every leaf, so every leaf sees a neighbour of degree 20.

Observation 3.4

The corpus lies between them. A factor of 2.1498—neither regular nor a star, but a structure with a spine.

Computation 3.5: Compare with the buses of Paper 308. For gaps [1, 1, 1, 1, 6] of mean 2, the mean gap a passenger experiences is 4.000000 and the variance over the mean is 2.000000.

Observation 3.6

The same identity holds for the buses. An excess of 2.000000 against $\sigma^2/\langle k \rangle = 2.000000$ —the object is an interval in time, not a graph.

Observation 3.7

Equal gaps give zero excess. Still 2.000000—for the same reason as the ring.

4. The Separator

The separator: is one counting vertices, or edges.

Observation 4.1

Counting vertices, the mean is $\langle k \rangle$. 11.799197—one vote per paper.

Observation 4.2

Counting edges, the mean is $\langle k^2 \rangle / \langle k \rangle$. 25.366236—a paper of high degree is counted as many times as its degree.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both may be called “the average number of citations”—one averages over papers and the other over citations: the unit differs.

Observation 4.4

Paper 308’s buses and this paper differ in object yet share a root. Intervals in time and degrees in a graph—both are the same sampling in which larger things are more likely to be chosen, down to the same closed form.

Observation 4.5

Paper 500 measured the same graph by other quantities. Components, diameter, mean shortest path—there how it is connected, here how it is counted.

Observation 4.6

This is one of the separators of Paper 481. When “I only cite papers above average” is questioned—whether the choosing was biased or the unit was different sends the repair elsewhere.

Observation 4.7

To be honest—direction was discarded. Citing and being cited were not distinguished—real citations have direction, and keeping it makes “how often the cited paper is cited” and “how often the citing paper cites” different quantities. That distinction is not drawn here.

5. Roll Call and Closing

1. Mean degree **11.799197** (Computation 2.1): **over 499 papers**.
2. Mean neighbour **25.366236** (Observation 2.2): **a factor of 2.149827**.
3. Excess **13.567039** matches the closed form (Observation 2.3): **the core**.
4. Variance **13.6** times the mean (Observation 2.5): **the degrees vary widely**.
5. Exactly zero on a ring (Observation 3.2): **zero variance**.
6. **5.5125** on a star (Observation 3.3): **the other extreme**.
7. Excess **2.000000** for the buses (Observation 3.6): **the same identity**.
8. Equal gaps give zero (Observation 3.7): **as with the ring**.
9. The separator is vertices or edges (Observation 4.3): **distinct roots by Paper 300**.
10. Direction discarded (Observation 4.7): **keeping it changes the quantity**.

Closing: The papers one happens to cite are taken to have been the conspicuous ones by chance—a bias in the choosing, to be corrected away. But no “bias” appears in $\langle k^2 \rangle / \langle k \rangle = \langle k \rangle + \sigma^2 / \langle k \rangle$. It is an identity, and on any graph whose degrees vary it must happen. Measuring the corpus’s graph—reading the 499 numbered papers as an undirected graph gives a mean degree of **11.799197**, a mean neighbour degree of **25.366236** and a ratio of

2.149827. From every paper, the papers it cites look above average. The excess of **13.567039** matches the variance over the mean, **13.567039**, to six digits: an identity, not a coincidence. So the impression of being below average has no ground—everyone observes the same thing, and the comparison selects its own comparands. The variance is **160.080160**, about **13.6** times the mean, so the degrees vary widely. Checking both extremes—on a ring of equal degrees both averages are **2.000000** and the excess is exactly **0**: zero variance, zero excess. On a star, **1.904762** against **10.500000**, a factor of **5.5125**, since one centre neighbours every leaf and every leaf sees a neighbour of degree **20**. The corpus lies between them at **2.1498**: neither regular nor a star, but a structure with a spine. Compared with the buses of Paper 308, gaps $[1, 1, 1, 1, 6]$ of mean **2** give a passenger-experienced mean of **4.000000**, an excess of **2.000000** matching the variance over the mean. The object is an interval in time, not a graph, yet the same identity holds. Equal gaps give zero excess, for the same reason as the ring. One thing separates them—is one counting vertices, or edges. Counting vertices, the mean is $\langle k \rangle = 11.799197$, one vote per paper. Counting edges, it is $\langle k^2 \rangle / \langle k \rangle = 25.366236$, with a paper of high degree counted as many times as its degree. The two are distinct roots by the criterion of Paper 300: both may be called “the average number of citations”, but one averages over papers and the other over citations. Paper 308’s buses and this paper differ in object yet share a root, both being the sampling in which larger things are more likely chosen, down to the same closed form. Paper 500 measured the same graph by components, diameter and mean shortest path—there how it is connected, here how it is counted. This is one of the separators of Paper 481: when “I only cite papers above average” is questioned, whether the choosing was biased or the unit was different sends the repair elsewhere. To be honest—direction was discarded. Citing and being cited were not distinguished, yet real citations have direction, and keeping it makes “how often the cited paper is cited” and “how often the citing paper cites” different quantities. That distinction is not drawn here.

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