

Failing to Have a Transition for Want of Size Is Not the Same as Failing for Want of Dimension

— The free energy of the one-dimensional Ising model shows no jump in its second derivative at finite size (a largest step of **0.282856** at $n = 64$) — taking the limit is necessary for a transition, not sufficient — [Paper 502]

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September 2026

— *From thinking to syncing.* —

Abstract

The absence of a phase transition is put down to the system being finite. This paper shows that a system can fail to have one even in the thermodynamic limit—the one-dimensional Ising model, whose largest step stays at **0.510568**. No new theorem or law is claimed. First, at $n = 1024$ the free energy matches the closed form at **−1.626523375036** (Section 2). Second, at $n = 4$ it is still 7.782×10^{-2} away—so the test responds to n (Section 2). Third, at finite $n = 64$ the second derivative does not jump—the largest step is **0.282856** (Section 3). Fourth, this is the core of the paper. Nor does it jump in the thermodynamic limit—the closed form gives a largest step of **0.510568** (Section 3). Fifth, the same test on a kink jumps by 2.000×10^4 —so it does respond to non-analyticity (Section 3). Sixth, the separator is whether what is lacking is size or dimension (Section 4). **Keywords:** phase transition, one-dimensional Ising model, finite system, partition function, Lee-Yang theorem.

Scope of this paper (scope note): No new theorem or law is claimed—that the one-dimensional Ising model has no transition, and that finite partition functions are analytic, are standard. No theory of transitions—critical exponents, the renormalisation group and the distribution of Yang-Lee zeros are not the subject; only the second derivative of the free energy. Two dimensions were not computed—Onsager’s exact solution is not used, and nothing here shows that two dimensions do jump. No claim about numerical precision—these are finite differences at step 10^{-4} , not true second derivatives; what is compared is values taken at the same step. The kink is not physics— $|\beta - 1|$ is a control for seeing the test react, not a model. Relation to earlier papers: Paper 154 showed that “transitions occur only in the thermodynamic limit” is the non-commutation of two limits—this paper shows in numbers that the “only” is necessary and not sufficient. Paper 245 showed that two limits have no answer until the order is written—here, writing the order still leaves no jump when the dimension is too low. Paper 271 showed that mean field does not see the dimension—on this side, the dimension settles the answer itself. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is tabulating the free energy from $n = 4$ to 1024, measuring the finite-size step at 0.282856, showing the limit stays at 0.510568, giving the kink control at 2.000×10^4 , and placing the separator at size versus dimension.

1. Notation and the Question

Setting: For the one-dimensional Ising model with periodic boundaries and no field, the partition function is $Z = \lambda_1^n + \lambda_2^n$ with transfer-matrix eigenvalues $\lambda_1 = 2 \cosh \beta$ and $\lambda_2 = 2 \sinh \beta$. Write the free energy per site as

$$f(n, \beta) = -\frac{1}{n\beta} \ln Z = -\frac{1}{n\beta} [n \ln \lambda_1 + \ln (1 + (\lambda_2/\lambda_1)^n)]$$

and compute it in the second form, since forming λ^n directly overflows. As $n \rightarrow \infty$, $f \rightarrow -\ln(2 \cosh \beta)/\beta$.

Observation 1.1

The absence of a phase transition is put down to the system being finite. A finite sum is a polynomial and analytic everywhere—so take the limit and a transition appears, runs the picture.

Observation 1.2

But taking the limit guarantees no jump. It only makes non-analyticity possible—whether anything jumps is another matter.

Question: Does taking the limit produce a jump.

2. Measuring the Finite System

Computation 2.1: Evaluate $f(n, \beta)$ by n .

n	$\beta = 0.5$	$\beta = 2.0$
4	-1.648821002	-1.086894756
16	-1.626523916	-1.022900335
64	-1.626523375	-1.009790254
$\rightarrow \infty$ (closed form)	-1.626523375	-1.009074964

Observation 2.2

Raising n approaches the closed form. At $n = 1024$ and $\beta = 0.5$, -1.626523375036 against -1.626523375036 —they agree.

Observation 2.3

At $n = 4$ it is still away. A difference of 7.782×10^{-2} —the test responds to n , confirming that convergence is what is being watched.

Observation 2.4

The rate depends on β . Six digits by $n = 16$ at $\beta = 0.5$, only three by $n = 64$ at $\beta = 2.0$ —the low-temperature side is slower.

3. Taking the Limit Produces No Jump

Computation 3.1: Take second differences in β at step 10^{-4} .

Object	Second differences ($\beta = 0.90$ to 1.10)
$n = 64$	$-1.661210, -1.378354, -1.150643, -0.965664, -0.814208$
$n \rightarrow \infty$	$-1.661210, -1.150642, -0.814202$
Kink $- \beta - 1 $	$0, -2.000 \times 10^4, 0$

Observation 3.2

At finite $n = 64$ there is no jump. A largest step of 0.282856 —of the same order as the values themselves: smooth.

Observation 3.3

Nor in the thermodynamic limit. A largest step of 0.510568 —with n sent to infinity, it stays the same order as at finite n .

Observation 3.4

The same test does respond to a kink. 2.000×10^4 —four orders above the Ising model's 0.510568 : the test picks up non-analyticity.

Observation 3.5

So “finite, therefore no jump” is not the reason here. The limit was taken and still nothing jumped—the reason lies elsewhere.

Observation 3.6

That elsewhere is the dimension. In one dimension no order forms at finite temperature—the cost of one domain wall is finite, so arbitrarily many enter.

4. The Separator

The separator: is what is lacking the size of the system, or the dimension of space.

Observation 4.1

If size, the limit should produce a jump. $n \rightarrow \infty$ —but it stayed at 0.510568.

Observation 4.2

If dimension, the limit changes nothing. One dimension—and nothing jumped.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both may be stated as “there is no phase transition”—one a reason that the limit removes, the other a reason it does not.

Observation 4.4

Paper 154 spoke to the first. Transitions occur only in the thermodynamic limit—a necessary condition, not a sufficient one, and this paper gives the difference in numbers.

Observation 4.5

It neighbours Paper 245 as well. Two limits have no answer until the order is written—here, writing the order still gives the answer “no jump” when the dimension is too low.

Observation 4.6

This is one of the separators of Paper 481. When “the model shows no transition” is reported—whether the system is small, the dimension too low, or the interaction too short-ranged sends the repair elsewhere.

Observation 4.7

To be honest—two dimensions were not computed. That one dimension does not jump is shown; that two do is not—Onsager’s exact solution would show it, but it does not enter the numbers here. So “raise the dimension and it jumps” is not claimed.

5. Roll Call and Closing

1. $n = 1024$ matches the closed form (Observation 2.2): **−1.626523375036**.
2. $n = 4$ differs by 7.782×10^{-2} (Observation 2.3): **the test responds**.
3. The rate depends on β (Observation 2.4): **slower at low temperature**.
4. Largest step **0.282856** at $n = 64$ (Computation 3.1): **smooth**.
5. **0.510568** in the limit (Observation 3.3): **the core**.
6. 2.000×10^4 for the kink (Observation 3.4): **four orders above**.
7. Not “because it is finite” (Observation 3.5): **the reason is elsewhere**.
8. The reason is the dimension (Observation 3.6): **a finite wall cost**.
9. The separator is size or dimension (Observation 4.3): **distinct roots by Paper 300**.
10. Two dimensions uncomputed (Observation 4.7): **no claim that they jump**.

Closing: The absence of a phase transition is put down to the system being finite—a finite sum is a polynomial and analytic everywhere, so take the limit and a transition appears. But taking the limit guarantees no jump: it only makes non-analyticity possible, and whether anything jumps is another matter. Measuring the finite system—the free energy per site at $\beta = 0.5$ runs **−1.648821002**, **−1.626523916** and **−1.626523375** for $n = 4, 16$ and 64 , against a closed form of **−1.626523375**. At $n = 1024$ the two agree at **−1.626523375036**. At $n = 4$ the difference is still 7.782×10^{-2} , so the test responds to n . The rate depends on β : six digits by $n = 16$ at $\beta = 0.5$, only three by $n = 64$ at $\beta = 2.0$. Taking the limit produces no jump—second differences in β at step 10^{-4} run from **−1.661210** to **−0.814208** at $n = 64$, with a largest step of **0.282856**: smooth. The closed form in the thermodynamic limit runs from **−1.661210** to **−0.814202**, a largest step of **0.510568**—with n sent to infinity, the same order as at finite n . The same test applied to the kink $-|\beta - 1|$ jumps by 2.000×10^4 , four orders above, so it does pick up non-analyticity. So “finite, therefore no jump” is not the reason here: the limit was taken and still nothing jumped. That elsewhere is the dimension: in one dimension the cost of a single domain wall is finite, so arbitrarily many enter and no order forms at finite temperature. One thing separates them—is what is lacking the size of the system, or the dimension of space. If size, the limit should produce a jump—but it stayed at 0.510568. If dimension, the limit changes nothing—and nothing jumped. The two are distinct roots by the criterion of Paper 300: both are stated as

“there is no phase transition”, but one is a reason the limit removes and the other a reason it does not. Paper 154 spoke to the first, that transitions occur only in the thermodynamic limit—a necessary condition and not a sufficient one, and this paper gives the difference in numbers. It neighbours Paper 245 as well: two limits have no answer until the order is written, yet here writing the order still gives “no jump” when the dimension is too low. This is one of the separators of Paper 481: when a model shows no transition, whether the system is small, the dimension too low, or the interaction too short-ranged sends the repair elsewhere. To be honest—two dimensions were not computed. That one dimension does not jump is shown; that two do is not. Onsager’s exact solution would show it, but it does not enter the numbers here, so “raise the dimension and it jumps” is not claimed.

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