

The Extrapolation Crosses Zero Because the Heat-Capacity Difference Was Held Constant

— With a melting entropy of **10** and a heat-capacity difference of **30**, the extrapolated entropy difference crosses zero at $T = \mathbf{214.959393}$, or **0.716531** of the melting point — let it vanish as the third law requires and it need not cross at all — [Paper 503]

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— *From thinking to syncing.* —

Abstract

The Kauzmann paradox is spoken of as a property of supercooled liquids. This paper shows that crossing zero follows from holding the heat-capacity difference constant—held constant, however small, it always crosses. No new theorem or law is claimed. First, the entropy difference is $+4.530353$ at $T = 250$ and -2.163953 at $T = 200$ (Section 2). Second, it crosses zero at **214.959393**, or **0.716531** of the melting point (Section 2). Third, that matches the closed form $\exp(-\Delta S_m / \Delta C_p)$ at **0.716531310574** (Section 2). Fourth, this is the core of the paper. Lowering the difference to 1 still crosses, at $T_K = 1.36 \times 10^{-2}$ —only the crossing temperature falls (Section 3). Fifth, let the difference vanish with temperature and it crosses only when the total exceeds ΔS_m (Section 3). Sixth, the separator is whether the heat-capacity difference was held constant or made to vanish (Section 4).

Keywords: Kauzmann paradox, glass transition, supercooled liquid, entropy, heat capacity.

Scope of this paper (scope note): No new theorem or law is claimed—the Kauzmann paradox and the candidate resolutions are standard. No theory of glasses—whether an ideal glass transition exists, or dynamic arrest suffices, is not the subject. No real substance—a melting point of 300, melting entropy of 10 and heat-capacity difference of 30 are model values, not measurements of any material. The temperature dependence was not measured—only two cases are compared: held constant, and vanishing in proportion to T . The real dependence differs by substance. The third law is not proved—that heat capacities vanish as $T \rightarrow 0$ is used as a premise, not derived. Relation to earlier papers: Paper 352 showed that the glass transition temperature moves with the speed of measurement—that is the kinetic side; this is the thermodynamic one. Paper 498 showed continuity in composition and discontinuity in identity—both are cases where entropy shows a different face depending on which variable is moved. Paper 481 showed that a refutation can be blamed on any assumption—“the entropy goes negative” can be blamed on the extrapolation. Paper 300 showed that a shared root can be decided—holding constant and letting vanish are distinct roots. What is added is solving $T_K = 214.959393$ by bisection, confirming the closed form to twelve digits, showing that lowering the difference to 30, 5 and 1 still crosses, showing that with a vanishing difference the total decides, and placing the separator in the extrapolation.

1. Notation and the Question

Setting: A liquid that gained a melting entropy ΔS_m at T_m is cooled without crystallising. With ΔC_p for the heat-capacity difference between liquid and crystal, the entropy difference at T is

$$\Delta S(T) = \Delta S_m - \int_T^{T_m} \frac{\Delta C_p}{T'} dT'$$

and holding ΔC_p constant gives $\Delta S(T) = \Delta S_m - \Delta C_p \ln(T_m/T)$. Below, $T_m = 300$, $\Delta S_m = 10$ and $\Delta C_p = 30$, all of the order of J/(mol K).

Observation 1.1

The Kauzmann paradox is spoken of as a property of supercooled liquids. Cool far enough and the liquid's entropy falls below the crystal's—a picture of matter behaving that way.

Observation 1.2

But the crossing lies outside the measured range. Measurement stops at the glass transition—beyond it is extrapolation, and extrapolation needs assumptions.

Question: Does the crossing come from the substance, or from the assumption.

2. The Extrapolation Crosses Zero

Computation 2.1: Tabulate $\Delta S(T)$.

Temperature T	$\Delta S(T)$
300.0	+10.000000
250.0	+4.530353
214.0	−0.134194
200.0	−2.163953
150.0	−10.794415

Observation 2.2

The sign changes between 214 and 250.From +4.530353 to −0.134194—the boundary is bracketed.

Computation 2.3: Bisection gives $T_K = 214.959393$, or **0.716531** of the melting point.Recomputing ΔS there returns 0.000000000000.

Observation 2.4

It matches the closed form.0.716531310574 against $\exp(-\Delta S_m/\Delta C_p) = 0.716531310574$ —to twelve digits: an independent check.

Observation 2.5

It matches what is reported.For many glass formers T_K/T_m is about 0.7—the model’s 0.7165 lies in that range.

Observation 2.6

Below T_K it is negative. -3.160815 —the liquid’s entropy falls below the crystal’s, colliding with the third law.

Observation 2.7

Above T_K it is positive. $+2.859305$ —the other side of the boundary, so T_K really is where the sign turns.

3. It Crosses Because the Difference Was Held Constant

Computation 3.1: Lower ΔC_p and see where T_K goes.

ΔC_p	T_K	As a fraction of T_m
30.0	2.149594×10^2	7.165×10^{-1}
5.0	4.060058×10^1	1.353×10^{-1}
1.0	1.361998×10^{-2}	4.540×10^{-5}

Observation 3.2

A smaller difference still crosses. At $\Delta C_p = 1$, $T_K = 1.36 \times 10^{-2}$ —only the crossing temperature falls.

Observation 3.3

The reason is that the logarithm diverges. $\Delta C_p \ln(T_m/T)$ —it grows without bound as $T \rightarrow 0$, so any finite ΔS_m is eventually exceeded.

Observation 3.4

So “it must cross” follows from holding the difference constant. Not from the substance—the assumption contains the conclusion.

Computation 3.5: Let $\Delta C_p(T) = \Delta C_{p,0}(T/T_m)$, as the third law requires, and the total lost down to $T = 0$ saturates at $\Delta C_{p,0}$.

$\Delta C_{p,0}$	Total lost	Against $\Delta S_m = 10$
30.0	30.000000	crosses
5.0	5.000000	does not cross

Observation 3.6

A vanishing difference need not cross. $5.000000 < 10$ —it crosses only when the total exceeds ΔS_m .

Observation 3.7

And the third law requires heat capacities to vanish as $T \rightarrow 0$. They cannot stay constant—the extrapolation’s assumption is incompatible with the third law itself.

4. The Separator

The separator: was the heat-capacity difference held constant, or made to vanish as the third law requires.

Observation 4.1

Held constant, it must cross. The logarithm diverges—the paradox is built into the extrapolation.

Observation 4.2

Made to vanish, the total decides. 30 crosses, 5 does not—the answer may differ by substance.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both may be called “the Kauzmann temperature”—one a number the assumption always produces, one a number the substance decides.

Observation 4.4

Paper 352 did the same thing on the kinetic side. The glass transition temperature moves with the speed of measurement—there, “written like a material constant but not constant”; here, “written like a property of matter but a consequence of an assumption”.

Observation 4.5

It neighbours Paper 498. Entropy showing a different face depending on which variable is moved—there composition against identity, here temperature against the assumption.

Observation 4.6

This is one of the separators of Paper 481. When “the entropy goes negative” arrives as a refutation—whether the third law failed or the extrapolation was poor sends the repair elsewhere.

Observation 4.7

To be honest—the real temperature dependence was not measured. Only constant and linearly vanishing were compared—the real dependence differs by substance, and knowing its form would settle whether T_K exists. This paper does not have that form.

5. Roll Call and Closing

1. $+4.530353$ at $T = 250$ (Computation 2.1): -2.163953 at $T = 200$.
2. $T_K = 214.959393$ (Computation 2.3): 0.716531 of T_m .
3. Matching the closed form to twelve digits (Observation 2.4): $\exp(-\Delta S_m / \Delta C_p)$.
4. Matching the reported 0.7 (Observation 2.5): the model gives 0.7165.
5. $\Delta C_p = 1$ still crosses (Computation 3.1): the core.
6. The logarithm diverges (Observation 3.3): any ΔS_m is exceeded.
7. A vanishing difference lets the total decide (Computation 3.5): $5 < 10$ does not cross.
8. The third law requires vanishing (Observation 3.7): incompatible with the assumption.

9. The separator is constant or vanishing (Observation 4.3): **distinct roots by Paper 300.**

10. The real dependence is unmeasured (Observation 4.7): **that form is not held here.**

Closing: The Kauzmann paradox is spoken of as a property of supercooled liquids—cool far enough and the liquid’s entropy falls below the crystal’s. But the crossing lies outside the measured range: measurement stops at the glass transition, and beyond it is extrapolation, which needs assumptions. The extrapolation crosses zero—with $T_m = 300$, $\Delta S_m = 10$ and $\Delta C_p = 30$, ΔS runs +10.000000, +4.530353, **−0.134194** and **−10.794415** at $T = 300, 250, 214$ and 150, changing sign between 214 and 250. Bisection gives $T_K = \mathbf{214.959393}$, or **0.716531** of the melting point, and recomputing there returns 0.000000000000. The ratio matches the closed form $\exp(-\Delta S_m/\Delta C_p) = 0.716531310574$ to twelve digits. Reported values of T_K/T_m for glass formers are about 0.7, and the model’s 0.7165 lies in that range. Below T_K it is **−3.160815**, colliding with the third law; above it, **+2.859305**. It crosses because the difference was held constant—lowering ΔC_p to 30, 5 and 1 gives $T_K = 2.149594 \times 10^2, 4.060058 \times 10^1$ and **1.361998×10^{-2}** . A smaller difference still crosses; only the crossing temperature falls. The reason is that $\Delta C_p \ln(T_m/T)$ grows without bound as $T \rightarrow 0$, so any finite ΔS_m is eventually exceeded. So “it must cross” follows from holding the difference constant, not from the substance: the assumption contains the conclusion. Letting $\Delta C_p(T) = \Delta C_{p,0}(T/T_m)$ as the third law requires, the total lost saturates at $\Delta C_{p,0}$, so 30 crosses while **5** does not. And since the third law requires heat capacities to vanish as $T \rightarrow 0$, the extrapolation’s assumption is incompatible with the third law itself. One thing separates them—was the heat-capacity difference held constant, or made to vanish. Held constant, the logarithm diverges and it must cross: the paradox is built into the extrapolation. Made to vanish, the total decides—30 crosses, 5 does not—and the answer may differ by substance. The two are distinct roots by the criterion of Paper 300: both may be called “the Kauzmann temperature”, but one is a number the assumption always produces and one a number the substance decides. Paper 352 did the same on the kinetic side: there “written like a material constant but not constant”, here “written like a property of matter but a consequence of an assumption”. It neighbours Paper 498, where entropy showed a different face depending on which variable was moved—there composition against identity, here temperature against the assumption. This is one of the separators of Paper 481: when “the entropy goes negative” arrives as a refutation, whether the third law failed or the extrapolation was poor sends the repair elsewhere. To be honest—the real temperature dependence was not measured. Only constant and linearly vanishing were compared, while the real dependence differs by substance; knowing its form would settle whether T_K exists, and this paper does not have that form.

References

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1. Angell, C. A. (1995). Formation of Glasses from Liquids and Biopolymers. *Science*.
 2. Kauzmann, W. (1948). The Nature of the Glassy State and the Behavior of Liquids at Low Temperatures. *Chemical Reviews*.
 3. Stillinger, F. H., Debenedetti, P. G., & Truskett, T. M. (2001). The Kauzmann Paradox Revisited. *Journal of Physical Chemistry B*.

4. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
5. Yamagishi, Y. (2026). *The Glass Transition Temperature Moves with the Speed of Measurement* [Paper 352]. Zenodo. 10.5281/zenodo.22695723
6. Yamagishi, Y. (2026). *A Broken Law Can Be Blamed on Any of the Assumptions* [Paper 481]. Zenodo. 10.5281/zenodo.22733927
7. Yamagishi, Y. (2026). *Continuous in Composition, Discontinuous in Whether the Species Are the Same* [Paper 498]. Zenodo. 10.5281/zenodo.22760218

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