

Falling Noise and Non-Falling Noise Are Not Two Noises but Two Ends of One Expression

— the root-mean-square error is the single expression $\sqrt{b^2 + \sigma^2/N}$, whose two terms are equal at $N^* = \sigma^2/b^2 = 10^4$ — a hundred times past that point the bias term holds **0.990099** of the total, and a hundredth of the way to it the fluctuation term holds the same — [Paper 504]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

Noise that falls with longer measurement and noise that does not are taken to be two different noises. This paper shows that they are the two ends of one expression, and that the crossover can be computed—it sits at $N^* = \sigma^2/b^2$. No new theorem or law is claimed. First, without bias, multiplying N by 9 divides the root-mean-square by **3.0152**—and by **2.9386** on the next factor of 9 (Section 2). Second, with bias it settles at **0.014863576** by $N = 8100$ (Section 2). Third, this is the core of the paper. Both follow the single expression $\sqrt{b^2 + \sigma^2/N}$, matching to 0.0013, 0.0130 and **0.0057** (Section 3). Fourth, the two terms are equal at $N^* = 10^4$, where the value is $\sqrt{2}$ times the bias, **0.014142136** (Section 4). Fifth, the measured range $N \leq 8100$ lies short of the crossover, which is why both terms were still at work (Section 4). Sixth, the separator is whether there are two noises or one expression with two sides (Section 5). **Keywords:** noise, averaging, bias, Allan variance, law of large numbers.

Scope of this paper (scope note): No new theorem or law is claimed—that independent noise falls as $1/\sqrt{N}$, and that a bias does not average away, are both standard. Noise types are not enumerated—the classification into thermal, shot and $1/f$ noise belongs to Paper 312 and is not entered here. $1/f$ is excluded—the “non-falling” here is a constant bias, not correlated noise. The two are different things. The generator is not assessed—a linear congruential generator with Box-Muller is used; the quality of the pseudo-random numbers is not the subject. The origin of the bias is not treated— b is supplied, and why anything is biased is not examined. Relation to earlier papers: Paper 312 showed that some noise falls when you measure longer and some does not—this paper shows that the two are one expression, and gives the crossover. Paper 322 showed that one sensor error reaches position by three different powers—likewise a case of one source wearing several faces. Paper 300 showed that a shared root can be decided—the two terms live inside one expression. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is confirming the $1/\sqrt{N}$ law at 3.0152 and 2.9386, measuring the biased side at 0.014863576, comparing with the closed form at three points, giving the crossover $N^* = 10^4$ and the factor $\sqrt{2}$ there, and showing that the measured range fell short of it.

1. Notation and the Question

Setting: Let independent noise of spread σ carry a constant bias b . Averaging N samples gives a root-mean-square error of

$$\text{RMS}(N) = \sqrt{b^2 + \frac{\sigma^2}{N}}$$

Below, $\sigma = 1$ and $b = 0.01$, measured at $N = 100, 900$ and 8100 over 400 seeds each. The root mean square is taken over seeds rather than a single realisation, since the $1/\sqrt{N}$ law is a statement about averages and cannot be confirmed on one run.

Observation 1.1

Noise that falls with longer measurement and noise that does not are taken to be two different noises. One averages away, the other remains—two kinds, runs the picture.

Observation 1.2

But the expression above has only two terms. b^2 and σ^2/N —one noise, with only the question of which term is larger.

Question: Where does it change over.

2. Measuring Both Sides

Computation 2.1: Measure with and without bias.

<i>N</i>	Bias 0	Bias 0.01
100	0.100658455	0.100632328
900	0.033383439	0.034350201
8100	0.011360408	0.014863576

Observation 2.2

Without bias it falls as $1/\sqrt{N}$. A factor of 9 in N divides it by 3.0152, and the next by 2.9386—both near 3, so not a fit at one point.

Observation 2.3

With bias it settles. 0.014863576 at $N = 8100$ —above the bias of 0.01, but of the same order.

Observation 2.4

The improvement differs. 2.9386 without bias against 2.3110 with—the biased side gains less from the same operation.

Observation 2.5

Yet at $N = 100$ the two are nearly identical. 0.100658455 against 0.100632328—at small N , the presence of a bias cannot be told.

3. One Expression

Computation 3.1: Compare the biased side with $\sqrt{b^2 + \sigma^2/N}$.

N	Measured	Predicted	Relative difference
100	0.100632328	0.100498756	0.0013
900	0.034350201	0.034801022	0.0130
8100	0.014863576	0.014948471	0.0057

Observation 3.2

All three agree to within 1.3 per cent.0.0013, 0.0130, 0.0057—the remainder is estimation noise from a finite number of seeds.

Observation 3.3

The unbiased side is the same expression.Set $b = 0$ and it is $\sigma/\sqrt{N}$ —0.011360408 against 0.011111111 at $N = 8100$.

Observation 3.4

So these are not two kinds of noise.One expression, with b zero or not—what separates them is a parameter value, not a kind.

4. The Crossover Can Be Computed

Computation 4.1: Solve for the N at which the two terms are equal.

$$b^2 = \frac{\sigma^2}{N^*} \implies N^* = \frac{\sigma^2}{b^2} = 10^4$$

and the value there is $\sqrt{2}b = \mathbf{0.014142136}$.

Observation 4.2

At the crossover the value is $\sqrt{2}$ times the bias. 0.014142136 against 0.014142136 —the two terms being equal, the sum is $2b^2$.

Observation 4.3

The measured range fell short of it. $8100 < 10000$ —which is why both terms were still at work at $N = 8100$ and no clean plateau appeared.

Observation 4.4

A hundred times past it, the bias term holds 0.990099 . $N = 10^6$ —only there does it look like “noise that does not fall”.

Observation 4.5

A hundredth of the way to it, the fluctuation term holds the same 0.990099 . $N = 100$ —the other side of the boundary, where it looks like “noise that falls”.

Observation 4.6

One noise shows two faces across N . $N = 100$ and $N = 10^6$ —which face is seen is fixed by the length of the measurement, not by the noise.

5. The Separator

The separator: are there two noises, or one expression with two sides.

Observation 5.1

Two noises, and one of them can be removed. A separate mechanism—one ought to be addressable on its own.

Observation 5.2

One expression, and changing N alone changes the face. With b and σ unchanged—and indeed both faces appeared in the same sequence.

Observation 5.3

The two are distinct roots by the criterion of Paper 300. Both may be called “noise that does not fall”—one the presence of another mechanism, the other one term of a single expression winning.

Observation 5.4

Paper 312 divided them on the first side. Thermal, shot and $1/f$ noise—different mechanisms are indeed different things; what this paper shows is that one mechanism already yields two faces.

Observation 5.5

It has the shape of Paper 322 as well. One sensor error arriving by three powers—one source, several ways of arriving.

Observation 5.6

This is one of the separators of Paper 481. When “measuring longer stops helping” is reported—whether another noise entered or the crossover was simply passed sends the repair elsewhere.

Observation 5.7

To be honest—only a constant bias was treated. Correlated noise such as $1/f$ is not included—it too fails to average away, but for a different reason: with correlation the term σ^2/N does not hold and this paper's expression cannot be used. “Does not fall” contains at least two things.

6. Roll Call and Closing

1. Divided by **3.0152** without bias (Observation 2.2): and by **2.9386** next.
2. **0.014863576** with bias (Computation 2.1): it settles.
3. Indistinguishable at $N = 100$ (Observation 2.5): **0.100658** against **0.100632**.
4. Within **1.3** per cent of the closed form (Computation 3.1): the core.
5. The unbiased side is the same expression (Observation 3.3): set $b = 0$.
6. Crossover $N^* = 10^4$ (Computation 4.1): σ^2/b^2 .
7. $\sqrt{2}$ times the bias there (Observation 4.2): **0.014142136**.
8. The measured range fell short (Observation 4.3): $8100 < 10000$.
9. The separator is two noises or one expression (Observation 5.3): distinct roots by Paper 300.
10. $1/f$ was excluded (Observation 5.7): “does not fall” has at least two senses.

Closing: Noise that falls with longer measurement and noise that does not are taken to be two different noises—one averages away and the other remains. But $\text{RMS}(N) = \sqrt{b^2 + \sigma^2/N}$ has only two terms: one noise, with only the question of which term is larger. Measuring both sides—with $\sigma = 1$, $b = 0.01$ and 400 seeds, the unbiased case gives 0.100658455, 0.033383439 and **0.011360408** at $N = 100, 900$ and 8100, the biased case 0.100632328, 0.034350201 and **0.014863576**. Without bias, a factor of 9 in N divides the value by **3.0152** and the next factor by **2.9386**, both near 3. With bias it settles at 0.014863576, and the improvement is 2.3110 against the unbiased 2.9386. Yet at $N = 100$ the two are 0.100658455 against 0.100632328: at small N the presence of a bias cannot be told. One expression—comparing the biased side with the closed form gives relative differences of 0.0013, 0.0130 and **0.0057**, all within 1.3 per cent, the remainder being estimation noise from a finite number of seeds. The unbiased side is the same expression with $b = 0$, giving 0.011360408 against 0.011111111 at $N = 8100$. So these are not two kinds of noise: what separates them is a parameter value, not a kind. The crossover can be computed—the two terms are equal at $N^* = \sigma^2/b^2 = 10^4$, where the value is $\sqrt{2}b = \mathbf{0.014142136}$, since the sum is then $2b^2$. The measured range $N \leq 8100$ fell short of it, which is why both terms were still at work and no clean plateau appeared. A hundred times past it, at $N = 10^6$, the bias term holds **0.990099** of the total, and only there does it look like noise that does not fall. A hundredth of the way to it, at $N = 100$, the fluctuation term holds the same 0.990099, and there

it looks like noise that falls. One noise shows two faces across N , and which face is seen is fixed by the length of the measurement. One thing separates them—are there two noises, or one expression with two sides. Two noises, and one ought to be addressable on its own. One expression, and changing N alone changes the face—and indeed both faces appeared in the same sequence. The two are distinct roots by the criterion of Paper 300: both may be called “noise that does not fall”, but one is the presence of another mechanism and the other one term winning. Paper 312 divided them on the first side—thermal, shot and $1/f$ —and different mechanisms are indeed different things; what this paper shows is that one mechanism already yields two faces. It has the shape of Paper 322 as well: one source, several ways of arriving. This is one of the separators of Paper 481: when “measuring longer stops helping” is reported, whether another noise entered or the crossover was simply passed sends the repair elsewhere. To be honest—only a constant bias was treated. Correlated noise such as $1/f$ is not included; it too fails to average away, but for a different reason, since with correlation the term σ^2/N does not hold and this paper’s expression cannot be used. “Does not fall” contains at least two things.

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Yuuki YAMAGISHI (山岸 勇輝)

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