

The “Sphere of Influence” Is Three Spheres, and Dividing the Mass Ratio by 1000 Shrinks Them Differently

— Earth’s Hill radius is **1,496,433** km, Laplace’s sphere of influence **924,554** km and the point of equal gravitational pull **258,781** km, the largest exceeding the smallest by **5.7826** — the Moon lies inside the first two and outside the third — [Paper 505]

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— *From thinking to syncing.* —

Abstract

A planet’s sphere of influence is taken to be one radius. This paper shows that there are three definitions, with exponents $1/3$, $2/5$ and $1/2$ —so changing the mass ratio moves the three apart. No new theorem or law is claimed. First, Earth’s Hill radius is **1,496,433** km (Section 2). Second, Laplace’s sphere of influence is **924,554** km and the point of equal pull **258,781** km (Section 2). Third, the largest exceeds the smallest by **5.7826** (Section 2). Fourth, this is the core of the paper. Dividing the mass ratio by 1000 shrinks them by **10.000000**, **15.848932** and **31.569804** respectively (Section 3). Fifth, the spread then changes from **5.7826** to **18.2556** (Section 3). Sixth, the separator is what the radius was balanced against (Section 4). **Keywords:** sphere of influence, Hill radius, Laplace sphere, celestial mechanics, mass ratio.

Scope of this paper (scope note): No new theorem or law is claimed—the Hill radius and Laplace’s sphere of influence are standard, as is the difference in their exponents.No orbital mechanics is built—perturbations, escape conditions and spacecraft navigation are not the subject; only the three radii and their dependence on the mass ratio.Circular orbits only—Earth’s orbital eccentricity is ignored and the distance from the Sun taken as fixed.No definition is preferred—which to use depends on the purpose, and this paper does not say which is right.Only three are counted—the Roche limit and capture radius are not included, and it is not claimed that three exhausts the list.Relation to earlier papers: Paper 337 showed that the edge of the solar system is five edges spanning a factor of 5106—that is five around the Sun; this is three around the Earth.Paper 490 showed that the fence for the Lagrange points is a mass ratio of 24.96—the same mass ratio moves three radii apart here.Paper 300 showed that a shared root can be decided—the three are distinct roots, and the exponents are the evidence.Paper 481 showed that without separators a refutation has no destination—this paper writes one of them.What is added is placing the three radii in one table, giving the spread as 5.7826, comparing the shrinkage against closed forms, showing the spread itself becoming 18.2556, and showing that the three verdicts split on the Moon.

1. Notation and the Question

Setting: Take a planet of mass m at distance a from a primary of mass M , and write $q = m/M$.“Sphere of influence” has at least three definitions.

$$r_H = a \left(\frac{q}{3} \right)^{1/3}, \quad r_S = a q^{2/5}, \quad \frac{r_E}{a - r_E} = \sqrt{q}$$

From the left: the Hill radius, Laplace’s sphere of influence, and the point where the two gravitational pulls are equal.Below, Earth and Sun with $q = 3.002735 \times 10^{-6}$.

Observation 1.1

A planet’s sphere of influence is taken to be one radius. Inside it the planet rules, outside it the primary—one boundary, runs the picture.

Observation 1.2

But the three above differ in exponent. $1/3, 2/5, 1/2$ —one mass ratio, three numbers.

Question: How far apart are the three, and how do they move with the mass ratio.

2. Three Radii

Computation 2.1: Evaluate all three for the Earth.

Definition	Exponent	Radius
Hill radius	1/3	1,496,433 km
Laplace’s sphere of influence	2/5	924,554 km
Point of equal pull	1/2	258,781 km

Observation 2.2

The Hill radius matches what is measured. 1.496433×10^9 m—agreeing with the familiar figure of about 1.5 million km.

Observation 2.3

The sphere of influence matches too. 9.245539×10^8 m—the roughly 924,000 km used in spacecraft navigation.

Observation 2.4

The three span a factor of 5.7826.Largest against smallest—a sixfold width in the answer to “how far does Earth’s influence reach”.

Observation 2.5

The order is fixed.Hill > sphere of influence > equal pull—for small q , a smaller exponent gives a larger radius.

Observation 2.6

The Moon lies inside two of the three. 3.844×10^8 m—inside the Hill radius and the sphere of influence, outside the point of equal pull. “Does the Moon belong to the Earth” is answered differently by different definitions.

3. Changing the Mass Ratio Shrinks Them Differently

Computation 3.1: Divide q by 1000 and see how far each shrinks.

Definition	Shrinkage	Closed form
Hill radius	10.000000	$1000^{1/3} = 10.000000$
Laplace’s sphere of influence	15.848932	$1000^{2/5} = 15.848932$
Point of equal pull	31.569804	$1000^{1/2} = 31.622777$

Observation 3.2

The first two match their closed forms. 10.000000000 and 15.848931925—the exponent becomes the shrinkage directly.

Observation 3.3

Only the point of equal pull does not match exactly. 31.569804 against 31.622777, differing by 5.297×10^{-2} —because of the $1 + \sqrt{q}$ in its denominator; the smaller q is, the closer it comes to $1000^{1/2}$.

Observation 3.4

The three shrinkages differ from one another. 10.0000, 15.8489, 31.5698—one operation, three answers.

Observation 3.5

So the spread itself changes. From 5.7826 to 18.2556—“the three definitions differ only slightly” holds only at the present mass ratio.

Observation 3.6

The lighter the satellite, the more the definition matters. Smaller q , larger spread—for small bodies, the choice of definition changes the answer greatly.

4. The Separator

The separator: what the radius was balanced against.

Observation 4.1

Balance against the tidal force and the Hill radius results. Exponent $1/3$ —an effective balance in the rotating frame, centrifugal force included.

Observation 4.2

Cut on the relative size of the perturbation and the sphere of influence results. Exponent $2/5$ —a comparison of which body it costs less error to treat as primary.

Observation 4.3

Compare the gravitational pulls themselves and the point of equal pull results. Exponent $1/2$ —the most naive, and the innermost.

Observation 4.4

The three are distinct roots by the criterion of Paper 300. All are called a sphere of influence—but each balances against something different, and that difference shows as the exponent. The exponent is the fingerprint of a distinct root.

Observation 4.5

Paper 337 counted five around the Sun. Spanning a factor of 5106—this counts three around the Earth, spanning 5.7826: the same shape at two nested scales.

Observation 4.6

The same mass ratio as Paper 490's fence is at work. There, one threshold at 24.96—here, the same q moving three radii apart.

Observation 4.7

To be honest—it is not shown that three exhausts the list. The Roche limit and capture radius were not counted—other quantities may also be called a sphere of influence. What is shown is that there are at least three, with different exponents.

5. Roll Call and Closing

1. Hill radius 1,496,433 km (Computation 2.1): **matching measurement.**
2. Sphere of influence 924,554 km (Observation 2.3): **used in navigation.**
3. Equal pull at 258,781 km (Computation 2.1): **the innermost.**
4. A spread of 5.7826 (Observation 2.4): **a sixfold width.**
5. Shrinkages 10.0000, 15.8489, 31.5698 (Computation 3.1): **the core.**
6. Only equal pull misses its closed form (Observation 3.3): **the $1 + \sqrt{q}$ denominator.**
7. The spread becomes 18.2556 (Observation 3.5): **the difference itself moves.**
8. The Moon splits the verdicts (Observation 2.6): **inside two, outside one.**

9. The separator is what was balanced (Observation 4.4): the exponent is the fingerprint.

10. Three is not the whole list (Observation 4.7): others may exist.

Closing: A planet's sphere of influence is taken to be one radius—inside it the planet rules and outside it the primary, one boundary. But there are three definitions with exponents $1/3$, $2/5$ and $1/2$: one mass ratio, three numbers. Three radii—for the Earth, the Hill radius is **1,496,433** km, Laplace's sphere of influence **924,554** km and the point of equal pull **258,781** km. The Hill radius agrees with the familiar 1.5 million km, and the sphere of influence with the roughly 924,000 km used in navigation. The three span a factor of **5.7826**, a sixfold width in the answer to “how far does Earth's influence reach”. The order is fixed—Hill, then sphere of influence, then equal pull—since for small q a smaller exponent gives a larger radius. And the Moon, at 3.844×10^8 m, lies inside the first two and outside the third: “does the Moon belong to the Earth” is answered differently by different definitions. Changing the mass ratio shrinks them differently—dividing q by 1000 shrinks the Hill radius by **10.000000**, the sphere of influence by **15.848932** and the point of equal pull by 31.569804. The first two match $1000^{1/3}$ and $1000^{2/5}$ exactly, the exponent becoming the shrinkage directly. Only the point of equal pull differs, by 5.297×10^{-2} from $1000^{1/2} = 31.622777$, because of the $1 + \sqrt{q}$ in its denominator. The three shrinkages differ from one another: one operation, three answers. So the spread itself changes from 5.7826 to **18.2556**, and “the three definitions differ only slightly” holds only at the present mass ratio. The lighter the satellite, the more the choice of definition matters. One thing separates them—what the radius was balanced against. Against the tidal force gives the Hill radius, exponent $1/3$, an effective balance in the rotating frame with centrifugal force included. Against the relative size of the perturbation gives the sphere of influence, exponent $2/5$, a comparison of which body it costs less error to treat as primary. Against the gravitational pulls themselves gives the point of equal pull, exponent $1/2$, the most naive and the innermost. The three are distinct roots by the criterion of Paper 300: all are called a sphere of influence, but each balances against something different, and that difference shows as the exponent—the fingerprint of a distinct root. Paper 337 counted five around the Sun spanning 5106; this counts three around the Earth spanning 5.7826, the same shape at two nested scales. And the same mass ratio as Paper 490's fence is at work: there one threshold at 24.96, here the same q moving three radii apart. To be honest—it is not shown that three exhausts the list. The Roche limit and capture radius were not counted, and other quantities may also be called a sphere of influence. What is shown is that there are at least three, with different exponents.

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