

The Same Fuel Buys 3.90 Times the Energy Depending on Where It Is Burned

— The fuel fraction required is **0.283469** wherever it is burned — on an orbit of eccentricity **0.6** the gap between perihelion and aphelion is **3.9025**, closing to **1.019865** at eccentricity **0.01** — [Paper 506]

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— *From thinking to syncing.* —

Abstract

Fuel is taken to buy a fixed return. This paper shows that the same fuel yields energies differing by **3.90** depending on where it is burned—the fuel's own equation contains no place. No new theorem or law is claimed. First, a Δv of 1 km/s requires a fuel fraction of **0.283469**, matching $1 - e^{-\Delta v/v_e}$ (Section 2). Second, this is the core of the paper. With the same fuel, the specific energy gained differs by **3.000000** between $v = 3$ km/s and $v = 10$ km/s (Section 2). Third, that ratio matches the closed form $(v_2 + \Delta v/2)/(v_1 + \Delta v/2)$ (Section 2). Fourth, burning from rest gains only $\Delta v^2/2$, which is the floor (Section 2). Fifth, on an orbit of eccentricity **0.6** the gap is **3.9025**—closing to **1.019865** at eccentricity **0.01** (Section 3). Sixth, the separator is whether velocity or energy is being added (Section 4). **Keywords:** Oberth effect, Tsiolkovsky rocket equation, spaceflight, orbital mechanics, propellant.

Scope of this paper (scope note): No new theorem or law is claimed—the Oberth effect and the Tsiolkovsky equation are both standard. No mission is planned—real trajectory design, gravity assists and finite burn times are not the subject; only impulsive burns. Energy conservation is not doubted—including the energy carried off by the propellant, the total is conserved. What is counted here is only what remains with the vehicle. One exhaust velocity— $v_e = 3$ km/s throughout; the choice of specific impulse is not treated. Relativity is excluded—speeds are taken small compared with light. Relation to earlier papers: Paper 483 showed that a radius cancels when two forces share an exponent—this is the converse: a place that does not cancel. Paper 505 showed that the sphere of influence is three spheres—in the same field, here one quantity depends on place. Paper 300 showed that a shared root can be decided—an increment of velocity and an increment of energy are distinct roots. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is giving the fuel fraction as 0.283469 against its closed form, showing a factor of 3.000000 from the same fuel, confirming the ratio against its closed form, giving 3.9025 between perihelion and aphelion, and showing the gap closing to 1.019865.

1. Notation and the Question

Setting: A vehicle moving at speed v receives an impulsive Δv . The gain in energy per unit mass is

$$\Delta E = \frac{1}{2}(v + \Delta v)^2 - \frac{1}{2}v^2 = v \Delta v + \frac{\Delta v^2}{2}$$

while the fuel required follows Tsiolkovsky’s $\Delta v = v_e \ln(m_0/m_1)$, which contains no v . Below, $\Delta v = 1 \text{ km/s}$ and $v_e = 3 \text{ km/s}$.

Observation 1.1

Fuel is taken to buy a fixed return. The same fuel, the same gain—one ledger, runs the picture.

Observation 1.2

But the expression above contains v . The term $v\Delta v$ —absent from the fuel equation, present in the energy equation.

Question: By how much does the same fuel differ in return.

2. The Fuel Is the Same and the Return Is Not

Computation 2.1: Apply $\Delta v = 1 \text{ km/s}$ at three speeds.

Speed v at the burn	Gain in specific energy
3 km/s	$3.500000 \times 10^6 \text{ J/kg}$
10 km/s	$1.050000 \times 10^7 \text{ J/kg}$
30 km/s	$3.050000 \times 10^7 \text{ J/kg}$

Observation 2.2

The fuel required is the same in all three. A fraction of **0.283469**—matching $1 - e^{-\Delta v/v_e} = 0.283468689426$, with v appearing nowhere.

Observation 2.3

Yet the return differs by $3.0000001.050000 \times 10^7$ against 3.500000×10^6 —the same fuel, burned in a different place.

Observation 2.4

The ratio matches its closed form 3.000000000 against $(v_2 + \Delta v/2)/(v_1 + \Delta v/2) = 3.000000000$ —an independent check.

Observation 2.5

Burning from rest gains only $\Delta v^2/2.5.000000 \times 10^5$ J/kg—the floor: even at $v = 0$ the gain is not zero.

Observation 2.6

The faster the burn, the smaller that floor is in relative terms 0.016393 at $v = 30$ km/s—only 1.6 per cent, the rest being $v\Delta v$.

Observation 2.7

The fuel side responds to Δv . Doubling Δv takes the fraction to 0.486583 —it does not respond to v but does to Δv , so the equation is doing its work.

3. Measuring Inside an Orbit

Computation 3.1: Apply the same Δv at perihelion and aphelion of an orbit of eccentricity 0.6.

Position	Speed	Gain in specific energy
Perihelion	59.5694 km/s	6.006938×10^7 J/kg
Aphelion	14.8923 km/s	1.539235×10^7 J/kg
Ratio	4.0000	3.9025

Observation 3.2

Burning at perihelion is 3.9025 times better. The same fuel—only the timing differs.

Observation 3.3

The energy ratio is nearer to one than the speed ratio. 3.9025 against 4.0000—the $\Delta v^2/2$ term sits equally on both sides and dilutes the ratio.

Computation 3.4: Lower the eccentricity.

Eccentricity	Perihelion	Aphelion	Ratio
0.60	59.5694 km/s	14.8923 km/s	3.902549
0.30	40.5897 km/s	21.8560 km/s	1.837973
0.10	32.9282 km/s	26.9413 km/s	1.218173
0.01	30.0840 km/s	29.4883 km/s	1.019865

Observation 3.5

Lowering the eccentricity moves the ratio monotonically to one.
 3.902549 → 1.837973 → 1.218173 → 1.019865—with no step rising.

Observation 3.6

At 0.01 it is still not one. A difference of 1.987×10^{-2} —the other side of the boundary: approaching a circle does not extinguish the effect.

Observation 3.7

So the size of the effect is set by the shape of the orbit. The eccentricity—not the fuel and not the vehicle.

4. The Separator

The separator: is velocity being added, or energy.

Observation 4.1

Velocity, and the place is irrelevant. Δv —Tsiolkovsky's equation contains no v .

Observation 4.2

Energy, and the place matters. $v\Delta v + \Delta v^2/2$ — v is in it.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both may be called “what the burn bought”—one an increment of velocity, one of energy, differing even in units.

Observation 4.4

Conflate them and energy appears from nowhere. The same fuel buying 3.9025 times more—in fact the propellant carries off a different amount depending on where the burn happens.

Observation 4.5

Paper 483 runs the other way. There, matching exponents made the radius cancel—here, the v one might wish to cancel does not. What cancels is decided by where it sits in the expression.

Observation 4.6

This is one of the separators of Paper 481. When “the same fuel gave a different result” is reported—whether the measurement is poor or the quantities compared were different sends the repair elsewhere.

Observation 4.7

To be honest—only what remains with the vehicle is counted. The energy carried off by the propellant is not in the ledger—including it, the total is conserved. The “gain” belongs to the vehicle alone, and no energy appears from nowhere.

5. Roll Call and Closing

1. Fuel fraction **0.283469** (Observation 2.2): **containing no v .**
2. A factor of **3.000000** from the same fuel (Observation 2.3): **the core.**
3. The ratio matches its closed form (Observation 2.4): **an independent check.**
4. **5.000000×10^5 J/kg** even from rest (Observation 2.5): **the floor.**
5. The floor is **1.6 per cent** at **30 km/s** (Observation 2.6): **the rest is $v\Delta v$.**
6. **3.9025** between perihelion and aphelion (Computation 3.1): **eccentricity 0.6.**
7. Nearer to one than the speed ratio (Observation 3.3): **diluted by $\Delta v^2/2$.**
8. **1.019865** at eccentricity **0.01** (Computation 3.4): **monotone approach.**
9. The separator is velocity or energy (Observation 4.3): **distinct roots by Paper 300.**
10. Only the vehicle’s side was counted (Observation 4.7): **the total is conserved.**

Closing: Fuel is taken to buy a fixed return—the same fuel, the same gain, one ledger. But $\Delta E = v\Delta v + \Delta v^2/2$ contains v , which is absent from the fuel equation and present in the energy equation. The fuel is the same and the return is not—applying $\Delta v = 1$ km/s gives **3.500000×10^6 J/kg** at $v = 3$ km/s, **1.050000×10^7** at 10 km/s and **3.050000×10^7** at 30 km/s. The fuel required is the same throughout, a fraction of **0.283469** matching

$1 - e^{-\Delta v/v_e} = 0.283468689426$, with v appearing nowhere. Yet the return differs by **3.000000**, and the ratio matches $(v_2 + \Delta v/2)/(v_1 + \Delta v/2)$. Burning from rest gains only $\Delta v^2/2 = 5.000000 \times 10^5$ J/kg, the floor. The faster the burn, the smaller that floor in relative terms—only 1.6 per cent at 30 km/s. The fuel fraction does not respond to v but does to Δv , moving to 0.486583 when Δv doubles. Measuring inside an orbit—on an orbit of eccentricity 0.6, perihelion at 59.5694 km/s gains **6.006938** $\times 10^7$ J/kg and aphelion at 14.8923 km/s gains **1.539235** $\times 10^7$, a ratio of **3.9025**. The same fuel; only the timing differs. The energy ratio 3.9025 is nearer to one than the speed ratio 4.0000, the $\Delta v^2/2$ term sitting equally on both sides and diluting it. Lowering the eccentricity through 0.60, 0.30, 0.10 and 0.01 gives 3.902549, 1.837973, 1.218173 and **1.019865**, monotonically approaching one. At 0.01 a difference of 1.987×10^{-2} remains, so approaching a circle does not extinguish the effect. The size of the effect is set by the shape of the orbit, not by the fuel or the vehicle. One thing separates them—is velocity being added, or energy. Velocity, and the place is irrelevant: Tsiolkovsky’s equation contains no v . Energy, and the place matters: $v\Delta v + \Delta v^2/2$ has v in it. The two are distinct roots by the criterion of Paper 300—both may be called “what the burn bought”, but one is an increment of velocity and one of energy, differing even in units. Conflate them and the same fuel buying 3.9025 times more looks like energy from nowhere, when in fact the propellant carries off a different amount depending on where the burn happens. Paper 483 runs the other way: there matching exponents made the radius cancel, while here the v does not. What cancels is decided by where it sits in the expression. This is one of the separators of Paper 481: when “the same fuel gave a different result” is reported, whether the measurement is poor or the quantities compared were different sends the repair elsewhere. To be honest—only what remains with the vehicle is counted. The energy carried off by the propellant is not in the ledger, and including it the total is conserved. The “gain” belongs to the vehicle alone, and no energy appears from nowhere.

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