

The Yamagishi Gravitational Multipole Aggregation Theory

— 4th ed.: The Licence Is the Exponent $n=2$ and 4π Does Not Survive into the Answer, but That Licence
Reaches Only as Far as the Shell Theorem — [Paper 51]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

August 2026 (Fourth Edition)

— *From thinking to syncing.* —

ABSTRACT

A distant spherically symmetric mass distribution can be reproduced, as far as its external gravitational field is concerned, by just two numbers: the total mass and the centre of mass — **with zero error**. This is Newton's shell theorem. This paper **pins down the single source** of that aggregation licence. [4th ed.] **That line was too strong**. Through the third edition this paper also called the shell theorem “the ground on which the Barnes--Hut and fast multipole methods compress $O(N^2)$ in N -body computation”. **The shell theorem is an exact statement about spherically symmetric shells, whereas both methods handle asymmetric clumps whose error is never zero**. Indeed **Corollary 4.3 of this very paper already wrote that error as $O(\theta^2)$: the abstract and the body disagreed**. The attribution **reaches the shell theorem and stops there**. FMM's licence is low-rank separability, and the same compression works for kernels whose exponent is not **2** (**$\log r$** , Yukawa, Helmholtz, **$1/r^4$** , a Gaussian); the existence of kernel-independent FMM is itself the counterexample (Paper 124). FMM is moreover $O(N)$, while $O(N \log N)$ is Barnes--Hut — a distinction Theorem 7.1 below already drew correctly. [4th ed.] **Conversely, “ 4π does not survive into the answer” has gained a second leg**. **4π is an overall factor** of the kernel, and the complexity of a hierarchical low-rank method is unchanged by scaling the kernel. Hence **4π cannot enter the complexity** — a proof from computational cost, independent of the geometric cancellation (Paper 124). The first edition attributed it to **4π** , and more precisely to the Yamagishi patch **$G = 4\pi$** (Paper 35). The second edition corrected this: the licence is granted by the exponent **$n = 2$** (flux conservation), not by the value of **G** . The third edition **carries that correction into the theorems themselves**, and then pushes it one step further. The central claim is a single line: **4π does not survive into the answer of the shell theorem**. The flux integral over the sphere is **$g(r) \cdot 4\pi r^2$** ; the right-hand side of Gauss's law is **$-4\pi GM$** ; and **the two 4π 's are the same object appearing twice — the total solid angle Ω_2 — so they cancel exactly** (giving **$g = -GM/r^2$** ; verified symbolically). Hence **4π is not merely “not the licence”; it is absent from the conclusion**. The **4π** that remains in **$g = -4\pi M/r^2$** once the patch **$G = 4\pi$** is substituted is **the convention**, the geometric one having already cancelled. What the first edition pointed at and called geometry was a convention. What licenses aggregation is the exponent. **4π** is what that exponent tells you may be discarded.

Scope of this paper: The theorems used here — the shell theorem, multipole expansion, causal finitisation, the Barnes--Hut method — are **all established content in physics and computer science**. This paper claims no new physical law; it examines only **which quantity supports which conclusion**. **On the $O(N^2)$ paradox (important):** The combinatorial blow-up described in Section 1 is **not an open problem in physics**. In standard field theory no summation over all pairs is performed anywhere; each point responds only to the local field. **This “paradox” arises from the premise that the universe is an N-body simulator, and does not arise outside that premise.**

1. Introduction

Newton's law of universal gravitation prescribes an attraction inversely proportional to the square of the separation r . Applied without restriction to the universe as a whole, it generates a computational paradox. The number of elementary particles in the visible universe is estimated at $N \approx 10^{80}$; computing all pairwise interactions directly at each Planck tick would require

$$\text{Operations per tick} = \frac{N(N-1)}{2} \approx O(N^2) \approx 10^{160} \text{ [ops/tick]}$$

In actual computational practice (large-scale N -body simulation and 3D rendering), the Barnes--Hut spatial partitioning method and the fast multipole method (FMM) are indispensable precisely to avoid this blow-up.

The standing of this “paradox”: The blow-up above is **not an open problem in physics**. It arises as a problem only under the premise that **the universe is a computer that sequentially sums the forces of all particle pairs**. In standard field theory no such computation is performed **anywhere**. Gravity obeys local field equations, and each point **responds only to the field at that point**. Summation over all pairs is not a physical process, and therefore the figure 10^{160} does not exist either.

The problem posed here is thus not a physical paradox but a question internal to the framework of “how things look if the universe is read as a computer”. **The theorems treated here (shell theorem, multipole expansion, causal finitisation) are established physics that holds whether or not that framework is adopted; they do not depend on it.**

1.1 Aggregation, not culling

The first edition of this paper (July 2026) proposed, as the load-reduction mechanism, a Yukawa-type exponential decay

$$V_{\text{screened}}(r) = -\frac{4\pi M}{r} \exp\left(-\frac{r}{r_H}\right) \quad (\text{first ed., withdrawn})$$

The second edition withdrew this entirely. Gravity admits no screening, and introducing an exponential decay conflicts head-on with galactic rotation curves, gravitational lensing, and large-scale structure (Section 2).

However, the conclusion the first edition stated in prose — “**the universe maintains an ideal distributed-computing environment: high precision locally, coarse coupling at distance**” — is correct. What was wrong was the choice of formula expressing it. Indeed, the Barnes--Hut method cited by the first edition is an algorithm that **does not discard force**: it collapses a distant cell to a single centre of mass while preserving the contribution itself. The first edition's own reference supported the revised formula, not its own.

2. Why Gravity Cannot Be Screened: The Mediator Mass

Theorem 2.1 (Yukawa screening does not apply to gravity)

The exponential decay of the Yukawa potential originates in the finite mass of the mediating particle. So long as the mediating field of gravity is massless, no exponential decay term appears in the gravitational potential.

Proof In the potential introduced by Hideki Yukawa (1935) for the nuclear force,

$$V_Y(r) = -g^2 \frac{e^{-m_g c r / \hbar}}{r}$$

the decay length $\bar{\lambda} = \hbar / (m_g c)$ is the **reduced Compton wavelength** of the mediator, and the exponential decay is solely a consequence of the mediator carrying a mass $m_g \neq 0$. For the nuclear force, the pion mass $m_\pi \approx 140 \text{ MeV}/c^2$ gives the finite range $\bar{\lambda}_\pi \approx 1.4 \text{ fm}$. Conversely, in the limit $m_g \rightarrow 0$ one has $\bar{\lambda} \rightarrow \infty$ and

$$\lim_{m_g \rightarrow 0} V_Y(r) = -\frac{g^2}{r}$$

so that the pure inverse-square law is recovered. The mediating field of gravity (the graviton) is theoretically massless, and observationally a strong upper bound follows from dispersion measurements of gravitational waves.

Remark 2.2 (Third edition: which Compton wavelength?)

The second edition wrote the decay length of this theorem as $\lambda = \hbar/(m_g c)$ and then quoted the GW170104 figure $\lambda_g \gtrsim 1.6 \times 10^{13}$ km. **These are not the same quantity.** On recomputation, for $m_g = 7.7 \times 10^{-23}$ eV/ c^2 ,

$$\frac{\hbar}{m_g c} = 2.56 \times 10^{12} \text{ km}, \quad \frac{h}{m_g c} = 1.61 \times 10^{13} \text{ km}$$

and the quoted figure matches **the latter**: the ordinary Compton wavelength $\lambda = h/(mc)$, larger by a factor of 2π . What enters the Yukawa exponent is the **reduced** Compton wavelength $\bar{\lambda} = \hbar/(mc)$, so the formula was right and **only the name and the quoted figure were off by 2π** . The third edition writes the formula as $\bar{\lambda}$ and labels the quoted figure explicitly as $\lambda = h/(mc)$.

This 2π changes no conclusion. The bounds are $m_g \leq 7.7 \times 10^{-23}$ eV/ c^2 from GW170104 ($\lambda_g \gtrsim 1.6 \times 10^{13}$ km) and $m_g \lesssim 1.3 \times 10^{-23}$ eV/ c^2 from the GWTC-3 combined analysis; on either reading the range comfortably exceeds 10^{12} km, and exponential screening of gravity on cosmological scales is not permitted. **Even where it does not affect the conclusion, a quantity is to be called by its right name** — a check this corpus has used repeatedly (Paper 16).

Corollary 2.3 (Conflict with observation)

Assuming a screening e^{-r/r_H} with the Hubble radius r_H as decay length conflicts with the following established observations:

1. **Cluster lensing**: the strength with which Mpc-scale mass distributions deflect background light agrees with an undamped $1/r^2$ law.
2. **Growth of large-scale structure**: the linear growth rate of density perturbations matches observation on the premise that long-range gravity is undamped.
3. **Galactic rotation curves**: were the contribution of distant mass lost exponentially, the flatness of rotation curves would fall into even sharper conflict.

The route of achieving computational savings by modifying the law of gravity is therefore physically closed. **The resolution must be sought in the geometric structure of the law itself.**

3. Flux Structure and the Aggregation Licence

This section is the core of the paper. The ground for coarse-graining distant mass is not an extrapolative approximation: it **follows exactly from gravity being a flux-conserving law**. The third edition decomposes that “flux conservation” to the end, asking **which quantity is actually doing the work**.

3.1 The licence is granted by $n = 2$

The first edition stated that “the geometric structure $G = 4\pi$ functions as the licence for aggregation”. The second edition corrected this attribution. The check is simple — **delete the constant from the formula and see whether the conclusion changes**:

Conclusion	Setting $G = 1$
Shell theorem (Thm. 3.2)	$g = -GM/r^2$. Holds independently of the value of G
Multipole error $O(\theta^2)$ (Thm. 4.1)	G does not appear
$O(N \log N)$ compression (Thm. 7.1)	G does not appear
Causal finitisation (Thm. 5.1)	Depends on c . Unrelated to G

What, then, grants the licence? **The inverse-square law itself: the exponent $n = 2$** . The flux through a sphere is $\Phi(r) = 4\pi r^2 g(r)$, so with $g \propto r^{-n}$,

$$\Phi(r) \propto 4\pi r^{2-n}$$

and Φ is conserved independently of r only for $n = 2$ (verified numerically: of $n = 1, 3/2, 2, 5/2, 3$, only $n = 2$ is r -independent). And it is because the flux is conserved that the details of the distribution inside a closed surface do not appear outside it — which is the shell theorem.

This is the same condition as the foundation of the corpus. Paper 01 (the $n = d - 1$ uniqueness theorem), Paper 10 ($dm/dr = 4\pi r^{2-n}$), and Paper 41 Section 2 (flux conservation of spherical waves) all treat the same algebraic condition r^{2-n} . **The aggregation licence of this paper reduces to that same condition.**

3.2 [Third edition] 4π does not survive into the answer

The second edition stopped here, saying only that “ 4π is not what grants the licence”. **One step further can be taken. 4π is not merely not the licence: it is absent from the conclusion of the shell theorem.**

Theorem 3.1 (Gauss flux capacity)

Gauss's law for gravity gives the flux through a closed surface as

$$\oint_{\partial V} \mathbf{g} \cdot d\mathbf{A} = -4\pi G M_{enc}$$

The 4π appearing here is **the total solid angle Ω_2 of three-dimensional space**, a **fact** independent of the choice of unit system (Paper 35, third edition). The value of G is a **convention** fixed by the unit system, and constrains nothing in this equation.

Theorem 3.2 (Exactness of shell aggregation)

When a spherically symmetric mass distribution $\rho(\mathbf{r}')$ is confined within radius R , the gravitational field at $r > R$ **coincides exactly** (zero error) with that of a point mass $M = \int \rho dV$ at the centre of mass.

Proof By spherical symmetry $\mathbf{g}$ is radial and takes a constant value $g(r)$ on the sphere of radius r . The surface integral on the left yields the area $4\pi r^2$ of the sphere; equating with the right-hand side of Theorem 3.1,

$$g(r) \cdot 4\pi r^2 = -4\pi G M \implies g(r) = -\frac{GM}{r^2}$$

The 4π on the left comes from **the area of the sphere**, the 4π on the right from **the normalisation of Gauss's law**. The result depends in no way on the details of $\rho(\mathbf{r}')$, being fixed by the total mass M alone. This is precisely the shell theorem Newton proved in the *Principia* (1687), Book I, Proposition LXXI.

Corollary 3.3 (The centre of the third edition: the cancellation of 4π)

In the derivation of Theorem 3.2, **4π appears on both sides and cancels completely**. The conclusion $g = -GM/r^2$ contains no π (verified symbolically).

This cancellation is no coincidence. **The two 4π 's are the same object appearing twice**: the one on the left is the solid angle contained in the area of the sphere of radius r ; the one on the right is the solid angle contained in the normalisation of Gauss's law. Both are the same geometric quantity $\Omega_2 = 4\pi$. Placing the same thing on both sides makes it cancel, as it must — so **in the shell theorem 4π is not a quantity that does work, but a quantity that balances on both sides and leaves**.

Corollary 3.4 (The attribution was inverted)

Substituting the Yamagishi patch $G = 4\pi$ (Paper 35) gives

$$g(r) = -\frac{4\pi M}{r^2}$$

so that a 4π does appear in the answer. **But this 4π is a convention, not geometry.** It is the substituted value of G itself; the solid angle has already cancelled and gone.

The first edition pointed at the 4π in this expression and called it “the geometric structure that licenses aggregation”. The 4π it pointed at is the convention; the geometric 4π had left the equation by then. **The attribution was not merely wrong: it was inverted.** This is the heart of the third edition's correction.

Remark 3.5 (The two 4π 's must not be multiplied)

The second edition's Theorem 3.1 wrote $-4\pi GM = -(4\pi)(4\pi)M = -16\pi^2 M$, stating that “the first 4π is the total solid angle, the second is the mass patch” — **naming them correctly and then multiplying them together.** If the naming is right but the product is formed, the names are lost on the next line. What Paper 35's third edition forbids is exactly this merger — **fold a fact and a convention into one number and the record of provenance disappears** — and so the third edition forms no product, keeping 4π (fact) and G (convention) apart. Only the former cancels in Theorem 3.2; the latter remains in the conclusion as the letter G . It is because this separation was maintained that the inversion of Corollary 3.4 became visible.

Remark 3.6 (The cancellation depends on $n = 2$)

4π can cancel only because there is something for it to cancel against, namely Gauss's law; and Gauss's law holds only when the flux is r -independent, that is, only for $n = 2$ (Section 3.1). **The exponent is the condition; 4π is what that condition tells you may be discarded.** The cancellation is moreover dimension-independent: in d spatial dimensions Ω_{d-1} appears identically on both sides and cancels for every d , while what changes with d is the exponent $n = d - 1$ (Paper 01).

Corollary 3.7 (What the licence means)

Theorem 3.2 has a decisive meaning for a universe OS. In handling a distant mass aggregate such as a galaxy, the OS need not retain the coordinates of the constituent stars individually: **two quantities, the total mass and the centre of mass, reproduce exactly the same external gravitational field**. This aggregation is not an approximation but an exact consequence of flux conservation. **And its condition is $n = 2$.**

4. Multipole Aggregation and Error Control

Real mass distributions are not perfectly spherically symmetric, so aggregation incurs a controlled error. We quantify its structure by multipole expansion.

Theorem 4.1 (Multipole expansion and the $O(\theta^2)$ error): The potential exerted at an observation point at distance $d \gg s$ by a mass group distributed within a cell of size s is, by multipole expansion about the cell's centre of mass,

$$\Phi(d) = -\frac{G}{d} \left[M_{\text{cell}} + \frac{1}{2d^2} Q_{ij} \hat{n}_i \hat{n}_j + O\left(\frac{s^3}{d^3}\right) \right]$$

where the dipole term vanishes identically by choosing the centre of mass as expansion centre. With the opening angle $\theta \equiv s/d$, the relative error of the monopole approximation is bounded by

$$\frac{|\Delta\Phi|}{|\Phi|} = O(\theta^2)$$

Proof The quadrupole moment $Q_{ij} = \sum_k m_k (3x_{k,i}x_{k,j} - r_k^2 \delta_{ij})$ satisfies $|Q| \lesssim M_{\text{cell}} s^2$. The ratio of the second term to the first is therefore $\frac{Ms^2/d^2}{2M} \sim \frac{1}{2} (s/d)^2 = \frac{1}{2} \theta^2$.

Remark 4.2 (Third edition: here too G is an overall factor)

In the expression above, G stands outside the bracket as an **overall factor**, and cancels when the relative error $|\Delta\Phi|/|\Phi|$ is taken. That is, **the value of G enters neither the error estimate nor the acceptance criterion**. The second edition wrote this G as 4π ; written or not, the θ criterion is the same thing. **A quantity that enters a normalised quantity only as an overall factor is, with respect to that quantity, untestable** (Paper 35).

Corollary 4.3 (Acceptance criterion)

The universe OS applies monopole aggregation only to cells whose opening angle falls below a threshold ($\theta < \theta_{\text{crit}}$). At the standard Barnes--Hut value $\theta_{\text{crit}} \approx 0.5$, the error of an individual interaction stays at $O(\theta^2)$, and after statistical cancellation across many cells the relative error of the net force is in practice within 10^{-2} .

Crucially, this mechanism discards no force. The contribution of a distant cell is **fully retained** as a monopole (and, as needed, a quadrupole). What is reduced is “the number of degrees of freedom to be retained”, not “the strength of the interaction”. This is the decisive difference from the first edition's Yukawa screening.

5. Exact Finitisation of the Source Set by the Causal Horizon

Independently of aggregation, a mechanism exists that finitises the source set exactly. This is not an approximation but an exact consequence of relativity.

Theorem 5.1 (Finiteness of the causal source set)

Because gravity propagates at the speed of light c , the sources that can contribute to the gravitational field at time t and position $\mathbf{x}$ are confined to the interior of the past light cone:

$$\Phi(\mathbf{x}, t) = -G \int \frac{\rho(\mathbf{x}', t_{\text{ret}})}{|\mathbf{x} - \mathbf{x}'|} d^3x', \quad t_{\text{ret}} = t - \frac{|\mathbf{x} - \mathbf{x}'|}{c}$$

By this retarded-time structure (Paper 49), the total number of contributing sources is strictly limited to the finite number within the particle horizon.

Remark 5.2

This horizon is **a boundary of the source set**, not something that attenuates contributions inside the boundary. The first edition's e^{-r/r_H} was wrong precisely in misusing the horizon as “a decay length that gradually weakens the interior”. Correctly, it is “a causal excision that excludes the exterior”. **What does the finitising is the finiteness of c ; G plays no part here either.**

6. A Significance Threshold from Planck-Length Displacement

Theorem 6.1 (Threshold of unobservable force)

If a force $\mathbf{F}$ acting on a particle of mass m produces, accumulated over the whole age of the universe $T \approx 4.35 \times 10^{17}$ s, a displacement smaller than the Planck length l_P , then that force is unobservable in principle. The threshold is

$$\frac{1}{2} \frac{F}{m} T^2 < l_P \implies F_{\min} < \frac{2m l_P}{T^2}$$

Substituting the proton mass $m_p = 1.673 \times 10^{-27}$ kg,

$$F_{\min} \approx \frac{2(1.673 \times 10^{-27})(1.616 \times 10^{-35})}{(4.35 \times 10^{17})^2} \approx 2.858 \times 10^{-97} \text{ N}$$

That is, a force below $\sim 10^{-97}$ N produces no displacement at the minimal length scale even over the entire history of the universe (verified numerically).

Remark 6.2 (Relation to the first edition's figure)

The first edition stated that its $\epsilon_{\text{cutoff}} \approx 10^{-100}$ N “corresponded in order of magnitude to this physical lower bound”. **That assessment is too generous.** The ratio is **2858**, a gap of **3.46 orders of magnitude** (verified numerically) — beyond what can be called “corresponding in order of magnitude”. Precisely, **the first edition's 10^{-100} is a figure without foundation, derived independently of the 2.858×10^{-97} of this section.** That the two landed at nearby orders does not rise above coincidence.

Remark 6.3 (The standing of this section)

More importantly, **this section is not necessary to the revised argument.** As Sections 4 and 5 showed, **the revised mechanism discards no force at all** — distant contributions are fully retained as monopoles. A “threshold of unobservable force” is therefore **a tool the first edition needed in order to justify discarding**, and is not needed for compression by aggregation. The section is retained because the bound $\sim 10^{-97}$ N is **an independently meaningful physical scale, not because it supports any claim of this paper.**

7. Compression of Computational Complexity

Theorem 7.1 (Complexity compression)

Combining hierarchical spatial partitioning by octree with the acceptance criterion of Corollary 4.3, the interaction cost per step of an N -body system is compressed as

$$O(N^2) \xrightarrow[\text{Multipole Aggregation}]{\text{Octree}+\theta\text{-criterion}} O(N \log N)$$

Using in addition the fast multipole method (Greengard & Rokhlin, 1987), the interconversion of local and multipole expansions achieves $O(N)$, i.e. **linear in the particle count**.

Proof The depth of the octree is $O(\log N)$. For each particle, the set of cells satisfying the acceptance criterion is bounded at each depth of the tree (by a constant depending on θ_{crit}), so the number of interactions per particle is $O(\log N)$. Summing over all particles gives $O(N \log N)$.

8. Discussion

The first edition reasoned that “since computation breaks down, the law of gravity must be modified”. What the revisions show is the fact that **“the law of gravity is already written in a computable form, requiring no modification”**. The exactness of the shell theorem, the rapid convergence of the multipole expansion, the causal excision by the finiteness of c — all are established physics. [4th ed.] **What makes the $O(N \log N)$ compression possible, however, is the second of the three — the convergence of the multipole expansion — and not the shell theorem.** What the shell theorem grants is zero error for a symmetric shell, and no cell of the tree is symmetric.

8.1 Where the attribution moved across three editions

	What licenses aggregation	Standing of 4π
1st ed.	$G = 4\pi$ (Yamagishi patch)	geometric structure serving as licence
2nd ed.	exponent $n = 2$ (flux conservation)	a solid angle that is “not the licence”
3rd ed.	exponent $n = 2$ (flux conservation)	absent from the conclusion (cancels)
4th ed.	exponent $n = 2$ (as far as the shell theorem)	absent (cancels + an overall factor)

The move from the second to the third edition looks small but differs in kind. The second edition **demoted 4π** : “it is not the licence”. What the third edition shows is **absence** — 4π never appears in the conclusion of the shell theorem at all.

Demotion is a matter of degree; absence is a matter of presence or not.

Further, the identity of the 4π the first edition was pointing at has been settled (Cor. 3.4). The 4π that remains in the answer once the patch $G = 4\pi$ is inserted is **the convention**; the geometric 4π cancelled before that. **The first edition was pointing at a convention and calling it geometry**. The second edition correctly said “the value of G does not contribute”, but **had not yet shown why, as a cancellation between the two sides**.

8.2 What this correction returns to the corpus

This correction is not a retreat. $n = 2$ is the very thing treated by the uniqueness theorem of Paper 01, so **this paper connects to the foundation**. The connection is stronger than under the first edition's claim of connecting to the patch $G = 4\pi$.

In addition, the paper returns one check to the corpus. **On seeing an expression containing 4π , first distinguish whether it is a solid angle that appears on both sides and cancels, or a number substituted as a convention**. The corpus carries many papers about 4π (Papers 17, 35, 45, 67). What this paper experienced is the failure of confusing the two and reading it as “geometry at work” — and **the confusion happened after the naming** (Remark 3.5). Writing the name correctly is not enough; **not forming a product after writing the name is itself part of the check**.

9. Conclusion

Against the $O(N^2)$ computational paradox generated by the long-range character of universal gravitation, this paper has shown the following.

1. A resolution by Yukawa-type exponential screening is physically untenable, conflicting with the masslessness of the mediator and with cosmological observations (Thm. 2.1, Cor. 2.3). The corresponding formulation of the first edition stands withdrawn.
2. By Gauss's law and the shell theorem, aggregating a distant spherically symmetric mass at its centre of mass is **licensed exactly, with zero error** (Thms. 3.1, 3.2). **What grants this licence is the exponent $n = 2$ (flux conservation), not the value of G** (Section 3.1).
3. **4π does not survive into the conclusion of the shell theorem**. The same solid angle Ω_2 appears in the area of the sphere and in the normalisation of Gauss's law, and cancels completely (Cor. 3.3, verified symbolically). The 4π remaining in the answer once the patch $G = 4\pi$ is substituted is **a convention, not geometry**, and **the first edition's attribution was inverted** (Cor. 3.4).

4. For general distributions, the aggregation error is controlled at $O(\theta^2)$ by multipole expansion. The mechanism conserves force and reduces only the number of degrees of freedom (Thm. 4.1, Cor. 4.3). G cancels as an overall factor and does not enter the criterion (Rem. 4.2).
5. The causal horizon finitises the source set exactly, but does not attenuate its interior (Thm. 5.1).
6. A threshold of unobservable force $\sim 2.858 \times 10^{-97}$ N follows from Planck-length displacement (Thm. 6.1, verified numerically). **It is however 3.46 orders from the first edition's 10^{-100} and gives it no retrospective support; and the threshold is not necessary to the revised argument** (Section 6).
7. Degrees of freedom are thereby compressed from $O(N^2)$ to $O(N \log N)$ (to $O(N)$ with FMM) (Thm. 7.1).

9.1 Scope

- **What this paper shows:** that if the flux is conserved ($n = 2$) the shell theorem licenses the aggregation of distant mass exactly; that the error is controlled at $O(\theta^2)$; that causal structure finitises the source set; **and that 4π survives into none of these conclusions. The first three are established physics and computer science, organised and re-verified here. The last is a correction to this paper's own first edition.**
- **What this paper does not show:** (i) that $G = 4\pi$ contributes to aggregability — every theorem holds with $G = 1$, and 4π cancels away (Section 3); (ii) the correctness of the premise that the universe actually performs an all-pairs summation — **the $O(N^2)$ paradox arises only inside the framework** (Section 1); (iii) any justification of the first edition's 10^{-100} N (Section 6); (iv) any change whatever to the law of gravity.
- **What remains:** that coarse-graining is exactly licensed and that gravity obeys $n = 2$ are the same algebraic fact. This $n = 2$ is the condition treated in Papers 01, 10 and 41, so **this paper connects to the foundation of the corpus.**

The implication is that one need not “cut gravity off” to make computation light: **that gravity is an inverse-square law is itself the built-in mechanism that compresses degrees of freedom without any truncation.** And what issues the licence is the exponent, not a constant.

Revision Record

Corrections added in the fourth edition (August 2026)

1. **The reach of the aggregation licence was measured (Abstract, Section 8):** through the third edition the abstract called the shell theorem “the ground on which Barnes--Hut and the fast multipole method compress the cost”. **That line was too strong.** The shell theorem is a **zero-error** statement about **spherically symmetric** shells, whereas both methods handle asymmetric clumps whose error is never zero. The attribution reaches the shell theorem and stops.
2. **The disagreement between abstract and body was resolved:** Corollary 4.3 already wrote “the error of an individual interaction is $O(\theta^2)$ ”. The fact the fourth edition points out was thus **already inside this paper**; only the abstract still claimed zero error.

3. **FMM's licence was named:** low-rank separability. The same compression works for kernels whose exponent is not **2**, and **the existence of kernel-independent FMM is itself the counterexample** (Paper 124).
4. **$O(N)$ and $O(N \log N)$ were separated in the abstract too:** FMM is $O(N)$, Barnes--Hut is $O(N \log N)$. Theorem 7.1 already drew the distinction in the body.
5. **Evidence from complexity was added to “ 4π does not survive”:** 4π is an overall factor of the kernel and does not change the numerical rank at a relative threshold. **The claim is not weakened but given a second leg** (Paper 124).

Corrections added in the third edition (August 2026)

1. **Subtitle:** the second edition's subtitle read “How the Gaussian Solid Angle 4π Licenses ...”. **The subtitle still advertised an attribution the second edition's own body (Section 3.1) had already withdrawn.** The third edition keeps the main title and changes only the subtitle, to what the paper shows: the exponent $n = 2$.
2. **Theorems 3.1 and 3.2 rewritten free of G :** immediately after stating in Section 3.1 that “every theorem holds with $G = 1$ ”, the second edition substituted $G = 4\pi$ in Theorems 3.1 and 3.2 and proved the shell theorem using $16\pi^2$. **The correction had not been carried into the body's theorems.** The third edition keeps G as a letter.
3. **The cancellation of 4π newly shown (Cors. 3.3, 3.4):** 4π appears on both sides as the same solid angle, cancels completely, and does not survive into the conclusion. The 4π remaining in the answer under the patch is **a convention**. **The first edition's attribution was not “wrong” but “inverted”.**
4. **The product of the two 4π 's dissolved (Rem. 3.5):** the second edition's $-4\pi G M = -16\pi^2 M$ multiplied a fact (solid angle) and a convention (patch) together after naming both correctly. This is the merger forbidden by Paper 35's third edition; the third edition forms no product.
5. **The 2π of the Compton wavelength corrected (Rem. 2.2):** the second edition wrote the decay length as $\hbar/(m_g c)$ while quoting figures corresponding to $h/(m_g c)$. The formula was right; the name and quoted figure were off by 2π . No conclusion changes.
6. **G restored as a letter in Theorems 4.1 and 5.1:** the second edition had substituted 4π here too. G cancels from the relative error as an overall factor (Rem. 4.2).
7. **Numbers re-verified:** $F_{\min} = 2.858 \times 10^{-97}$ N; the gap to the first edition's figure is **3.46** orders (refining the second edition's “3.5”); flux conservation only at $n = 2$; the cancellation of 4π (symbolic).
8. **DOIs added to the self-citations in the bibliography.**

Claims withdrawn or corrected in the second edition (unchanged)

1. **Yukawa-type exponential screening e^{-r/r_H} :** withdrawn (Section 2). Gravity admits no screening.
2. **“The licence for aggregation is $G = 4\pi$ ”:** corrected (Section 3.1). The value of G contributes to none of the conclusions.
3. **“The 10^{-100} N was given order-of-magnitude support”:** withdrawn (Section 6). The two are independent figures.

References

1. Yamagishi, Y. (2026). *Three Distinct Roots of Aggregation* [Paper 124]. Zenodo. 10.5281/zenodo.22057728
2. Newton, I. (1687). *Philosophiæ Naturalis Principia Mathematica*. Book I, Proposition LXXI (Shell Theorem). London: Royal Society.
3. Yukawa, H. (1935). On the Interaction of Elementary Particles. I. *Proceedings of the Physico-Mathematical Society of Japan. 3rd Series*, 17, 48--57.
4. Barnes, J., & Hut, P. (1986). A hierarchical $O(N \log N)$ force-calculation algorithm. *Nature*, 324(6096), 446--449.
5. Greengard, L., & Rokhlin, V. (1987). A fast algorithm for particle simulations. *Journal of Computational Physics*, 73(2), 325--348.
6. Abbott, B. P. et al. (LIGO Scientific and Virgo Collaborations) (2017). GW170104: Observation of a 50-Solar-Mass Binary Black Hole Coalescence at Redshift 0.2. *Physical Review Letters*, 118, 221101.
7. Abbott, R. et al. (LIGO Scientific, Virgo and KAGRA Collaborations) (2021). Tests of General Relativity with GWTC-3. arXiv:2112.06861.
8. Yamagishi, Y. (2026). *The Uniqueness Theorem of $\mathbf{n} = \mathbf{2}$ for Spherically Symmetric Density Distributions* [Paper 01]. Zenodo. 10.5281/zenodo.21403186
9. Yamagishi, Y. (2026). *Only Dimensionless Numbers Can Be Fine-Tuned* [Paper 10], 3rd ed.. Zenodo. 10.5281/zenodo.21404556
10. Yamagishi, Y. (2026). *The Equals Sign Has Distinct Roots* [Paper 16], 3rd ed.. Zenodo. 10.5281/zenodo.21600471
11. Yamagishi, Y. (2026). *The 8π Is an Artefact of the Units* [Paper 17], 3rd ed.. Zenodo. 10.5281/zenodo.21600813
12. Yamagishi, Y. (2026). *The Yamagishi Gravitational Constant Unpacking Theory* [Paper 35], 3rd ed.. Zenodo. 10.5281/zenodo.21430118
13. Yamagishi, Y. (2026). *The Fringes Do Not Remember the Exponent* [Paper 41], 3rd ed.. Zenodo. 10.5281/zenodo.21436048
14. Yamagishi, Y. (2026). *An Area Is Not a Solid Angle* [Paper 45], 3rd ed.. Zenodo. 10.5281/zenodo.21602337
15. Yamagishi, Y. (2026). *2.5PN Is the First Dissipative Order Because Mass Conservation Kills $\mathbf{n} = \mathbf{1}$* [Paper 49], 3rd ed.. Zenodo. 10.5281/zenodo.21602658
16. Yamagishi, Y. (2026). *The Euler-Characteristic Origin of the 4π Patch* [Paper 67]. Zenodo. 10.5281/zenodo.21642170
17. Yamagishi, Y. (2026). *Convention and Fact* [Paper 95]. Zenodo. 10.5281/zenodo.21758621

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)