

The Same 19.471221 Degrees Arises from Two Distinct Roots

— the half-angle of a ship's wake, $\arcsin(1/3) = 19.471221$ degrees, is a constant containing no speed, while the Mach angle $\arcsin(1/M)$ reaches the same value at the single point $M = 3$ — [Paper 515]

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September 2026

— *From thinking to syncing.* —

Abstract

When two angles agree, one mechanism is readily assumed to be at work. This paper shows that 19.471221 degrees arises for two separate reasons—one independent of speed, the other taking that value only at $M = 3$. No new theorem or law is claimed. First, a ship's wake opens at $\arcsin(1/3) = 19.471221$ degrees (Section 2). Second, the expression contains no speed— $\arcsin(1/3)$ is a constant (Section 2). Third, its ground is that the phase speed is exactly **2** times the group speed, confirmed at three wavenumbers (Section 2). Fourth, the Mach angle depends on speed, giving 41.810315 degrees at $M = 1.5$ and 11.536959 at $M = 5$ (Section 3). Fifth, this is the core of the paper. At $M = 3$ the Mach angle is **19.471220634** degrees, differing from the wake's half-angle by 0.000×10^0 rad (Section 3). Sixth, the separator is whether the angle is a constant or one point of a variable (Section 4).

Keywords: Kelvin wake angle, Mach angle, ship wake, shock wave, fluid dynamics.

Scope of this paper (scope note): No new theorem or law is claimed—the Kelvin wake angle and the Mach angle are both standard. No fluid dynamics is built—the wake pattern itself, viscosity, finite depth and hull shape are not the subject; only the dispersion of deep-water gravity waves. Shock waves are excluded—the Mach angle is used as the geometry of sound propagation; shock structure and strength are not entered. Shallow water is excluded—in shallow water the dispersion changes and so does the half-angle; only the deep case is examined. No mechanism is offered for the agreement—it is not claimed that the coincidence at $M = 3$ has a common cause; only that the numbers are the same. Relation to earlier papers: Paper 485 showed that Michell’s radius was right for a different reason—there one expression came from two derivations and broke at the neighbouring quantity; here one number comes from two roots and breaks when the speed is varied. Paper 300 showed that a shared root can be decided—another instance of the criterion that numerical agreement alone establishes nothing. Paper 491 showed that the classical Doppler effect is two formulas—both are cases where what the wave speed depends on splits the answer. Paper 481 showed that without separators a refutation has no destination—this paper writes one of them. What is added is giving the half-angle as 19.471221 degrees, confirming the phase-to-group ratio as 2.000000000 at three wavenumbers, showing the Mach angle moving across four speeds, giving the agreement at $M = 3$ to 0.000×10^0 rad, and placing the separator at constant versus a point of a variable.

1. Notation and the Question

Setting: A ship moving through deep water leaves a wake that opens as a wedge, of half-angle

$$\theta_K = \arcsin \frac{1}{3}$$

while a body moving supersonically leaves a cone of half-angle

$$\theta_M = \arcsin \frac{1}{M}$$

with M the Mach number. The forms match, but one carries a speed and the other does not. The two are set side by side below.

Observation 1.1

When two angles agree, one mechanism is readily assumed to be at work. The same number—so presumably from the same cause, runs the picture.

Observation 1.2

But one is a constant and the other a variable. $\arcsin(1/3)$ against $\arcsin(1/M)$ —they agree only at $M = 3$.

Question: What does the agreement mean.

2. The Wake Angle Contains No Speed

Computation 2.1: Evaluate the half-angle.

$$\theta_K = \arcsin \frac{1}{3} = 0.339836909 \text{ rad} = 19.471221 \text{ degrees}$$

Observation 2.2

It matches the known value. 19.471221 degrees—agreeing with the familiar 19.47.

Observation 2.3

No speed enters the expression. $\arcsin(1/3)$ —a rowing boat and a large ship open the same wedge.

Computation 2.4: For deep-water gravity waves, evaluate the ratio of phase speed to group speed at three wavenumbers.

Wavenumber k	Phase speed	Group speed	Ratio
0.5	4.428691	2.214345	2.000000000
2.0	2.214345	1.107173	2.000000000
10.0	0.990285	0.495143	2.000000002

Observation 2.5

All three ratios are 2.2.000000000—the trailing digit comes from the finite-difference step, and the ratio does not depend on k .

Observation 2.6

That 2 produces the $1/3 \sin \theta_K = 0.33333333333333$ against $1/3 = 0.33333333333333$ —following from the dispersion relation $\omega = \sqrt{gk}$ alone.

Observation 2.7

So the disappearance of the speed is no accident. Because the dispersion goes as $\sqrt{k}$ —the speed decides which wavenumbers remain in the wake, not how wide it opens.

3. The Mach Angle Depends on Speed

Computation 3.1: Evaluate the Mach angle at four speeds.

Mach number M	Half-angle
1.5	41.810315 degrees
2.0	30.000000 degrees
3.0	19.471221 degrees
5.0	11.536959 degrees

Observation 3.2

It moves greatly with speed. From 30.0000 to 11.5370 degrees—sharper the faster it goes.

Observation 3.3

At $M = 3$ it is exactly the wake's half-angle. 19.471220634 against 19.471220634 degrees, differing by 0.000×10^0 rad—not even within rounding, because it is the same expression $\arcsin(1/3)$.

Observation 3.4

Yet only one of them moves. The Mach angle by 0.1838 rad, the wake's half-angle not at all—the agreement occurs at the single point $M = 3$.

Observation 3.5

The physics upholding them differs. Sound is a non-dispersive wave, water gravity waves are dispersive—in the first a signal travels at one speed, in the second every wavenumber travels at its own.

Observation 3.6

The $1/3$ comes from different places. For Mach, a ratio of speeds $1/M$; for the wake, a $1/3$ arising from the group-to-phase ratio of 2—both sit inside an $\arcsin$, but their origins differ.

4. The Separator

The separator: is the angle a constant, or one point of a variable.

Observation 4.1

A constant does not move with speed. The wake—the same angle at every speed.

Observation 4.2

One point of a variable departs when moved. The Mach angle—step away from $M = 3$ and the value is different at once.

Observation 4.3

The two are distinct roots by the criterion of Paper 300. Both are $\arcsin(1/3) = 19.471221$ degrees—one a property of dispersive waves, the other the geometry of non-dispersive ones: the physics has nothing in common.

Observation 4.4

Varying the speed is the test. Moving means a point of a variable; not moving means a constant—one point alone cannot tell them apart.

Observation 4.5

It has the shape of Paper 485. Michell's radius was right and broke at the neighbouring quantity—there one expression from two derivations, broken by light bending; here one number from two roots, broken by varying the speed.

Observation 4.6

This is one of the separators of Paper 481. When “the same angle came out, so it is the same phenomenon” is questioned—whether the measurement was wrong or the agreement held at one point only sends the repair elsewhere.

Observation 4.7

To be honest—no mechanism is shown for the agreement. They agree at $M = 3$ because both arguments of the $\arcsin$ become $1/3$ —no common reason has been found for why the water side selects $1/3$ and why $M = 3$ is what meets it on the sound side. What is shown is that the agreement occurs at one point only.

5. Roll Call and Closing

1. A half-angle of 19.471221 degrees (Computation 2.1): **matching the known value.**
2. No speed in the expression (Observation 2.3): **the same for any ship.**

3. Phase speed twice the group speed (Computation 2.4): at three wavenumbers.
4. That 2 produces the $1/3$ (Observation 2.6): because dispersion goes as $\sqrt{k}$.
5. Mach from 41.81 to 11.54 degrees (Computation 3.1): moving with speed.
6. Agreement to 0.000×10^0 rad at $M = 3$ (Observation 3.3): the core.
7. Only one of them moves (Observation 3.4): by 0.1838 rad.
8. Different physics upholds them (Observation 3.5): dispersive or not.
9. The separator is constant or a point (Observation 4.3): distinct roots by Paper 300.
10. No mechanism for the agreement (Observation 4.7): it holds at one point.

Closing: When two angles agree, one mechanism is readily assumed to be at work—the same number, so presumably the same cause. But one is a constant and the other a variable: $\arcsin(1/3)$ and $\arcsin(1/M)$ agree only at $M = 3$. The wake angle contains no speed—it is $\arcsin(1/3) = 0.339836909$ rad, or **19.471221** degrees, agreeing with the familiar 19.47. No speed enters, so a rowing boat and a large ship open the same wedge. The ground is the dispersion: for deep-water gravity waves the phase speed is exactly 2 times the group speed, giving 2.000000000 at $k = 0.5, 2.0$ and 10.0 alike. That 2 gives $\sin \theta_K = 0.3333333333333333$, matching $1/3$, and follows from $\omega = \sqrt{gk}$ alone. So the disappearance of the speed is no accident: the speed decides which wavenumbers remain in the wake, not how wide it opens. The Mach angle depends on speed—it is 41.810315 degrees at $M = 1.5$, 30.000000 at 2.0, **19.471221** at 3.0 and 11.536959 at 5.0, sharper the faster it goes. At $M = 3$ it is exactly the wake's half-angle: 19.471220634 against 19.471220634 degrees, differing by 0.000×10^0 rad, because it is the same expression $\arcsin(1/3)$. Yet only one of them moves—the Mach angle by 0.1838 rad, the wake's not at all—so the agreement occurs at the single point $M = 3$. The physics upholding them differs: sound is non-dispersive and water gravity waves are dispersive. And the $1/3$ comes from different places: a ratio of speeds for Mach, and the group-to-phase ratio of 2 for the wake. One thing separates them—is the angle a constant, or one point of a variable. A constant does not move with speed, and the wake gives the same angle at every speed. One point of a variable departs when moved, and stepping away from $M = 3$ changes the value at once. The two are distinct roots by the criterion of Paper 300: both are $\arcsin(1/3) = 19.471221$ degrees, but one is a property of dispersive waves and the other the geometry of non-dispersive ones. Varying the speed is the test, and one point alone cannot tell them apart. It has the shape of Paper 485: there one expression from two derivations, broken by light bending; here one number from two roots, broken by varying the speed. This is one of the separators of Paper 481: when “the same angle came out, so it is the same phenomenon” is questioned, whether the measurement was wrong or the agreement held at one point only sends the repair elsewhere. To be honest—no mechanism is shown for the agreement. They agree at $M = 3$ because both arguments of the $\arcsin$ become $1/3$, and no common reason has been found for why the water side selects $1/3$ and why $M = 3$ meets it on the sound side. What is shown is that the agreement occurs at one point only.

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