

The Disagreement Between Priors Has an Expiry, and It Falls One Short of the Reciprocal of the Tolerance

— after n straight successes the probability of another splits by prior, but the span is exactly $1/(n + 2)$, so bringing it under $1/m$ takes $m - 1$ observations — [Paper 518]

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— *From thinking to syncing.* —

Abstract

Replacing the prior changes the answer for the same observations. What this paper shows is that the span of that disagreement has a closed form, and the number of observations that ends it can be computed. No new theorem or law is claimed. First, after ten straight successes the answers are **0.916667** under Laplace, **0.954545** under Jeffreys and **1.000000** under Haldane (Section 2). Second, the naive count returns **1.000000** regardless of n —the same value at $n = 1$ and at $n = 10^6$, throwing away the amount of evidence (Section 2). Third, this is the core of the paper. The span of the three answers is exactly $1/(n + 2)$: **0.333333333333** at $n = 1$, **0.009803921569** at $n = 100$, **0.000998003992** at $n = 1000$ (Section 3). Fourth, the observations needed to bring the span under a tolerance $1/m$ number $m - 1$ —**9, 99** and **999** (Section 3). Fifth, for finite n the span never reaches zero—at $n = 10^6$ it still stands at 1.000×10^{-6} (Section 3). Sixth, the separator is the tolerance—until one fixes how many digits count as the same answer, there is no expiry (Section 4). **Keywords:** Laplace’s rule of succession, Bayesian inference, prior distribution, beta distribution, Jeffreys prior.

Scope of this paper (scope note): No new theorem or law is claimed—Laplace’s rule of succession and the conjugacy of the beta prior are standard. No prior is declared correct—whether to take the uniform, Jeffreys or Haldane prior is not the subject; only how far the value moves when they are exchanged, and when it stops moving. No verdict is passed on the positions—which of frequentism and Bayesianism is right is not entered. Only the all-successes case is treated—the single form of n successes in n trials. With failures mixed in the span takes a different expression. Only the binomial case—no extension to multinomial data or Dirichlet priors. No asymptotic theory is built—general convergence results such as Bernstein–von Mises are not treated; one expression’s span is computed exactly. Relation to earlier papers: Paper 164 showed that “probability” has four addresses, giving 0 for the frequency, 0.083333 for a uniform prior and 0.476190 for a Beta(10, 1) prior on the same data—it stops at “all three satisfy the axioms”; this paper goes on to the number of observations at which the disagreement ends. Paper 516 showed that freedom from units had merely handed the choice to the covariance—the prior here is likewise a relocated choice, but this one has an expiry. Paper 517 separated a vanishing choice from a surviving one by rotating the coordinates—this paper supplies the third kind, a choice that expires. Paper 300 showed that whether two things share a root is decidable—“it depends on the prior” and “it depends on the prior forever” have distinct roots. Paper 481 showed that without a separator the blame for a refutation has no address—this paper places that separator at the tolerance. What is added is setting the three priors’ answers side by side at the same n , confirming at six values of n that the span is exactly $1/(n + 2)$, obtaining in exact rationals that the expiry for a tolerance $1/m$ is $m - 1$, measuring that a tolerance sitting on the boundary makes the floating-point search stop one short, and placing the separator at the tolerance.

1. Notation and the Question

Setting: An event has been observed n times and occurred every time. What probability does one give for the next?

Three answers: Under a beta prior Beta(a, b) the answer is

$$P = \frac{n + a}{n + a + b}.$$

Set beside one another are $a = b = 1$ (uniform, Laplace), $a = b = 1/2$ (Jeffreys) and $a = b = 0$ (Haldane). The last agrees with the naive count $n/n = 1$.

Observation 1.1

The expression contains more than the observations n . It contains a and b as well—and the answer changes with the choice.

Observation 1.2

Paper 164 has already written this much. One dataset carries three answers and all three satisfy the axioms—stopping there leaves nothing but the arbitrariness of the choice.

Question: How long does that arbitrariness last?

2. The Three Answers at the Same n

Computation 2.1: The three answers as n varies.

n	Naive n/n	Laplace	Jeffreys	Haldane
0	undefined	0.500000	0.500000	undefined
1	1.000000	0.666667	0.750000	1.000000
10	1.000000	0.916667	0.954545	1.000000
100	1.000000	0.990196	0.995050	1.000000

Observation 2.2

Laplace's values can be checked independently. $2/3$ at $n = 1$, $11/12$ at $n = 10$, $101/102$ at $n = 100$ —written as fractions, they can be verified without a machine.

Observation 2.3

The naive answer does not depend on n . The same **1.000000** at $n = 1$ and at $n = 10^6$ —one observation and a million are treated alike, and the amount of evidence is thrown away.

Observation 2.4

Laplace's does depend on n . **0.666667** at $n = 1$ and **0.999999** at $n = 10^6$ —the other side of the boundary: the evidence is used.

Observation 2.5

Laplace's answer never reaches 1. At $n = 10^9$ it is **0.999999999000**—a billion straight successes still do not amount to certainty.

Observation 2.6

Haldane's has no value at $n = 0$. The denominator becomes $0 + 0 + 0 = 0$ —not a matter of implementation: the prior itself is undefined with no observations. Laplace's returns **0.500000** there, and that is the boundary.

Observation 2.7

Laplace's prior weighs as much as one success and one failure seen in advance. At $n = 2$, $(2 + 1)/(2 + 2) = \mathbf{0.750000}$ —the prior weighs 2, so at $n = 2$ it balances the evidence.

3. The Span Is Exactly $1/(n + 2)$ and the Expiry Is $m - 1$

Computation 3.1: Take the span to be the difference between the strongest prior (Haldane) and the weakest (Laplace).

n	Span	$1/(n + 2)$
1	0.333333333333	0.333333333333
10	0.083333333333	0.083333333333
100	0.009803921569	0.009803921569
1000	0.000998003992	0.000998003992

Observation 3.2

The span has a closed form. $1 - \frac{n + 1}{n + 2} = \frac{1}{n + 2}$ —agreement inside 10^{-12} at six values from $n = 1$ to $n = 10^5$.

Observation 3.3

The span never reaches zero. At $n = 10^6$ it still stands at 1.000×10^{-6} —finitely many observations never erase the prior. What ends is only the visible disagreement.

Computation 3.4: Search in exact rationals for the least n at which the span falls below a tolerance $1/m$.

Tolerance	Exact n	Closed form $m - 1$	Floating-point search
1/10	9	9	8
1/100	99	99	99
1/1000	999	999	999

Observation 3.5

The expiry is $m - 1.1/(n + 2) < 1/m$ is equivalent to $n > m - 2$, so in integers $n = m - 1$ —cutting the tolerance tenfold multiplies the observations needed by about **10.090909**.

Observation 3.6

One step earlier the span is still too wide. At $n = 98$ the span is $1/100$ and at $n = 99$ it is $1/101$ —the other side of the boundary was checked.

Observation 3.7

When the tolerance sits exactly on the boundary the answers differ by one. At $1/10$ the exact answer is **9** and the floating-point search gives **8**— $1/10$ has no binary representation, so the span at $n = 8$ is judged to fall just below 0.1 . At $1/100$ there is no discrepancy.

Observation 3.8

This is not an aside about rounding. A tolerance of $1/4$ is exact in binary and produces no discrepancy—the discrepancy appears only when the tolerance lands on the boundary, and that is precisely where the answer to “how many observations” shifts by one.

4. The Separator

Observation 4.1

The separator is the tolerance. Until one fixes how many digits count as the same answer, there is no expiry—move the tolerance and the expiry moves; leave it and the expiry stands.

Observation 4.2

The prior itself does not serve as a separator. Whichever two are compared, the span shrinks with n —exchanging priors does not touch the fact that the disagreement ends.

Observation 4.3

“It depends on the prior” and “it depends on the prior forever” have distinct roots. By the criterion of Paper 300, the first holds for every n and the second only for small n —one phrase covers both, and only one of them carries an expiry.

Observation 4.4

A scale is added to the four addresses of Paper 164. That paper stopped at “all three satisfy the axioms”—stopping was right, since the axioms do not settle which is correct. But how long the dispute lasts comes out without the axioms.

Observation 4.5

Set beside Papers 516 and 517 the count becomes four. The choice moves (the covariance of Paper 516), the choice vanishes (the longitudinal split of Paper 517), the choice remains (the xy split there), the choice expires (here)—a charge of arbitrariness is an unbacked note until it names one of the four.

Observation 4.6

In the language of Paper 481, doubting “the prior changed the conclusion” has two addresses. Doubt the prior, or doubt that there were enough observations—a number was given to the second, so whether $m - 1$ has been reached decides where the blame falls.

Observation 4.7

Which prior is right is not said. What can be said is that the dispute has an expiry and that the expiry can be computed—before it arrives, there is nothing to do but state the prior.

5. Roll Call and Closing

1. At $n = 10$: **0.916667**, **0.954545**, **1.000000** (Computation 2.1): **three answers, one dataset.**
2. Laplace’s values are fractions (Observation 2.2): **2/3**, **11/12**, **101/102**.
3. The naive answer ignores n (Observation 2.3): **the evidence thrown away.**
4. Haldane’s has no value at $n = 0$ (Observation 2.6): **the boundary.**
5. The span is exactly **$1/(n + 2)$** (Observation 3.2): **the core.**
6. At $n = 10^6$, **1.000×10^{-6}** remains (Observation 3.3): **never zero for finite n .**
7. The expiry is **$m - 1$: 9, 99, 999** (Computation 3.4): **in exact rationals.**
8. On the boundary the float search returns **8** (Observation 3.7): **1/10 is not binary.**
9. The separator is the tolerance (Observation 4.1): **not the prior.**
10. No prior is declared right (Observation 4.7): **only that the dispute expires.**

Closing: That exchanging priors changes the answer for the same observations has already been written. The disagreement has an expiry—after ten straight successes the probability of another splits into **0.916667** under Laplace, **0.954545** under Jeffreys and **1.000000** under Haldane. Laplace’s values are **2/3**, **11/12** and **101/102**,

verifiable without a machine. The naive count returns **1.000000** at $n = 1$ and at $n = 10^6$ alike, throwing away the amount of evidence. Laplace's stands at **0.999999999000** even at $n = 10^9$ and never calls the matter certain. Haldane's has no value at $n = 0$, where the denominator vanishes, while Laplace's returns **0.500000**. The span has a closed form—it is exactly $1/(n + 2)$: **0.333333333333** at $n = 1$, **0.009803921569** at $n = 100$, **0.000998003992** at $n = 1000$, agreeing inside 10^{-12} at six values of n . For finite n it never reaches zero, standing at 1.000×10^{-6} even at $n = 10^6$. Bringing it under a tolerance $1/m$ takes $m - 1$ observations, that is **9, 99** and **999**. Cutting the tolerance tenfold multiplies the requirement by about **10.090909**. At $1/10$, where the tolerance lands on the boundary, the exact answer **9** becomes **8** in a floating-point search, since $1/10$ has no binary representation; at $1/100$ there is no discrepancy. One thing parts the two—the separator is the tolerance: until one fixes how many digits count as the same answer, there is no expiry. The prior itself will not serve, since the span shrinks with n whichever two are compared. By the criterion of Paper 300, “it depends on the prior” and “it depends on the prior forever” have distinct roots. Paper 164 stopped at “all three satisfy the axioms”. Stopping was right, for the axioms do not settle which is correct; but how long the dispute lasts comes out without them. Beside Papers 516 and 517 the count becomes four: the choice moves, the choice vanishes, the choice remains, the choice expires. In the language of Paper 481, doubting “the prior changed the conclusion” falls either on the prior or on the shortage of observations, and whether $m - 1$ has been reached decides which. To be honest—only the single form of n successes in n trials was treated. With failures mixed in the span takes a different expression, and a multinomial setting gives the prior more freedom still. Nor is any prior declared correct. What can be said reaches as far as: the dispute has an expiry, and the expiry can be computed. Before it arrives there is nothing to do but state the prior.

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