How Many Frequencies Can Be Nulled Is Fixed by the Number of Coefficients

— five coefficients null two frequencies, a third raises the count to **7**, and truncating back to five leaves none of the three at zero — [Paper 520]

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— *From thinking to syncing.* —

Abstract

How many demands of the form "null this frequency" can be met?What this paper shows is that the ceiling is set by the number of coefficients, and that falling short does not cost one demand but all of them.No new theorem or law is claimed. First, placing zeros at two frequencies gives 5 coefficients, with responses 2.217×10^{-16} and 3.608×10^{-16} (Section 2). **Second, frequencies not asked for are not nulled**—**at $w = 1.35$ the response is 1.036220, so the check is not saying that everything vanishes** (Section 2). **Third, adding one more zero raises the count to 7**—**a real frequency needs a conjugate pair, so coefficients come 2 at a time** (Section 3). **Fourth, this is the core of the paper.Truncating back to 5 coefficients gives responses 1.811720, 1.144454 and 0.527118: none of the three is a zero any more** (Section 3). **Fifth, before the truncation all three stood below 1.335×10^{-15}**—**the boundary** (Section 3). **Sixth, the separator is the number of coefficients**—**changing which frequencies are chosen leaves the ceiling where it is; only changing the count moves it** (Section 4). **Keywords**: FIR filter, zeros, fundamental theorem of algebra, frequency nulling, digital signal processing.

Scope of this paper (scope note): **No new theorem or law is claimed—the fundamental theorem of algebra and the placing of zeros in a finite impulse response filter are standard.No filter is designed—passband flatness, transition width and group delay are not entered; only where zeros can be placed.No optimality is claimed—there are other ways to shorten a filter, such as applying a window, and the naive truncation used here is not put forward as the best.No extension to infinite impulse responses—a filter with poles requires a different count.Implementation precision is not treated—how far the zeros move when coefficients are held in finite word length is not the subject.Approximate attenuation is not called a zero—a zero here means vanishing at the order of 10^{-15}, not a drop of twenty decibels.Relation to earlier papers**: **Paper 347 showed that "cannot be constructed" is a property of the tools, not of the figure—"cannot be nulled" is likewise a property of the tools, and the tool is the number of coefficients.Paper 128 showed that the sum of the indices is not the number of zeros—there a count failed to serve as a certificate; here the count of roots with multiplicity is exactly the degree and serves as one. The same act of counting works in one place and not the other.Paper 519 showed that four paradoxes fall to four different tools—the fourth demand here does not fall unless the tools are increased.Paper 300 showed that whether two things share a root is decidable—"cannot be nulled" has distinct roots: a bad set of frequencies, or too few coefficients.Paper 481 showed that without a separator the blame for a refutation has no address—this paper places that separator at the number of coefficients.**What is added is **building the coefficients from prescribed zeros and confirming the responses at the order of 10^{-16}, placing a frequency that was not asked for as a control, counting that one further zero costs two coefficients, measuring that truncation destroys all three**, and **placing the separator at the number of coefficients**.

1. Notation and the Question

Setting: **The frequency response of a finite impulse response filter with coefficients $h_0, \ldots, h_N$ is**

$$H(w) = \sum_{i=0}^{N} h_i \, e^{-\mathrm{i}wi}.$$

A frequency w with $|H(w)| = 0$ is one that can be nulled.

Placing a zero: **To null an angular frequency w with real coefficients, multiply by**

$$1 - 2\cos(w)\, z^{-1} + z^{-2},$$

whose roots are a conjugate pair on the unit circle.

Observation 1.1

The response is a polynomial in the coefficients.Read as a polynomial in z^{-1}, its degree is one less than the number of coefficients—and by the fundamental theorem of algebra the number of roots, counted with multiplicity, is exactly that degree.

Observation 1.2

So the number of frequencies that can be nulled has a ceiling.Without roots to spare there is nowhere to place one—a circumstance not of the demand but of the tool.

Question: **What happens when the demand exceeds the ceiling?**

2. Up to Two, and They Vanish

Computation 2.1: **Place zeros at $w = 0.8$ and $w = 1.9$.This gives $\mathbf{5}$ coefficients.**

Quantity	Value
Coefficients	$1.000000,\ -0.746834,\ 1.099048,\ -0.746834,\ 1.000000$
Response at $w = 0.80$	$\mathbf{2.217 \times 10^{-16}}$
Response at $w = 1.90$	$\mathbf{3.608 \times 10^{-16}}$

Observation 2.2

The coefficients come out symmetric.$1.000000,\ -0.746834,\ 1.099048,\ -0.746834,\ 1.000000$—real coefficients with roots in conjugate pairs read the same from either end. A property one can check by eye.

Observation 2.3

The response vanishes at the two prescribed frequencies.At the order of 10^{-16}—which is the rounding, not a small approximate value.

Observation 2.4

Frequencies not asked for are not nulled.At $w = 1.35$ the response is $\mathbf{1.036220}$—the boundary: the check is not saying that everything vanishes.

Observation 2.5

The count balances.$\mathbf{4}$ zeros in conjugate pairs, $\mathbf{5}$ coefficients, degree $\mathbf{4}$—the number of roots is exactly the degree. The fundamental theorem of algebra is doing the bookkeeping.

3. Add a Third and Truncation Destroys Them All

Computation 3.1: **Place a zero at $w = 2.6$ as well.The count rises to $\mathbf{7}$ coefficients.**

Frequency	Response with 7 coefficients
$w = 0.80$	$\mathbf{2.822 \times 10^{-16}}$
$w = 1.90$	$\mathbf{1.335 \times 10^{-15}}$
$w = 2.60$	$\mathbf{8.887 \times 10^{-16}}$

Observation 3.2

The increase is $\mathbf{2}$.From $\mathbf{5}$ to $\mathbf{7}$—each real frequency needs a conjugate pair of roots, so coefficients arrive two at a time, not one.

Computation 3.3: **Truncate to $\mathbf{5}$ coefficients and measure the response at the same three frequencies.**

Frequency	7 coefficients	Truncated to 5
$w = 0.80$	$\mathbf{2.822 \times 10^{-16}}$	**1.811720**
$w = 1.90$	$\mathbf{1.335 \times 10^{-15}}$	**1.144454**
$w = 2.60$	$\mathbf{8.887 \times 10^{-16}}$	**0.527118**

Observation 3.4

None of the three remains a zero.The smallest is 0.527118—before running this I expected two to survive and one to break. That expectation was wrong. Truncation does not discard one root; it replaces the polynomial with a different polynomial.

Observation 3.5

Before the truncation all three did vanish.The largest was 1.335×10^{-15}—the other side of the boundary: what broke them was the truncation, not a bad choice of third frequency.

Observation 3.6

Why all of them break can be seen from the roots.Dropping coefficients lowers the degree, and the roots all move together—roots do not stand in one-to-one correspondence with coefficients, so the operation “delete one of them” does not exist.

Observation 3.7

This is not a problem where compromise buys a near answer.Giving up one demand does not leave the rest standing—choosing two demands afresh and rebuilding with 5 coefficients does work, but that is a redesign, not a truncation.

4. The Separator

Observation 4.1

The separator is the number of coefficients.Changing which frequencies are chosen leaves the ceiling where it is—0.8 and 1.9, or 0.3 and 2.9: with 5 coefficients the answer is two.

Observation 4.2

Only the count moves the ceiling.Two with 5, three with 7—what one varies to change the situation is pinned to a single quantity.

Observation 4.3

“Cannot be nulled” has distinct roots.By the criterion of Paper 300, failing because the set of frequencies is bad differs from failing because the coefficients are too few—the first is cured by choosing again, the second is not.

Observation 4.4

The shape is that of Paper 347.There, “cannot be constructed” belonged to the tools and not to the figure—here, “cannot be nulled” belongs to the number of coefficients and not to the frequency. What forbids is the equipment, not the object.

Observation 4.5

Against Paper 128 the counting runs the other way.There the sum of the indices was not the number of zeros, and counting gave no certificate—here the number of roots with multiplicity is exactly the degree, and the count is the certificate. The same act of counting serves in one place and not in the other.

Observation 4.6

Set beside Paper 519 the demand and the tool make a pair.There four paradoxes needed four tools—here a fourth demand needs a further tool. Both ask whether one resolution is enough.

Observation 4.7

How far one must null is not said.In practice nothing needs to reach 10^{-16}; small enough is enough—what can be said is that the number of exact zeros has a ceiling, and that the ceiling is set by the number of coefficients.

5. Roll Call and Closing

1. **Five coefficients give $\mathbf{2.217 \times 10^{-16}}$ and $\mathbf{3.608 \times 10^{-16}}$** (Computation 2.1): **two can be nulled.**
2. **The coefficients are symmetric** (Observation 2.2): **roots in conjugate pairs.**
3. **At $w = 1.35$, $\mathbf{1.036220}$** (Observation 2.4): **not everything vanishes.**
4. **Four zeros, five coefficients, degree four** (Observation 2.5): **the theorem does the bookkeeping.**
5. **A third zero raises the count to $\mathbf{7}$** (Observation 3.2): **two at a time.**
6. **Truncation gives $\mathbf{1.811720}$, $\mathbf{1.144454}$, $\mathbf{0.527118}$** (Computation 3.3): **the core.**
7. **All three break** (Observation 3.4): **the expectation that one would break was wrong.**
8. **Before truncation the largest was $\mathbf{1.335 \times 10^{-15}}$** (Observation 3.5): **the other side of the boundary.**
9. **The separator is the number of coefficients** (Observation 4.1): **not the choice of frequencies.**
10. **How far to null is not said** (Observation 4.7): **only the count of exact zeros.**

Closing: **How many demands of the form "null this frequency" can be met?The ceiling belongs to the tool—zeros at $w = 0.8$ and $w = 1.9$ give $\mathbf{5}$ coefficients with responses $\mathbf{2.217 \times 10^{-16}}$ and $\mathbf{3.608 \times 10^{-16}}$, vanishing at the order of 10^{-16}. The coefficients 1.000000, -0.746834, 1.099048, -0.746834, 1.000000 come out symmetric, since real coefficients place roots in conjugate pairs. At the unasked $w = 1.35$ a response of $\mathbf{1.036220}$ remains, so the check is not saying that everything vanishes. Four zeros in conjugate pairs, five coefficients, degree four: the number of roots is exactly the degree, and the fundamental theorem of algebra is doing the bookkeeping.Past the ceiling it is not one demand that fails but all—adding a zero at $w = 2.6$ raises the count to $\mathbf{7}$, since each real frequency needs a conjugate pair and coefficients arrive two at a time. Truncating back to $\mathbf{5}$ gives $\mathbf{1.811720}$, $\mathbf{1.144454}$ and $\mathbf{0.527118}$, the smallest still $\mathbf{0.527118}$, so none of the three remains a zero. Before running this I expected two to survive and one to break; that expectation was wrong. Before the truncation the largest was $\mathbf{1.335 \times 10^{-15}}$, so what broke them was the truncation and not a bad third frequency. The reason shows in the roots: dropping coefficients lowers the degree and the roots all move together, so roots do not correspond one-to-one with coefficients and the operation "delete one" does not exist. Hence no compromise buys a near answer; choosing two demands afresh and rebuilding with five coefficients works, but that is a redesign, not a truncation.The separator is a count—changing which frequencies are chosen leaves the ceiling untouched, while two with five coefficients and three with**

seven show that only the count moves it. By the criterion of Paper 300, “cannot be nulled” has distinct roots: a bad set of frequencies is cured by choosing again, too few coefficients is not. The shape is that of Paper 347, where “cannot be constructed” belonged to the tools and not the figure; here it belongs to the number of coefficients and not the frequency. Against Paper 128 the counting runs the other way: there the sum of the indices gave no certificate, here the count of roots with multiplicity is the certificate. Beside Paper 519, where four paradoxes needed four tools, a fourth demand here needs a further tool.To be honest—only exact zeros were treated. In practice nothing needs to reach 10^{-16}; small enough is enough. There are other ways to shorten a filter, such as applying a window, and the naive truncation used here is not put forward as the best. Neither the count for a filter with poles nor the movement of zeros under finite word length was treated. What can be said reaches as far as: the number of exact zeros has a ceiling, and that ceiling is set by the number of coefficients.

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