The ν^2, the T^3 and the Eigenvalue Count Are One and the Same Count

— counting the eigenvalues of a cube of side π, Weyl's leading term is off by **2.834** per cent and the boundary term shrinks the gap to **0.022** per cent — attaching Planck's weight to the same count gives a heat-capacity exponent of **3.000000**, and **2.000000** in two dimensions — [Paper 521]

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— *From thinking to syncing.* —

Abstract

The ν^2 of black-body radiation, the eigenvalue count of a drum and the T^3 of low-temperature specific heat look like three laws from three fields.What this paper shows is that they are one and the same count, and that only the attached weight differs.No new theorem or law is claimed. First, counting the eigenvalues of a cube of side π gives 260485 at $k = 80$——Weyl's leading term 268082.573 is off by 2.834 per cent (Section 2). **Second, adding the boundary term gives 260542.751 and the gap shrinks to 0.022 per cent——even at $k = 10$ it is 410 against 405.789, within 1.038 per cent** (Section 2). **Third, attaching the polarisation factor 2 to the same count yields $8\pi V\nu^3/(3c^3)$——the measured logarithmic slope is 3.000000000000, and the ν^2 density of states is simply its derivative** (Section 3). **Fourth, and this is the core.Attaching Planck's weight to the same count and measuring the low-temperature exponent gives 3.000000——the 3 in T^3 is not a number from thermodynamics but the dimension of the space being counted** (Section 3). **Fifth, repeating the procedure in two dimensions gives 2.000000——the other side of the boundary. The 3 is not a constant** (Section 3). **Sixth, the separator is the weight——weight 1 gives Weyl's law, $h\nu/(e^{h\nu/kT} - 1)$ gives Planck, its temperature derivative gives Debye, and the counting side never changes** (Section 4). **Keywords**: Weyl's law, eigenvalue counting, Rayleigh-Jeans law, Debye T3 law, density of states.

Scope of this paper (scope note): **No new theorem or law is claimed——Weyl's law, the Rayleigh--Jeans formula and the Debye model are all standard.No new asymptotic expansion is offered——terms beyond the boundary term (the curvature terms) are not entered. Only the leading and second terms are used.No proof of Weyl's law is given——what this paper does is count, in one concrete case, and compare.The validity of the Debye model is not assessed——how far T^3 holds in a real solid is not the subject.No historical priority is adjudicated——who asked first is not at issue; only that the count is identical.It is not claimed that the three are "the same law"——what is shared is the count, and all three weights differ.Relation to earlier papers**: **Paper 204 showed that the power in an eigenvalue count is half the dimension——this paper views that same count from radiation and specific heat.Paper 265 showed that what separated the two low-temperature models was the density of states, not quantisation——that paper asked what separates; this one asks what is shared.Paper 222 showed that the Sun peaks in the green, the orange and the infrared ——the splitting of the peak belongs to the weight, while the count stays single.Paper 299 showed that a drum has no pitch because of its boundary——that boundary is the second term here.Paper 102 treated the problem of hearing shape from spectrum——the leading term sees only volume, so hearing shape needs the second term and beyond.Paper 482 showed that the same words name two critical dimensions——the exponents 3 and 2 here also come from dimension.Paper 300 showed that sharing a root is decidable——the criterion used here is the correspondence that preserves identity.**What is added is **counting the eigenvalues explicitly and comparing them with the leading term, measuring that the boundary term brings even $k = 10$ within 1.038 per cent, measuring the heat-capacity exponent rather than assuming it, repeating it in two dimensions to obtain 2.000000**, and **placing the separator on the weight**.

1. Notation and the Question

Setting: **imposing Dirichlet conditions on a cube of side π, the eigenvalues are**

$$\lambda = n_1^2 + n_2^2 + n_3^2, \qquad n_i \in \{1, 2, 3, \dots\}$$

and $N(k)$ denotes the number of eigenvalues with $\sqrt{\lambda} \le k$.

Weyl's law: **the leading term is fixed by the volume alone,**

$$N(k) \;\simeq\; \frac{V}{6\pi^2} k^3 - \frac{S}{16\pi} k^2 + \cdots$$

with V the volume and S the surface area.For a cube of side π, $V = \pi^3$ and $S = 6\pi^2$.

Observation 1.1

the same $N(k)$ carries three different names in three fields.The number of radiation modes, the number of lattice vibrations, the number of eigenvalues——the objects counted are the same lattice points.

> **Observation 1.2**
>
> what differs is what is multiplied in afterwards.Mathematics multiplies nothing.Radiation multiplies the energy per mode.Specific heat then differentiates with respect to temperature——three layers, not three independent laws.

Question: if the count is shared, where does the 3 in T^3 come from?

2. The Count Is Single

Computation 2.1: without using a closed form, the lattice points are counted explicitly, over triples with $n_i \geq 1$ and $n_1^2 + n_2^2 + n_3^2 \leq k^2$.

k	counted	leading term	ratio
10	**410**	523.599	0.783042
20	**3719**	4188.790	0.887846
40	**31616**	33510.322	0.943471
80	**260485**	268082.573	0.971660

> **Observation 2.2**
>
> the leading term alone is still off by **2.834** per cent at $k = 80$.At $k = 10$ it is **21.696** per cent ——“asymptotically correct” is not the same as “already correct”.

Computation 2.3: adding the second term $-Sk^2/(16\pi)$.

k	counted	two terms	ratio
10	**410**	405.789	1.010377
20	**3719**	3717.551	1.000390
40	**31616**	31625.366	0.999704
80	**260485**	260542.751	0.999778

> **Observation 2.4**
>
> with the second term, **21.696** per cent at $k = 10$ becomes **1.038** per cent.At $k = 80$, **2.834** per cent becomes **0.022** per cent——two orders. The boundary term is not decoration.

> **Observation 2.5**
>
> the second term is negative.Dirichlet conditions force the field to vanish at the wall, so there are fewer modes than the volume would suggest——the boundary that Paper 299 called the determiner of pitch appears here as a deficit in the count.

> **Observation 2.6**
>
> the leading term sees only the volume.Feed it a different shape of equal volume and it returns the same value——which is why the question of Paper 102 lives in the second term and beyond.

3. Only the Weight Differs

Computation 3.1: attach the polarisation factor 2 to the same count and convert wavenumber to frequency,

$$N(\nu) = 2 \cdot \frac{V}{6\pi^2}\left(\frac{2\pi\nu}{c}\right)^3$$

which agrees with $8\pi V\nu^3/(3c^3)$ to 14 digits.The measured logarithmic slope is **3.000000000000**.

Observation 3.2

the density of states is merely its derivative.$dN/d\nu = 3N/\nu = 8\pi V\nu^2/c^3$——the ν^2 of Rayleigh--Jeans is one differentiation of an ν^3 count, with no new physics entering.

Computation 3.3: **attach Planck's weight to the same count, differentiate with respect to temperature, and measure the low-temperature exponent as a logarithmic slope rather than assuming it.**

quantity	value
Debye integral $\int_0^\infty x^4 e^x/(e^x-1)^2 dx$	**25.97575761**
independently known $4\pi^4/15$	25.97575761
low-temperature coefficient $12\pi^4/5$	**233.781818**
measured exponent (three dimensions)	**3.000000**
measured exponent (two dimensions)	**2.000000**

Observation 3.4

this is the core.The 3 in T^3 does not come from the thermodynamic side——it is 3 because the count went as k^3, and the same procedure in two dimensions returns 2.000000.

Observation 3.5

the exponent was not assumed.The specific heat was computed at $T/\Theta = 0.01$ and 0.02 and the exponent read off as a slope——T^3 was not put in to get 3 out. Had the count been different, the slope would have moved off 3.

Observation 3.6

all three weights differ.Weyl uses 1, Rayleigh--Jeans uses kT, Planck uses $h\nu/(e^{h\nu/kT}-1)$——the division is entirely in what is multiplied into the same count.

Computation 3.7: measuring the ratio of Planck's weight to that of Rayleigh--Jeans, namely $x/(e^x-1)$ with $x = h\nu/kT$.

$h\nu/kT$	ratio
0.01	**0.995008**
0.10	0.950833
1.00	**0.581977**
5.00	0.033918
10.00	$\mathbf{4.540199 \times 10^{-4}}$

> **Observation 3.8**
>
> at low frequency the difference in weight disappears.At $h\nu/kT = 0.01$ the ratio is **0.995008**——the two theories return the same answer there. The catastrophe is visible only on the high-frequency side, where at $h\nu/kT = 10$ the ratio falls to $\mathbf{4.540199 \times 10^{-4}}$.

4. The Separator

> **Observation 4.1**
>
> the separator is the weight.Hold the count fixed and swap weights, and Weyl, Rayleigh--Jeans, Planck and Debye appear in turn——hold the weight fixed and change the count instead, by changing dimension, and the exponent moves from **3.000000** to **2.000000**.

> **Observation 4.2**
>
> the second criterion of Paper 300 is met.Between the mode count and the eigenvalue count there is a single map $\lambda = k^2$, under which the two counts carry into each other——it is not that the numbers agree; the correspondence can be written down.

Observation 4.3

numerical agreement alone would not be evidence.As Definition 4.1 of Paper 56 states, obtaining the same number is not evidence of a shared root——what is shown here is that one counting procedure can be rewritten as the other.

Observation 4.4

there is a fence around what counts as shared.What is shared is the construction of $N(k)$, not the boundary conditions, not the polarisation factor 2, not the Debye cut-off——the 2 belongs to the electromagnetic field, and lattice vibrations carry a longitudinal branch and use 3.

Observation 4.5

the orientation differs from Paper 265.There the density of states was what separated two models——here the same density of states is what binds three fields. The same quantity works twice, once as a divider and once as a binder.

Observation 4.6

what is not said.How far T^3 holds in a real solid is not treated——the Debye model is itself an approximation, and the exponent was measured inside that approximation.

5. Roll Call and Closing

1. **260485 counted at $k = 80$** (Computation 2.1): **leading term 268082.573.**
2. **leading term off by 2.834 per cent** (Observation 2.2): **21.696 per cent at $k = 10$.**
3. **with the boundary term, 0.022 per cent** (Computation 2.3): **1.038 per cent even at $k = 10$.**
4. **the second term is negative** (Observation 2.5): **Dirichlet removes modes.**
5. **slope of the count 3.000000000000** (Computation 3.1): **ν^2 is its derivative.**
6. **Debye integral 25.97575761** (Computation 3.3): **matches $4\pi^4/15$.**
7. **measured exponent 3.000000** (Observation 3.4): **the core.**

8. **2.000000 in two dimensions** (Observation 3.4): **the 3 is not a constant.**
9. **weight ratio 0.995008 at 0.01, 4.540199×10^{-4} at 10** (Computation 3.7): **both sides.**
10. **the separator is the weight** (Observation 4.1): **not the count.**
11. **the polarisation 2 is not shared** (Observation 4.4): **lattice vibrations use 3.**

Closing: **The ν^2 of black-body radiation, the eigenvalue count of a drum and the T^3 of low-temperature specific heat look like three laws from three fields.The count is single——counting the eigenvalues of a cube of side π explicitly, rather than through a closed form, gives 260485 at $k = 80$, against Weyl's leading term 268082.573, a gap of 2.834 per cent. Adding $-Sk^2/(16\pi)$ gives 260542.751 and shrinks the gap to 0.022 per cent; even at $k = 10$, 21.696 per cent becomes 1.038 per cent. The second term is negative because Dirichlet conditions delete modes at the wall, so the boundary that Paper 299 named as the determiner of pitch appears here as a deficit. The leading term sees only volume and returns the same value for any shape of equal volume, which is why the question of Paper 102 lives beyond it.Only the weight differs——attaching the polarisation 2 gives $8\pi V\nu^3/(3c^3)$ with measured slope 3.000000000000, and the ν^2 of Rayleigh--Jeans is one differentiation of it. Attaching Planck's weight and measuring, rather than assuming, the low-temperature exponent gives 3.000000; the Debye integral is 25.97575761, matching the independently known $4\pi^4/15$, and the coefficient is $12\pi^4/5 = 233.781818$. The same procedure in two dimensions gives 2.000000, so the 3 in T^3 is the dimension of the space being counted, not a number from thermodynamics.The separator is the weight——weight 1 gives Weyl, kT gives Rayleigh--Jeans, $h\nu/(e^{h\nu/kT} - 1)$ gives Planck, and its temperature derivative gives Debye. The ratio $x/(e^x - 1)$ is 0.995008 at $x = 0.01$ and 4.540199×10^{-4} at $x = 10$, so the two theories agree at low frequency and part at high. By the second criterion of Paper 300, the map $\lambda = k^2$ can be written down, so this is a shared root rather than a numerical coincidence. Paper 265 used the same density of states as what separates two models; the same quantity works twice, once dividing and once binding.To be honest——only one case was treated, a cube of side π, and Weyl's law was not proved. Terms beyond the boundary term were not touched. What is shared is the construction of the count, not the boundary conditions, not the polarisation 2, not the Debye cut-off; lattice vibrations carry a longitudinal branch and use 3. How far T^3 holds in a real solid was not treated. What can be said is that three laws sit on one count of lattice points, and that the exponent comes from the dimension of that count, and no further.**

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