# The Exponential Is Shared, but the Prefactor Comes from Three Different Roots

— Eyring's prefactor $k_B T/h$ is $\mathbf{6.250986 \times 10^{12}}$ per second at 300 K, inverting $Q_{10} = 2$ for $E_a$ gives $\mathbf{53597.3}$ against $\mathbf{51061.8}$, and Kramers' rate peaks in friction at $\gamma = \mathbf{0.562341}$ — [Paper 523]


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— *From thinking to syncing.* —


## Abstract

**That a reaction rate goes as $e^{-E_a/kT}$ is shared by three theories.What this paper shows is that only the exponential is shared, and that the prefactor comes from three different places: something measured, something counted, and something fluctuating.No new theorem or law is claimed. First, Eyring's prefactor is not a measured quantity——$k_B T/h$ is $\mathbf{5.834253 \times 10^{12}}$ per second at 280 K and $\mathbf{6.250986 \times 10^{12}}$ at 300 K, fixed by temperature and two universal constants** (Section 2). **Second, $Q_{10}$ is not a constant——holding $E_a$ at 50000 joules per mole, it is $\mathbf{2.097179}$ at 280 K and $\mathbf{1.657559}$ at 340 K** (Section 3). **Third, and yet $Q_{10}$ can see the prefactor——Eyring's prefactor is proportional to $T$, so at the same $E_a$ it returns a $Q_{10}$ larger by $\mathbf{3.3333}$ per cent at 300 K** (Section 3). **Fourth, and this is the core.Inverting a measured $Q_{10}$ for $E_a$ gives different answers for different prefactors——from $Q_{10} = 2$ one gets $\mathbf{53597.3}$ under Arrhenius and $\mathbf{51061.8}$ under Eyring, a gap of $\mathbf{2535.5}$ joules per mole, or $\mathbf{4.965}$ per cent** (Section 3). **Fifth, Kramers' prefactor is a function of friction and is not monotone——the rate peaks at $\gamma = \mathbf{0.562341}$ with $\mathbf{5.992964 \times 10^{-4}}$, falling to $\mathbf{8.898 \times 10^{-3}}$ and $\mathbf{1.789 \times 10^{-3}}$ of that at the two ends** (Section 4). **Sixth, the separator is where the prefactor comes from——a fit, a count containing $h$, or a fluctuation named $\gamma$** (Section 5). **Keywords**: Arrhenius equation, transition state theory, Kramers theory, pre-exponential factor, reaction rate.

**Scope of this paper (scope note)**: **No new theorem or law is claimed——the Arrhenius equation, Eyring's transition-state theory and Kramers' escape rate are all standard.No verdict is given on which theory is right——what is done here is to show that the three prefactors have different origins and to put numbers on the difference.Transmission coefficients are not treated——tunnelling and recrossing corrections are not entered.No claim is made about any real reaction's $E_a$——50000 joules per mole is an illustrative value, not a measurement.Biological temperature responses are not generalised——most systems where $Q_{10}$ is used are not elementary reactions and need not be describable by a single exponential.Kramers' interpolation is not called optimal——several conventions bridge the low- and high-friction limits, and the one used here is a standard choice made to exhibit the turnover.Relation to earlier papers**: **Paper 353 showed that adding noise can improve detection, and its scope note explicitly declined to enter Kramers' rate——this paper takes that seat: the optimal noise level of stochastic resonance cannot be written without Kramers' prefactor.Paper 443 and Paper 457 both used temperature coefficients——this paper puts numbers on the fact that $Q_{10}$ reads the exponential and barely reads the prefactor.Paper 372 treated the exponent of an empirical power law——the difference in standing between a fitted exponent and a counted one is the same issue there and here.Paper 300 showed that sharing a root is decidable——the three prefactors are separate roots under the criterion of route.Paper 481 showed a broken law can be blamed on any assumption——when $Q_{10}$ disagrees, either the exponential or the prefactor can be blamed, and this paper gives that choice a number.**What is added is **setting the three origins side by side, measuring the drift of $Q_{10}$ from 280 K to 340 K, putting the Arrhenius--Eyring gap at 3.3333 per cent, inverting $Q_{10} = 2$ to obtain a 2535.5 joule per mole discrepancy**, and **locating Kramers' turnover at $\gamma = 0.562341$**.

## 1. Notation and the Question

**Setting**: **every theory writes the rate in the same shape,**

$$k \;=\; A\,\exp\!\left(-\frac{E_a}{k_B T}\right)$$

**with the exponential common to all three.What differs is the prefactor $A$.**

**Observation 1.1**

**the three $A$'s have different origins.The $A$ of Arrhenius (1889) is measured and fitted.The $A$ of Eyring (1935) is $k_B T/h$, written without measurement.The $A$ of Kramers (1940) comes from friction $\gamma$ and the curvature of the barrier.**

**Observation 1.2**

**the three are a fit, a count and a fluctuation.They merely stand in front of the same exponential——by the route criterion of Paper 300, the road to the exponential is shared and the road to the prefactor is not.**

**Question**: **can measurement tell the three apart?**

## 2. Three Prefactors

**Computation 2.1**: **evaluating Eyring's prefactor $k_B T/h$ by temperature.**

| temperature (K) | $k_B T/h$ (per second) |
|---|---|
| 280 | $5.834253 \times 10^{12}$ |
| 300 | $6.250986 \times 10^{12}$ |
| 310 | $6.459352 \times 10^{12}$ |
| 320 | $6.667718 \times 10^{12}$ |

**Observation 2.2**

**this number contains nothing about the reaction.Only temperature, $k_B$ and $h$ enter——every reaction gets the same value. Here is a theory whose prefactor is not a measurement.**

**Observation 2.3**

**the presence of $h$ is this prefactor's signature.Planck's constant appears in neither the Arrhenius nor the Kramers prefactor——numbers standing in the same slot can carry different things.**

**Observation 2.4**

**Kramers' prefactor carries friction.Change the viscosity of the medium and the value moves——$k_B T/h$ does not. That is to say, one experiment can separate the two.**

## 3. What $Q_{10}$ Actually Reads

**Computation 3.1**: holding $E_a = 50000$ joules per mole, evaluating $Q_{10}$ by temperature, with the Eyring column carrying the extra factor $(T + 10)/T$ from its prefactor.

| temperature (K) | Arrhenius | Eyring | gap (per cent) |
|---|---|---|---|
| 280 | **2.097179** | **2.172079** | 3.5714 |
| 300 | **1.909088** | **1.972724** | **3.3333** |
| 320 | **1.767333** | **1.822562** | 3.1250 |
| 340 | **1.657559** | **1.706311** | 2.9412 |

> **Observation 3.2**
>
> $Q_{10}$ is not a constant.With $E_a$ untouched it falls from **2.097179** at 280 K to **1.657559** at 340 K——to write "this organism has $Q_{10} = 2$" without the temperature is to leave the number undetermined.

> **Observation 3.3**
>
> and yet $Q_{10}$ is sensitive to the prefactor.The gap at 300 K is **3.3333** per cent——not zero. Whether the prefactor depends on temperature does, in principle, show up in $Q_{10}$.

> **Observation 3.4**
>
> the gap is exactly $(T + 10)/T$.At 300 K that is $310/300 = 1.033333$——independent of the exponential, so the gap stays 3.3333 per cent whatever $E_a$ is chosen.

**Computation 3.5**: conversely, taking $Q_{10} = 2$ as measured and inverting for $E_a$ by bisection at 300 K.

| prefactor | inverted $E_a$ (joules per mole) |
|---|---|
| Arrhenius ($A$ constant) | **53597.3** |
| Eyring ($A \propto T$) | **51061.8** |
| gap | **2535.5** (**4.965** per cent) |

**Observation 3.6**

**this is the core.From one measurement, choosing the prefactor moves $E_a$ by $2535.5$ joules per mole——reporting an $E_a$ from a $Q_{10}$ without stating the assumed prefactor leaves the number non-unique.**

**Observation 3.7**

**the inversion is itself a check.Feeding either $E_a$ back returns $Q_{10} = 2$ to within $10^{-6}$——confirming separately that the solved values really are solutions.**

## 4. Kramers: a Prefactor That Is Not Monotone

**Computation 4.1**: **setting the barrier at $5 k_B T$ and sweeping friction $\gamma$ from $10^{-3}$ to $10^{3}$.**

| $\gamma$ | rate | fraction of peak |
|---|---|---|
| 0.001 | $\mathbf{5.332545 \times 10^{-6}}$ | $\mathbf{8.898 \times 10^{-3}}$ |
| **0.562341** | $\mathbf{5.992964 \times 10^{-4}}$ | 1 |
| 1000 | $\mathbf{1.072162 \times 10^{-6}}$ | $\mathbf{1.789 \times 10^{-3}}$ |

**Observation 4.2**

**the peak is interior.Both ends fall below one per cent of the maximum——neither “less friction is faster” nor “more friction is faster” holds.**

**Observation 4.3**

**the two sides have different powers.On the low side a factor of $10$ in $\gamma$ multiplies the rate by nearly $10$; on the high side it divides it by nearly $10$——two lines of slope $+1$ and $-1$ meeting at the turnover.**

**Observation 4.4**

**the turnover is not in the exponential.$e^{-E_a/k_BT}$ contains no $\gamma$——friction changes the rate because the prefactor alone is moving.**

**Observation 4.5**

**this is the seat left by Paper 353.Stochastic resonance can speak of an optimal noise level only because the rate is non-monotone in noise——that non-monotonicity comes from the prefactor, not the exponential. What that paper placed outside its scope is the subject here.**

## 5. The Separator

**Observation 5.1**

**the separator is the origin of the prefactor.A fit, a count, a fluctuation——hold the exponential fixed and swap prefactors, and $Q_{10}$ moves by $3.3333$ per cent while the inverted $E_a$ moves by $2535.5$ joules per mole.**

**Observation 5.2**

**they can be sorted by what they carry.Eyring's carries $h$, Kramers' carries $\gamma$, Arrhenius' carries neither——the criterion of Paper 300 concerning a quantity's belongings applies directly.**

**Observation 5.3**

**the same number does not make a shared root.Fitted well, all three return the same room-temperature rate——agreement holds at one point; move the temperature and they part by 3.3333 per cent, move the medium and they part by more.**

**Observation 5.4**

**the choice of Paper 481 acquires a number.When $Q_{10}$ disagrees with expectation, one measurement cannot say whether $E_a$ or the prefactor is at fault——sweep temperature widely and the prefactor's dependence shows as 3.3333 per cent; change the medium and the friction dependence shows. Do neither and the destination stays undetermined.**

**Observation 5.5**

**what is not said.No verdict is given on which theory is right——tunnelling and recrossing are not treated, and the fact that most systems using $Q_{10}$ are not elementary reactions is not entered.**

## 6. Roll Call and Closing

1. $k_B T/h = 6.250986 \times 10^{12}$ **per second at 300 K** (Computation 2.1): **no reaction information.**
2. **only Eyring carries $h$** (Observation 2.3): **sorted by belongings.**
3. $Q_{10}$ **is 2.097179 at 280 K, 1.657559 at 340 K** (Computation 3.1): **not a constant.**
4. **gap 3.3333 per cent at 300 K** (Observation 3.3): **not zero.**
5. **the gap is exactly $(T+10)/T$** (Observation 3.4): **independent of $E_a$.**
6. **inverted $E_a$: 53597.3 against 51061.8** (Computation 3.5): **the core.**
7. **a discrepancy of 2535.5 joules per mole** (Observation 3.6): **4.965 per cent.**
8. **feeding them back returns $Q_{10} = 2$** (Observation 3.7): **a check.**
9. **Kramers peaks at $\gamma = 0.562341$** (Computation 4.1): **ends below one per cent.**
10. **slopes $+1$ and $-1$** (Observation 4.3): **meeting at the turnover.**
11. **the turnover is in the prefactor** (Observation 4.4): **the exponential has no $\gamma$.**

**Closing**: That a rate goes as $e^{-E_a/k_BT}$ is shared by three theories.Only the exponential is shared——Eyring's prefactor $k_BT/h$ is $5.834253 \times 10^{12}$ per second at 280 K and $6.250986 \times 10^{12}$ at 300 K, containing only temperature, $k_B$ and $h$, and returning the same value for every reaction. Only this prefactor carries $h$; Kramers' carries friction $\gamma$ instead, and the Arrhenius prefactor carries neither. The criterion of Paper 300 concerning belongings separates all three directly.$Q_{10}$ mostly reads the exponential——with $E_a$ fixed at 50000 joules per mole, $Q_{10}$ is $2.097179$ at 280 K and $1.657559$ at 340 K, so it is not a constant. Its sensitivity to the prefactor is nevertheless non-zero: Eyring returns a $Q_{10}$ larger by $3.3333$ per cent at 300 K. That gap is exactly $(T+10)/T$ and so does not depend on the choice of $E_a$.Inverting splits the answer——from $Q_{10} = 2$ the inverted $E_a$ is $53597.3$ under Arrhenius and $51061.8$ under Eyring, a discrepancy of $2535.5$ joules per mole, or $4.965$ per cent. Both values return $Q_{10} = 2$ when fed back, so this is not a failure of solving. Report an $E_a$ from a $Q_{10}$ without stating the assumed prefactor and the number is not unique.Kramers' prefactor is not even monotone——with a barrier of $5k_BT$ the rate peaks at $\gamma = 0.562341$ with $5.992964 \times 10^{-4}$, falling to $8.898 \times 10^{-3}$ of that at $\gamma = 0.001$ and $1.789 \times 10^{-3}$ at $\gamma = 1000$. The low side is nearly proportional to $\gamma$ and the high side nearly inversely so, two slopes of $+1$ and $-1$ meeting at the turnover. The turnover is not in the exponential, which contains no $\gamma$; only the prefactor moves. The Kramers rate that Paper 353 placed outside its scope is exactly what supplies this non-monotonicity, so the "optimal noise" of stochastic resonance comes from the prefactor.The separator is where the prefactor comes from——a fit, a count containing $h$, or a fluctuation named $\gamma$. In the language of Paper 481, a disagreeing $Q_{10}$ can be blamed on the exponential or on the prefactor; sweeping temperature exposes the first as $3.3333$ per cent and changing the medium exposes the second, and without either experiment the destination is undetermined.To be honest——no verdict was given on which theory is right. Tunnelling and recrossing were not treated, 50000 joules per mole is illustrative rather than measured, and most systems where $Q_{10}$ is used are not elementary reactions describable by one exponential. Several conventions interpolate Kramers' two limits, and the one used here was chosen to exhibit the turnover. What can be said is that the three theories share only the exponential, that their prefactors have different origins, and that the difference surfaces as $2535.5$ joules per mole in an inverted $E_a$, and no further.

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