The Wind on a Sail and the Apparent Place of a Star Are One and the Same Formula

— the formula that turns a boat-to-wind ratio into an apparent wind gives an aberration of $\mathbf{20.492653}$ arcseconds when fed the Earth's $\beta = \mathbf{9.935118 \times 10^{-5}}$ — the two formulas part at $\beta = 0.5$, where they differ by $\mathbf{3.4349}$ degrees — [Paper 524]

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— From thinking to syncing. —

Abstract

The apparent wind of a sailing craft and the aberration of starlight are taught as separate matters.What this paper shows is that at low speed they are one and the same function, and that they part only where the rule for composing velocities changes.No new theorem or law is claimed. First, feeding the boat-to-wind ratio $\mathbf{0.533333}$ into the formula gives an apparent wind at $\mathbf{61.9275}$ degrees——a true wind of 15 knots becoming $\mathbf{17.000000}$ knots (Section 2). **Second, feeding the Earth's $\beta = \mathbf{9.935118 \times 10^{-5}}$ into the same formula gives $\mathbf{20.492653}$ arcseconds ——matching the independently known aberration constant 20.49552 with ratio $\mathbf{0.999860}$** (Section 2). **Third, the same input returns the same angle——whether β is a boat-to-wind ratio or a speed-of-light ratio, the function does not distinguish them** (Section 2). **Fourth, and this is the core.The two formulas part where the rule for composing velocities changes——at the Earth's β the Galilean and relativistic answers differ by $\mathbf{4.905 \times 10^{-13}}$ radians, or $\mathbf{0.000000}$ arcseconds, while at $\beta = 0.5$ they differ by $\mathbf{3.4349}$ degrees** (Section 3). **Fifth, Bradley's observation does not reach the separator——so long as β is of order 10^{-4}, no amount of precision decides the composition rule** (Section 3). **Sixth, the separator is the composition of velocities——the shape of the function is unchanged; Galilean composition leaves it alone, Lorentz composition brings in γ** (Section 4). **Keywords**: aberration of light, apparent wind, sailing, velocity addition, special relativity.

Scope of this paper (scope note): **No new theorem or law is claimed——aberration, apparent wind and relativistic velocity composition are all standard.Sailing performance is not discussed——no lift or drag coefficients, no polar diagrams; only the geometry of angle and speed.Not all aberration terms are treated——only annual aberration; diurnal and secular aberration and the separation from parallax are not entered.The history of the aether is not adjudicated——what Bradley believed is not at issue, only the precision his measurement reaches.Light in a medium is not treated——Fizeau's experiment and the drag coefficient are a different subject.They are not called "the same phenomenon"——what is shared is the shape of a function, and wind and light are not claimed to be one thing.Relation to earlier papers**: **Paper 390 showed that going faster makes the wind stronger, which lets a sailing craft outrun the wind——that its formula is the formula of aberration is not stated there; this paper adds that edge.Paper 328 showed that an ellipse three months out was evidence of a different cause, and treated aberration in doing so——that paper separated aberration from parallax; this one joins aberration to the apparent wind.Paper 83 showed that Galileo's relativity principle alone admits only two relativities——the separator here is which of the two is chosen, appearing as 3.4349 degrees at $\beta = 0.5$.Paper 300 showed that sharing a root is decidable——the verdict here is reached under the criterion of a correspondence that preserves identity.Paper 481 showed a broken law can be blamed on any assumption——when aberration disagrees, whether the speed or the composition rule is at fault stays undetermined while β is small.**What is added is **writing the two phenomena as one function with different arguments**, **building the aberration constant independently from orbital elements to obtain 20.492653 arcseconds**, **measuring the Galilean--relativistic gap by β and putting 3.4349 degrees at $\beta = 0.5$**, **showing that at the Earth's β the gap is 4.905×10^{-13} radians and unmeasurable**, and **placing the separator on velocity composition**.

1. Notation and the Question

Setting: **something arrives at speed V while the receiver moves at speed v.With θ the true direction, the apparent direction θ' satisfies**

$$\tan\theta' = \frac{\sin\theta}{\cos\theta + \beta}, \qquad \beta = \frac{v}{V}$$

If V is a wind speed this is the apparent wind; if V is the speed of light it is aberration.

Observation 1.1

neither "wind" nor "light" appears in the formula.Only the ratio β does——the function does not distinguish which speeds are being compared.

Observation 1.2

but the derivations share a premise.Both use Galilean composition, a plain vector subtraction——in the language of Paper 83, one of the two available relativities has been chosen.

Question: **if the formula is the same, where do they part?**

2. One Function, Two Arguments

Computation 2.1: **a true wind of 15 knots on the beam, with the boat making 8 knots, so that $\beta = 8/15 = \mathbf{0.533333}$.**

quantity	value
β (boat to wind)	$\mathbf{0.533333}$
apparent wind direction	$\mathbf{1.080839}$ radians $=$ $\mathbf{61.9275}$ degrees
apparent wind speed	$\mathbf{17.000000}$ knots
difference from the direct vector construction	$\mathbf{0.00 \times 10^{0}}$

Observation 2.2

the wind has become stronger.15 knots to $\mathbf{17.000000}$ knots——this is what Paper 390 described as going faster making the wind stronger.

Observation 2.3

formula and vector construction agree.The difference is $\mathbf{0.00 \times 10^{0}}$——subtracting the boat's velocity from the true wind and reading the angle directly, without trusting the $\arctan$ expression, returns the same number.

Computation 2.4: **feeding the same formula a β built from the Earth's orbit, using a semi-major axis of $1.495978707 \times 10^{11}$ metres and a sidereal year of 365.256363004 days.**

quantity	value
Earth’s orbital speed	**29784.7355** metres per second
β (orbital speed to c)	$\mathbf{9.935118 \times 10^{-5}}$
angle returned by the formula	**20.492653** arcseconds
independently known aberration constant	20.49552 arcseconds
ratio	**0.999860**

> **Observation 2.5**
>
> **this is a check.The aberration constant is fixed by observation, independently of anything computed here——agreement to 0.014 per cent is evidence that both the formula and the inputs are right, and it is the kind of check that can fail.**

> **Observation 2.6**
>
> **the same input returns the same angle.Entering $\beta = 0.533333$ as a boat-to-wind ratio or as an unlabelled ratio of speeds returns 1.0808390005 radians in both cases, agreeing to 12 digits——this is what it means to say the function is the same.**

> **Observation 2.7**
>
> **the second criterion of Paper 300 is met.The map from one setting to the other is a single relabelling of β——the numbers do not agree by accident.**

3. Where They Part

Computation 3.1: **holding $\theta = 90$ degrees and comparing the Galilean formula with relativistic aberration, $\cos\theta' = (\cos\theta + \beta)/(1 + \beta\cos\theta)$, across β.**

β	Galilean (radians)	relativistic (radians)	difference (radians)
$\mathbf{9.935118 \times 10^{-5}}$	1.570696976	1.570696976	$\mathbf{4.905 \times 10^{-13}}$
10^{-3}	1.569796327	1.569796327	5.000×10^{-10}
0.1	1.471127674	1.470628906	4.988×10^{-4}
0.5	1.107148718	1.047197551	$\mathbf{5.995 \times 10^{-2}}$
0.9	0.837981225	0.451026812	3.870×10^{-1}

Observation 3.2

this is the core.At the Earth's β the gap is $\mathbf{4.905 \times 10^{-13}}$ radians, which is **0.000000** arcseconds——unmeasurable. At $\beta = 0.5$ it is **3.4349** degrees.

Observation 3.3

the gap enters at second order in β.Multiplying β by 10 multiplies the gap by roughly 10^3, from 5.000×10^{-10} at 10^{-3} to 4.988×10^{-4} at 0.1——at small β, the wall of β^2 arrives before any gain in precision.

Observation 3.4

so Bradley's observation does not reach the separator.The existence of aberration is settled at first order in β; the composition rule lives at second——measuring the first order ever more finely does not reveal the second. One must go to a system with larger β, not to a finer instrument.

Observation 3.5

on the sailing side the gap cannot appear at all.The boat-to-wind ratio can be made large, but that is v/V, while v/c stays tiny——the same $\beta = 0.5$ means different things, and it is v/c that summons the Lorentz transformation.

Observation 3.6

here is the fence.Sameness of function is not sameness of physics——a craft sailing at half the wind speed, $\beta = 0.5$, is ordinary, and no γ enters it.

4. The Separator

Observation 4.1

the separator is velocity composition.Keep the shape of the function and swap only the composition rule, and $\beta = 0.5$ yields a gap of $\mathbf{3.4349}$ degrees——shrink β and both rules return the same answer.

Observation 4.2

the choice of Paper 83 is the separator itself.Galileo's relativity principle alone yields two relativities——aberration measurements are, so far, not material for choosing between them.

Observation 4.3

in the language of Paper 481 there are two destinations.If aberration disagrees with expectation, is the Earth's speed wrong or the composition rule——while β is of order 10^{-4} the second is not a candidate, because the gap is 10^{-13} radians.

Observation 4.4

the shared root has a fence around it.What is shared is the small-β formula, and at large β they are different objects——whether two things share a root depends on the range considered, and a verdict without a stated range is incomplete.

> **Observation 4.5**
>
> **what is not said.Sailing performance, light in a medium and diurnal aberration are not treated——only the geometry of angle and speed is examined.**

5. Roll Call and Closing

1. $\beta = \mathbf{0.533333}$ **gives** $\mathbf{61.9275}$ **degrees** (Computation 2.1): **one formula.**
2. **15 knots becomes** $\mathbf{17.000000}$ (Observation 2.2): **the wind strengthens.**
3. **difference from vectors** $\mathbf{0.00 \times 10^{0}}$ (Observation 2.3): **independent construction agrees.**
4. **orbital speed** $\mathbf{29784.7355}$ **metres per second** (Computation 2.4): **built from orbital elements.**
5. **the formula returns** $\mathbf{20.492653}$ **arcseconds** (Computation 2.4): **ratio** $\mathbf{0.999860}$ **to observation.**
6. **12-digit agreement on the same input** (Observation 2.6): **the function ignores provenance.**
7. **at the Earth's** β **the gap is** $\mathbf{4.905 \times 10^{-13}}$ **radians** (Computation 3.1): **the core.**
8. **at** $\beta = 0.5$ **the gap is** $\mathbf{3.4349}$ **degrees** (Observation 3.2): **the other side.**
9. **the gap is second order in** β (Observation 3.3): **precision cannot cross it.**
10. **no** γ **on the sailing side** (Observation 3.5): v/V **and** v/c **differ.**
11. **the separator is composition** (Observation 4.1): **not the shape of the function.**

Closing: **The apparent wind of a sailing craft and the aberration of starlight are taught as separate matters.They are one function——feeding** $\tan\theta' = \sin\theta/(\cos\theta + \beta)$ **the boat-to-wind ratio** $\mathbf{0.533333}$ **gives** $\mathbf{1.080839}$ **radians, or** $\mathbf{61.9275}$ **degrees, and turns 15 knots of true wind into** $\mathbf{17.000000}$ **knots. Subtracting the boat's velocity from the true wind directly, without the** $\arctan$ **expression, differs by** $\mathbf{0.00 \times 10^{0}}$**, so an independent construction agrees. Feeding the same formula the Earth's** $\beta = \mathbf{9.935118 \times 10^{-5}}$**, built from orbital elements, returns** $\mathbf{20.492653}$ **arcseconds against the independently known aberration constant** 20.49552**, a ratio of** $\mathbf{0.999860}$**. The same input returns the same angle to 12 digits: the function does not look at where its argument came from.What parts them is the rule for composing velocities——at** $\theta = 90$ **degrees the Galilean and relativistic answers differ by** $\mathbf{4.905 \times 10^{-13}}$ **radians at the Earth's** β**, which is** $\mathbf{0.000000}$ **arcseconds and unmeasurable. At** $\beta = 0.5$ **the gap is** $\mathbf{5.995 \times 10^{-2}}$ **radians, or** $\mathbf{3.4349}$ **degrees, and at** $\beta = 0.9$ **it opens to** 3.870×10^{-1} **radians. The gap enters at second order, so multiplying** β **by 10 multiplies it by roughly** 10^{3}**. However finely aberration is measured, the composition rule stays hidden: existence is settled at first order and the rule lives at second.There is a fence——on the sailing side no** γ **ever enters, however large** β **becomes, because** v/V **large is not** v/c **large, and a craft sailing at half the wind speed is ordinary. Sameness of function is not sameness of physics. The shared root covers the small-**β **formula only, and a verdict on shared roots without a stated range is incomplete.The separator is velocity**

composition——it is the choice between the two relativities of Paper 83, and the choice shows itself as $\mathbf{3.4349}$ degrees at $\beta = 0.5$. In the language of Paper 481, a disagreeing aberration measurement has two destinations, but while β is of order 10^{-4} the composition rule is not a candidate, the gap being 10^{-13} radians. The formula behind Paper 390 and the aberration of Paper 328 are the same one.To be honest——only the geometry of angle and speed was examined. Sailing performance, lift, drag and polar diagrams were not treated. Only annual aberration was considered, leaving diurnal and secular aberration and the separation from parallax aside. Light in a medium and the drag coefficient are a different subject, and the history of the aether was not adjudicated. What can be said is that the two phenomena are one function with different arguments, that they part where the composition rule changes, and that the gap is $\mathbf{4.905 \times 10^{-13}}$ radians for the Earth and $\mathbf{3.4349}$ degrees at $\beta = 0.5$, and no further.

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