Multiplying the Earth's Radius by $4/3$ Is a Convention, Not a Fact, and There Are Two of Them

— the standard atmosphere implies $\mathbf{1.330617}$, $\mathbf{0.2041}$ per cent off the conventional $\mathbf{4/3}$, and the optical and radio conventions return horizon distances differing by a factor of $\mathbf{1.077033}$ for the same atmosphere — [Paper 527]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— From thinking to syncing. —

Abstract

To compute radio line of sight one multiplies the Earth's radius by $4/3$ and then draws straight lines.What this paper shows is that this $4/3$ is a convention rather than a measured value, and that optics and radio use two different conventions, returning distances that differ by a factor of 1.077033 for one and the same atmosphere.No new theorem or law is claimed. First, the standard atmosphere does not give $4/3$——a gradient of 39 N units per kilometre gives 1.330617, differing from $4/3 = 1.333333$ by 0.2041 per cent (Section 2). **Second, obtaining exactly $4/3$ means pinning the gradient instead——inverting gives 3.924031×10^{-8} per metre, that is 39.2403 N units per kilometre** (Section 2). **Third, and this is the core.One atmosphere carries two conventions——optics takes a refraction coefficient of 0.13, giving $k = 1.1494253$, while radio takes $k = 4/3$, so the horizon stretches by 1.072113 and by 1.154701 respectively** (Section 3). **Fourth, the two conventions differ by a factor of 1.077033 ——from a height of 10 metres the distance is 11288.0 metres without refraction, 12102.1 under optics and 13034.3 under radio** (Section 3). **Fifth, the operation preserves angles——under the logarithmic map the intersection angle shifts by only 3.00×10^{-7} radians while lengths scale by 0.447214: conformal but not isometric** (Section 4). **Sixth, the separator is which convention was adopted——the refraction is not different; the agreement for folding it into a radius is** (Section 5). **Keywords**: effective Earth radius, four-thirds Earth radius, radio refraction, terrestrial refraction coefficient, line-of-sight distance.

Scope of this paper (scope note): **No new theorem or law is claimed——the effective Earth radius, the coefficient of terrestrial refraction and the flat-Earth transformation are all standard.No new atmospheric model is built——one standard gradient is assumed, and real atmospheric variability is not treated.No verdict is given on which convention is right——optics and radio differ because the wavelengths differ, and neither is in error.Ducting is not treated——gradients strong enough to make k negative, trapping radio near the surface, lie outside the frame.Diffraction is not treated——propagation beyond the geometric horizon is not the subject.Surveying practice is not described——how a refraction coefficient is determined in the field is not entered.Relation to earlier papers**: **Paper 349 showed that a mirage needs a rising temperature gradient——that paper looked at the gradient itself; this one looks at the operation folding it into a radius.Paper 404 showed that the horizon is not the limit of what can be seen——this paper puts numbers on that limit under each convention.Paper 395 showed that sound travels far because it does not travel straight——sound rays and radio rays share the geometry of refraction, and the folding is the same.Paper 95 separated convention from fact——$4/3$ falls on the convention side, and the 1.330617 from the standard atmosphere on the side of fact.Paper 409 showed that no undistorted map exists——this transformation is likewise conformal and not isometric, and the same choice appears.**What is added is **building the coefficient from the standard atmosphere to obtain 1.330617, inverting for the gradient that makes it exactly $4/3$, setting the optical and radio conventions side by side for a ratio of 1.077033, computing the 10-metre horizon three ways**, and **checking numerically that the transformation is conformal but not isometric**.

1. Notation and the Question

Setting: **the refractive index of air falls with height, so both light and radio bend downward.Rather than tracing curved rays, one multiplies the Earth's radius by k and draws straight lines,**

$$k = \frac{1}{1 + a\,\dfrac{dn}{dh}}$$

with a the Earth's radius and dn/dh the vertical gradient of refractive index.

Observation 1.1

k is not measured but constructed from a gradient.Move the gradient and k moves——"$k = 4/3$" is a declaration that a particular gradient has been chosen.

Observation 1.2

the gradients differ between optics and radio.Radio is strongly affected by water vapour and visible light hardly at all——the same air refracts them by different amounts, so there are two conventions.

Question: **which is $4/3$, a fact or a convention?**

2. $4/3$ Does Not Come from the Standard Atmosphere

Computation 2.1: **taking the commonly used standard gradient of 39 N units per kilometre, or 3.9×10^{-8} per metre, and computing k.**

quantity	value
standard gradient	3.9000×10^{-8} per metre
resulting k	1.330617
conventional $k = 4/3$	1.333333
difference	0.2041 per cent
gradient making k exactly $4/3$	3.924031×10^{-8} per metre

Observation 2.2

$4/3$ is a rounded number.The standard atmosphere gives 1.330617——using $4/3$ amounts to pinning the gradient at 39.2403 N units per kilometre instead.

Observation 2.3

the inversion serves as a check.Feeding the recovered gradient back returns $k = 4/3$ to within 10^{-9}——confirming separately that the solution is a solution.

Observation 2.4

the difference is small but not zero.0.2041 per cent——negligible in practice, which is why the convention took hold. It does not mean the atmosphere is like that.

Observation 2.5

in the terms of Paper 95 this lies on the convention side.The fact is the gradient; $4/3$ is an agreement made by rounding it——a convention may be convenient, but it must not be cited as a fact.

3. There Are Two Conventions

Computation 3.1: setting the optical coefficient of terrestrial refraction 0.13 beside the radio value $k = 4/3$. Horizon distance goes as $\sqrt{k}$.

convention	k	$\sqrt{k}$ (stretch of the horizon)
no refraction (geometric)	1.000000	1.000000
optics	**1.1494253**	**1.072113**
radio	**1.3333333**	**1.154701**

Observation 3.2

this is the core.The ratio of the two distances is 1.077033——for the same place and the same air, an optics textbook and a radio textbook return horizon distances differing by 7.7 per cent.

Observation 3.3

1.154701 is exact.It is $\sqrt{4/3}$ itself——once the convention is adopted, the stretch becomes a convention too.

Computation 3.4: computing the horizon from an eye height of 10 metres three ways.

convention	distance (metres)
no refraction	**11288.0**
optics	**12102.1**
radio	**13034.3**

> **Observation 3.5**
>
> **the spread is 1.7 kilometres.Optics and radio differ by 932.2 metres——and the gap between no refraction and radio is 1746.3 metres, a visible amount.**

> **Observation 3.6**
>
> **this continues Paper 404.There the horizon was shown not to be the limit of what can be seen——here the limit itself is computed and turns out to depend on which convention is adopted.**

4. The Folding Preserves Angles

Computation 4.1: **pushing a refractive gradient into curvature corresponds to a logarithmic map in the radial coordinate. Measuring numerically how far an intersection angle shifts under it.**

quantity	value
angle before the map	**1.570796327** radians
angle after the map	**1.570796027** radians
difference	$\mathbf{3.00 \times 10^{-7}}$ radians
ratio of lengths	**0.447214**

Observation 4.2

angles survive, lengths do not.A shift of 3.00×10^{-7} radians against a length ratio of 0.447214——conformal, not isometric.

Observation 4.3

so the method suits problems about angles and needs care with lengths.Elevation and incidence angles read directly——distances must be rescaled by $\sqrt{k}$.

Observation 4.4

the same choice as in Paper 409.There, no undistorted map exists——the same alternative, conformal or isometric, appears here, and the effective Earth radius takes the conformal branch.

5. The Separator

Observation 5.1

the separator is which convention was adopted.The refraction itself is not different——the agreement for folding it into a radius is, and that difference emerges as a factor of 1.077033 in distance.

Observation 5.2

there is a way to tell.A pure convention comes with a compensating quantity, so that predictions do not move——here, changing k does move the horizon, because changing k means assuming a different gradient. Convention and assumption are tied together in this case.

Observation 5.3

that is what makes $4/3$ awkward.Being convenient as a convention, its status as an assumption about the gradient is easy to miss——a rounding of 0.2041 per cent gets read as “the atmosphere is like this”.

Observation 5.4

the orientation is opposite to Paper 349.There the gradient reversed sign, with temperature rising——here the gradient is standard and the question is how to fold it: two treatments of the same gradient.

Observation 5.5

what is not said.Neither ducting nor diffraction is treated——gradients making k negative lie outside the frame, and propagation past the horizon is not examined.

6. Roll Call and Closing

1. **the standard atmosphere gives 1.330617** (Computation 2.1): **not $4/3$.**
2. **a difference of 0.2041 per cent** (Computation 2.1): **small, not zero.**
3. **$4/3$ needs 3.924031×10^{-8} per metre** (Computation 2.1): **by inversion.**
4. **feeding it back returns $4/3$** (Observation 2.3): **a check.**
5. **optics stretches by 1.072113** (Computation 3.1): **from a coefficient of 0.13.**
6. **radio stretches by 1.154701** (Computation 3.1): **exactly $\sqrt{4/3}$.**
7. **the ratio is 1.077033** (Observation 3.2): **the core.**
8. **11288.0, 12102.1, 13034.3 metres at 10 metres** (Computation 3.4): **three ways.**
9. **angles shift by 3.00×10^{-7} radians** (Computation 4.1): **conformal.**
10. **lengths scale by 0.447214** (Computation 4.1): **not isometric.**
11. **the separator is the convention** (Observation 5.1): **not the refraction.**

Closing: **To compute radio line of sight one multiplies the Earth's radius by $4/3$ and then draws straight lines.That $4/3$ is not a measured value——the commonly used standard gradient of 39 N units per kilometre gives 1.330617, differing from $4/3 = 1.333333$ by 0.2041 per cent. Using $4/3$ amounts to pinning the gradient at 39.2403 N units per kilometre, or 3.924031×10^{-8} per metre, and feeding that back returns $k = 4/3$, so it is not a failure of solving. The difference is negligible in practice, which is why the convention took hold, but the atmosphere is not thereby described. In the terms of Paper 95, the fact is the gradient and $4/3$ is the agreement.And there are two conventions——optics takes a coefficient of terrestrial refraction of 0.13, giving $k = 1.1494253$, while radio takes $k = 4/3$, and since horizon distance goes as $\sqrt{k}$ the stretches are 1.072113 and 1.154701. The radio value is exactly $\sqrt{4/3}$: adopt the convention and the stretch becomes conventional too. Their ratio is 1.077033, so for one**

place and one atmosphere the two literatures differ by 7.7 per cent. From 10 metres the distances are 11288.0, 12102.1 and 13034.3 metres, so optics and radio differ by 932.2 metres and the full spread is 1746.3 metres.The folding keeps angles and loses lengths——under the logarithmic map the intersection angle shifts by 3.00×10^{-7} radians while lengths scale by 0.447214. Elevation and incidence angles therefore read directly, while distances must be rescaled by $\sqrt{k}$. The alternative that Paper 409 identified, conformal or isometric, appears here too, and this method takes the conformal branch.The separator is which convention was adopted——not a difference in refraction but in the agreement for folding it, showing up as a factor of 1.077033. That is what makes $4/3$ awkward: being convenient, its status as an assumption about the gradient is easy to miss, and a rounding of 0.2041 per cent gets read as a statement about the air. Paper 349 looked at a gradient of reversed sign; this paper looks at how a standard gradient is folded, two treatments of one gradient. The limit that Paper 404 discussed turns out to depend on the convention, and the sound rays of Paper 395 share the same geometry.To be honest——no atmospheric model was built; one standard gradient was assumed and real variability was not treated. Optics and radio differ because their wavelengths differ, neither is in error, and no verdict was given. Ducting, where a strong gradient makes k negative, lies outside the frame, and diffraction past the horizon was not examined. How a refraction coefficient is fixed in the field was not entered. What can be said is that $4/3$ is a rounded convention, that one atmosphere carries two of them, and that horizon distances differ by a factor of 1.077033, and no further.

References

1. Bean, B. R., & Dutton, E. J. (1966). *Radio Meteorology*. National Bureau of Standards Monograph 92.
2. International Telecommunication Union (2019). *Recommendation ITU-R P.453: The radio refractive index*. ITU.
3. Schelleng, J. C., Burrows, C. R., & Ferrell, E. B. (1933). *Ultra-Short-Wave Propagation*. Proceedings of the Institute of Radio Engineers.
4. Torge, W., & Muller, J. (2012). *Geodesy* (4th ed.). De Gruyter.
5. Yamagishi, Y. (2026). *Convention and Fact* [Paper 95]. Zenodo. 10.5281/zenodo.21758621
6. Yamagishi, Y. (2026). *A Mirage Needs Temperature Rising with Height, Not Falling* [Paper 349]. Zenodo. 10.5281/zenodo.22695287
7. Yamagishi, Y. (2026). *Sound Travels Far Because It Does Not Travel Straight* [Paper 395]. Zenodo. 10.5281/zenodo.22699289
8. Yamagishi, Y. (2026). *The Horizon Is Not the Limit of What Can Be Seen* [Paper 404]. Zenodo. 10.5281/zenodo.22699418
9. Yamagishi, Y. (2026). *There Is No Undistorted Map, and the Projection Is Not to Blame* [Paper 409]. Zenodo. 10.5281/zenodo.22699616

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)