“Not Evolving” Is Stationary, and Stationary Is Not Equilibrium

— genotype frequencies settle in one generation and then shift by only $\mathbf{2.78 \times 10^{-17}}$, yet in a population of $N = 100$ the heterozygosity halves in **139** generations while the mean allele frequency stays at **0.503437** — [Paper 528]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

The Hardy--Weinberg law is taught as the state of a population that is not evolving.What this paper shows is that it is a stationary distribution rather than an equilibrium, and that a finite population necessarily drifts away from it.No new theorem or law is claimed. First, it is reached in one generation from any starting frequencies——$(0.9, 0, 0.1)$ becomes $(0.810000, 0.180000, 0.010000)$, and the second generation moves by only $\mathbf{2.78 \times 10^{-17}}$ (Section 2). **Second, where it arrives is not where it began——a population with 0 heterozygotes reaches 0.180000 in one generation, so it is not that nothing happens: it moves once, then stops** (Section 2). **Third, and this is the core.A finite population does not stay there——at $N = 100$ the heterozygosity halves in 139 generations, matching the closed form $2N \ln 2 = 138.6294$** (Section 3). **Fourth, the decay matches theory to within 3 standard errors——0.306252 at generation 100 against 0.302885, and 0.003381 at generation 1000 against 0.003327** (Section 3). **Fifth, 98.95 per cent of replicates are settled within 1000 generations——1033 fixations and 946 losses, and the fixation share 0.5165 matches the initial frequency 0.5** (Section 3). **Sixth, and yet the mean does not move——the mean allele frequency is 0.503437 even at generation 500: stationary in expectation while every realisation runs to an edge** (Section 4). **Keywords**: Hardy-Weinberg principle, genetic drift, stationarity and equilibrium, Wright-Fisher model, population genetics.

Scope of this paper (scope note): **No new theorem or law is claimed——the Hardy--Weinberg law, the Wright--Fisher model and the theory of genetic drift are all standard.Selection, mutation and migration are not included——the model here has none of them, and their consequences are not treated.No real population is discussed——$N = 100$ is illustrative, and the estimation of effective population size is not entered.Testing is not discussed——procedures for testing whether observed genotype frequencies depart from Hardy--Weinberg are not the subject.Linkage is not treated——only one locus with two alleles is considered; linkage disequilibrium is a different subject.It is not claimed that the law is wrong because it is not an equilibrium——the law is correct within its premise of an infinite population; what happens outside that premise is added.Relation to earlier papers**: **Paper 263 showed that stationary is not equilibrium and that detailed balance is only sufficient——this is the population-genetic version, putting numbers on the stationary side.Paper 478 showed that whether an advantage matters is decided by population size——that paper had selection; this one has none, and the two join in the limit of zero selection.Paper 307 showed that "fitness" is four quantities, one of which changes sign with the population——no fitness enters this model at all, and frequencies still move.Paper 300 showed that sharing a root is decidable——stationary and equilibrium are separate roots under the criterion of what a quantity carries.**What is added is **confirming one-generation arrival from three starting points**, **measuring the halving of heterozygosity at 139 generations**, **comparing the decay against theory within 3 standard errors**, **measuring that 98.95 per cent settle within 1000 generations**, and **setting the unmoving mean beside the drifting realisations**.

1. Notation and the Question

Setting: **one locus with two alleles at frequencies p and $q = 1 - p$. With an infinite population, random mating and no selection, mutation or migration, the genotype frequencies are**

$$(AA,\ Aa,\ aa) = (p^2,\ 2pq,\ q^2)$$

which is the Hardy--Weinberg law.

> **Observation 1.1**
>
> **"not evolving" can carry two meanings.Either the distribution does not move, or the forces that move it balance——as Paper 263 showed, these are different things.**

Observation 1.2

“infinite population” sits in the premises.Every real population is finite——seeing what survives when the premise is dropped tells which meaning was intended.

Question: **is Hardy--Weinberg stationary, or an equilibrium?**

2. Reached in One Generation

Computation 2.1: **computing the genotype frequencies one and two generations on from three starting points.**

start	after one generation	movement in the second
$(0.900, 0.000, 0.100)$	$(\mathbf{0.810000}, \mathbf{0.180000}, \mathbf{0.010000})$	$\mathbf{2.78 \times 10^{-17}}$
$(0.200, 0.300, 0.500)$	$(\mathbf{0.122500}, \mathbf{0.455000}, \mathbf{0.422500})$	$\mathbf{1.11 \times 10^{-16}}$
$(0.000, 1.000, 0.000)$	$(\mathbf{0.250000}, \mathbf{0.500000}, \mathbf{0.250000})$	$\mathbf{0.00}$

Observation 2.2

it arrives in one generation and then holds.The second generation moves at the rounding level——this is not fast convergence; it is finished in one step.

Observation 2.3

where it arrives is not where it began.A population with $\mathbf{0}$ heterozygotes reaches $\mathbf{0.180000}$, and one that is all heterozygotes falls to $\mathbf{0.500000}$——it is not that nothing happens: it moves once, then stops.

Observation 2.4

what holds still is the genotype frequency; the allele frequency never moved.Mating does not change p——the conserved quantity is p, and $(p^2, 2pq, q^2)$ is its consequence.

> **Observation 2.5**
>
> **nothing so far is about equilibrium.There are not two forces in balance——the detailed balance of Paper 263 is not needed. This is simply a fixed point of a map.**

3. A Finite Population Does Not Stay

Computation 3.1: **running a Wright--Fisher model with $N = 100$ and initial frequency 0.5, 2000 replicates over 1000 generations, and comparing mean heterozygosity with the closed form $2pq(1 - 1/2N)^t$.**

generation	measured	theory	gap (standard errors)
0	**0.500000**	0.500000	0.00
100	**0.306252**	0.302885	0.85
200	**0.185444**	0.183479	0.44
500	**0.043262**	0.040786	0.89
1000	**0.003381**	0.003327	0.07

> **Observation 3.2**
>
> **this is the core.Heterozygosity does not stay, falling to 0.003381 by generation 1000——one 148th of its initial value.**

> **Observation 3.3**
>
> **the comparison is in standard errors, not ratios.By generation 500 most replicates have already settled, and the standard error of the mean is around 6 per cent——a test on ratios alone ignores that error. In standard errors every generation falls within 3.**

Computation 3.4: **measuring the halving generation and the share settled.**

quantity	value
generation at which heterozygosity halves (measured)	**139**
closed form $2N \ln 2$	**138.6294**
fixations	**1033** / 2000
losses	**946** / 2000
settled within **1000** generations	**98.95** per cent

Observation 3.5

the halving generation matches the closed form.139 against 138.6294——an independently derived expression, so this checks the implementation.

Observation 3.6

the fixation share matches the initial frequency.0.5165 against 0.5——which edge is reached is decided by the initial frequency, but reaching an edge is unavoidable.

Observation 3.7

98.95 per cent settle within 1000 generations.Only about one per cent are still at Hardy--Weinberg——the state called “not evolving” is in fact a place passed through.

4. The Mean Holds While Realisations Run

Computation 4.1: **measuring the mean allele frequency itself, with a different seed and 400 replicates.**

generation	mean p
0	**0.500000**
100	**0.501162**
500	**0.503437**

> **Observation 4.2**
>
> **the mean has not moved.Still 0.503437 after 500 generations——in expectation it is indeed stationary.**

> **Observation 4.3**
>
> **and yet each population has reached an edge.98.95 per cent fixed or lost——the mean holds because those that went one way cancel those that went the other, not because anything stayed.**

> **Observation 4.4**
>
> **this is the difference between stationary and equilibrium.An unmoving mean is not a system that stays——the claim of Paper 263 appears here in the vocabulary of population genetics.**

> **Observation 4.5**
>
> **with absorbing states the word equilibrium is especially dangerous.Neither fixation nor loss can be left——these are dead ends, not balances.**

5. The Separator

> **Observation 5.1**
>
> **the separator is population size.Infinite N stays, finite N drifts——since halving takes $2N\ln 2$, multiplying N by 10 buys ten times longer, but not forever.**

Observation 5.2

the same quantity is at work as in Paper 478.There, with selection, Ns decided——here there is no selection and N alone decides; the two join in the limit of zero selection.

Observation 5.3

no fitness enters.The four quantities distinguished in Paper 307 appear nowhere in this model——and frequencies still move, so movement is no evidence of fitness.

Observation 5.4

separate roots under the criterion of Paper 300.Equilibrium needs the belonging "two balancing flows"——stationarity does not, and Hardy--Weinberg has nothing to balance against.

Observation 5.5

what is not said.The law is not called wrong——it holds within its premise of an infinite population, and only what happens outside that premise has been added.

6. Roll Call and Closing

1. **reached in one generation** (Computation 2.1): **second generation moves $\mathbf{2.78 \times 10^{-17}}$.**
2. **arrival differs from the start** (Observation 2.3): **$\mathbf{0}$ becomes $\mathbf{0.180000}$.**
3. **what is conserved is p** (Observation 2.4): **genotype frequencies follow.**
4. **halving at $\mathbf{139}$ generations for $N = 100$** (Computation 3.4): **$\mathbf{2N \ln 2 = 138.6294}$.**
5. **$\mathbf{0.003381}$ at generation 1000** (Computation 3.1): **one 148th of the start.**
6. **within $\mathbf{3}$ standard errors** (Observation 3.3): **ratios alone are not enough.**
7. **$\mathbf{1033}$ fixed, $\mathbf{946}$ lost** (Computation 3.4): **the share matches the initial frequency.**
8. **$\mathbf{98.95}$ per cent settled** (Computation 3.4): **the core.**
9. **mean p is $\mathbf{0.503437}$** (Computation 4.1): **stationary in expectation.**
10. **the mean holds by cancellation** (Observation 4.3): **nothing stayed.**

11. **the separator is population size** (Observation 5.1): **ten times N buys ten times longer.**

Closing: **The Hardy--Weinberg law is taught as the state of a population that is not evolving.It is reached in one generation and then holds——$(0.9, 0, 0.1)$ becomes $(0.810000, 0.180000, 0.010000)$ and the second generation moves by only 2.78×10^{-17}, so this is not fast convergence but a single step. Where it arrives is not where it began: a population with 0 heterozygotes reaches 0.180000, and one entirely heterozygous falls to 0.500000. What holds still is the genotype frequency, while the conserved quantity is the allele frequency p, and $(p^2, 2pq, q^2)$ is its consequence. There are not two forces in balance; this is a fixed point of a map, and detailed balance is not needed.A finite population does not stay——with $N = 100$ and initial frequency 0.5 over 2000 replicates, heterozygosity falls to 0.306252 by generation 100 and 0.003381 by generation 1000, one 148th of its start. It matches $2pq(1 - 1/2N)^t$ within 3 standard errors at every generation; comparing ratios alone would have missed the sampling error, which reaches about 6 per cent once most replicates have settled. Halving takes 139 generations against the independently derived $2N \ln 2 = 138.6294$. Within 1000 generations 98.95 per cent are settled, 1033 fixed and 946 lost, and the fixation share 0.5165 matches the initial frequency.And yet the mean does not move——the mean allele frequency is 0.503437 even at generation 500, so in expectation the process is stationary. The mean holds because populations that ran one way cancel those that ran the other, not because any stayed. Fixation and loss are absorbing, so these are dead ends rather than balances, and with absorbing states the word equilibrium is especially dangerous.The separator is population size——infinite N stays and finite N drifts, and since halving takes $2N \ln 2$, multiplying N by ten buys ten times longer but not forever. Paper 478 showed that with selection Ns decides; here there is none and N alone decides, the two joining at zero selection. None of the four fitness quantities of Paper 307 enters this model, and frequencies still move, so movement is no evidence of fitness. Under the criterion of Paper 300, equilibrium needs two balancing flows as a belonging and stationarity does not, and Hardy--Weinberg has nothing to balance against. What Paper 263 established appears here in the vocabulary of population genetics.To be honest——the law was not called wrong; it holds within its premise of an infinite population, and only what lies outside that premise was added. Selection, mutation and migration were left out, $N = 100$ is illustrative rather than estimated, tests for departure from Hardy--Weinberg were not treated, and only one locus with two alleles was considered, so linkage disequilibrium is a separate subject. What can be said is that Hardy--Weinberg is stationary rather than an equilibrium, that heterozygosity halves in 139 generations at $N = 100$, and that 98.95 per cent settle within 1000 generations, and no further.**

References

1. Hardy, G. H. (1908). *Mendelian Proportions in a Mixed Population*. Science.

2. Kimura, M. (1955). *Solution of a Process of Random Genetic Drift with a Continuous Model*. Proceedings of the National Academy of Sciences.

3. Weinberg, W. (1908). *Uber den Nachweis der Vererbung beim Menschen*. Jahreshefte des Vereins fur vaterlandische Naturkunde in Wurttemberg.

4. Wright, S. (1931). *Evolution in Mendelian Populations*. Genetics.

5. Yamagishi, Y. (2026). *Stationary Is Not Equilibrium, and Detailed Balance Is Only Sufficient* [Paper 263]. Zenodo. 10.5281/zenodo.22169689

6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

7. Yamagishi, Y. (2026). *"Fitness" Is Four Quantities, and One of Them Changes Sign When the Population Changes* [Paper 307]. Zenodo. 10.5281/zenodo.22172235

8. Yamagishi, Y. (2026). *Whether Being Advantageous Matters Is Decided by the Size of the Population* [Paper 478]. Zenodo. 10.5281/zenodo.22733726

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)