One and the Same “Proportional To” Carries Four Different Kinds of Constant

— a sphere’s $S/V^{2/3}$ is fixed by geometry, a pendulum’s $T\sqrt{g/L}$ moves with amplitude, a power law fitted to the primes moves with the window, and the Catalan coefficient reaches only $\mathbf{0.99988751}$ of its limit even at $n = 10000$ — [Paper 530]

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— From thinking to syncing. —

Abstract

When one writes “A is proportional to B”, the standing of the constant in front is rarely asked about.What this paper shows is that such constants come in four kinds, some of which move when something is changed and some of which do not.No new theorem or law is claimed. First, some constants are fixed by geometry——a sphere’s $S/V^{2/3}$ is $\mathbf{4.835976}$, agreeing with an independently assembled $(36\pi)^{1/3}$ to 12 digits, while a cube gives $\mathbf{6.000000}$ (Section 2). **Second, dimensional analysis fixes no constant——a pendulum’s $T\sqrt{g/L}$ is $\mathbf{6.283186}$ at small amplitude, matching $2\pi = 6.283185$, but $\mathbf{7.416299}$ at 90 degrees, a factor of $\mathbf{1.1803}$** (Section 3). **Third, and this is the core.A fitted constant moves with the window it is fitted in——fitting a power law to the prime counting function gives $\mathbf{0.827369}$ on $[100, 1000]$ and $\mathbf{0.912949}$ on $[10^5, 10^6]$** (Section 4). **Fourth, that is because the truth is not a power law——the local slope is $1 - 1/\ln x$, which is $\mathbf{0.855235}$ at $x = 10^3$ and $\mathbf{0.927618}$ at $x = 10^6$ and keeps moving** (Section 4). **Fifth, a limit is never reached at finite size——the Catalan coefficient is $\mathbf{0.89780266}$ of $1/\sqrt{\pi}$ at $n = 10$ and still only $\mathbf{0.99988751}$ at $n = 10000$** (Section 5). **Sixth, the separator is what must be changed to move the constant——shape, or range, or window, or n: the four answer to different things** (Section 6). **Keywords**: constant of proportionality, dimensional analysis, power law, Catalan numbers, scaling.

Scope of this paper (scope note): **No new theorem or law is claimed——dimensional analysis, power-law fitting and the asymptotics of Catalan numbers are all standard.The axis differs from Paper 117——that paper divided the ways an exponent is chosen into identity, dimension matrix, measurement and fixed point. This paper leaves the exponent alone and divides the standing of the constant. The two fourfold splits are not the same.The ubiquity of power laws is not discussed——why nature offers so many of them is not entered.Fitting methods are not compared——least squares against maximum likelihood is not the subject.No claim about prime distribution is made——the prime counting function is used as a specimen of fitting a power law to something that is not one; nothing in number theory is asserted.No asymptotic expansion is offered——the next term for Catalan numbers is not treated; only that the ratio tends to one.It is not claimed the four exhaust the possibilities——four were counted, and no claim is made that there are no others.Relation to earlier papers**: **Paper 117 showed that scale invariance fixes an exponent in four different ways——this paper looks at the coefficient instead, so the contents of the four differ, and that difference is declared at the outset.Paper 326 showed that the square-cube law is an identity——that is the first kind here, a constant fixed by geometry.Paper 372 treated an exponent relating area and species——that exponent is a fitted value, and the third kind here measures what such a value is like.Paper 144 showed that an exponent is the signature of what is conserved ——it can be read as a signature when the constant's standing is geometric or asymptotic.Paper 300 showed that sharing a root is decidable——the four standings are separate roots under the criterion of what a quantity carries.**What is added is **setting the four side by side and measuring what moves each**, **measuring the pendulum's group moving by $\mathbf{1.1803}$**, **measuring a fitted exponent moving from $\mathbf{0.827369}$ to $\mathbf{0.912949}$**, **showing a limit unreached at $\mathbf{0.99988751}$**, and **placing the separator on what must be changed**.

1. Notation and the Question

Setting: **"$A \propto B$" can be read as "there is a c with $A = cB$".This paper looks only at the c.**

Observation 1.1

the axis differs from Paper 117.That paper divided the ways an exponent is chosen into four——this one holds the exponent fixed and divides the standing of the constant in front. Both say four, and the contents differ.

Observation 1.2

standing shows itself in what must be changed to move the constant.The shape, the range, the window—— calling something a constant when it does not move is safe; calling something a constant when it does breaks predictions.

Question: **how many kinds of constant stand in front of that one symbol?**

2. First Kind: Fixed by Geometry

Computation 2.1: **computing $S/V^{2/3}$ for solids.**

solid	$S/V^{2/3}$
sphere	$\mathbf{4.835976}$
independently assembled $(36\pi)^{1/3}$	$\mathbf{4.835976}$
cube ($a = 1$)	$\mathbf{6.000000}$
cube ($a = 2$)	$\mathbf{6.000000}$
cube ($a = 7.3$)	$\mathbf{6.000000}$

Observation 2.2

changing the size does not move it.From $a = 1$ to $a = 7.3$ it stays $\mathbf{6.000000}$——not a measured agreement but an identity, as Paper 326 established for the square-cube law.

Observation 2.3

changing the shape does move it.$\mathbf{4.835976}$ for a sphere against $\mathbf{6.000000}$ for a cube——fixed by geometry means fixed again whenever the geometry changes. It is not a universal constant.

Observation 2.4

the agreement to 12 digits is a check.$(36\pi)^{1/3}$ can be assembled without the area and volume formulas—— disagreement would mean one of them is wrong. It is a check that can fail.

3. Second Kind: Dimensional Analysis Fixes Nothing

Computation 3.1: **computing the pendulum period by numerical integration without the small-angle approximation, and measuring the dimensionless group $T\sqrt{g/L}$.**

amplitude (radians)	$T\sqrt{g/L}$
0.001	**6.283186**
0.100	6.287115
0.500	6.382790
1.000	6.699976
1.571 (90 degrees)	**7.416299**

Observation 3.2

dimensional analysis stops here.The form $T \propto \sqrt{L/g}$ follows from dimensions, the number in front does not ——2π appears only once the equation of motion is solved.

Observation 3.3

and that 2π is itself conditional.6.283186 at small amplitude, 7.416299 at 90 degrees——a factor of 1.1803. It deserves the name constant only while the amplitude is small.

Observation 3.4

it moves because there is more than one dimensionless group.The amplitude is itself dimensionless, so a second Π group exists——Paper 117 imposed the condition that exactly one Π group exist precisely to exclude this case.

4. Third Kind: a Fitted Value Moves with the Window

Computation 4.1: **fitting $\pi(x) \propto x^{z}$ to the prime counting function. Here $\pi(10^{6}) = 78498$, agreeing with the independently known value.**

window	fitted exponent
$[10^{2}, 10^{3}]$	$\mathbf{0.827369}$
$[10^{3}, 10^{4}]$	$\mathbf{0.864243}$
$[10^{4}, 10^{5}]$	$\mathbf{0.892357}$
$[10^{5}, 10^{6}]$	$\mathbf{0.912949}$

Observation 4.2

this is the core.Moving only the window carries the value from $\mathbf{0.827369}$ to $\mathbf{0.912949}$——every window returns something below one, so the fit looks successful, and still the value does not settle.

Observation 4.3

the truth is not a power law.Since $\pi(x) \sim x/\ln x$, the local slope is $1 - 1/\ln x$——$\mathbf{0.855235}$ at $x = 10^{3}$ and $\mathbf{0.927618}$ at $x = 10^{6}$. The fitted values were averaging a moving quantity over a window.

Observation 4.4

a fitted value therefore obliges one to state the window.Write only “the exponent is 0.86” and a reader takes it for a constant——the z of Paper 372 is likewise a fitted value, and the range of areas measured is part of the number.

Observation 4.5

there is a procedure for telling.Split the window in two and fit both halves——a shift marks a fitted value; no shift leaves the other standings as candidates. This is a check that can actually be run.

5. Fourth Kind: a Limit Is Not Reached

Computation 5.1: **measuring the coefficient in $C_n \sim 4^n/(n^{3/2}\sqrt{\pi})$ at several n.**

n	$C_n n^{3/2}/4^n$	ratio to the limit
10	**0.5065309103**	**0.89780266**
100	0.5579057328	0.98886216
1000	0.5635555088	0.99887613
10000	**0.5641261186**	**0.99988751**
limit $1/\sqrt{\pi}$	0.5641895835	1

Observation 5.2

it approaches and does not arrive.Even at $n = 10000$ the ratio is $\mathbf{0.99988751}$——this is not an error. Failing to agree exactly at finite n is what defines this standing.

Observation 5.3

here the difference from a fitted value shows.A fitted value moves with the window and converges to nothing——a limit draws towards one value as n grows. Both fail to agree at finite size; only one has somewhere to go.

Observation 5.4

$n = 10$ was placed on the far side of the boundary.A ratio of 0.89780266, off by ten per cent rather than one——using an asymptotic formula at small n misses by that much.

6. The Separator

Observation 6.1

the separator is what must be changed to move the constant.

standing	what moves it	example
fixed by geometry	shape (not size)	**4.835976** against **6.000000**
not fixed by dimensions	value of the second group	**6.283186** against **7.416299**
fitted	the window	**0.827369** against **0.912949**
a limit	n (the destination does not move)	**0.89780266** against **0.99988751**

Observation 6.2

they part under the criterion of Paper 300.A geometric constant carries a shape; a fitted value carries a window——quote the number without its belongings and the standing disappears.

Observation 6.3

this connects to Paper 144.There an exponent was the signature of what is conserved——it reads as a signature when the standing is geometric or asymptotic; a fitted exponent speaks about the window, not about a conservation law.

Observation 6.4

no claim that four exhausts the list.Four were counted and no more are asserted——to say a list is exhaustive one must first fix how it is to be exhausted.

Observation 6.5

what is not said.Neither the ubiquity of power laws nor the merits of fitting methods are discussed——the prime counting function served only as a specimen.

7. Roll Call and Closing

1. **sphere gives $\mathbf{4.835976}$** (Computation 2.1): **12 digits with $(36\pi)^{1/3}$.**
2. **cube stays $\mathbf{6.000000}$** (Observation 2.2): **an identity.**
3. **shape moves it** (Observation 2.3): **not universal.**
4. **small amplitude gives $\mathbf{6.283186}$** (Computation 3.1): **2π comes from solving.**
5. **90 degrees gives $\mathbf{7.416299}$** (Observation 3.3): **a factor of $\mathbf{1.1803}$.**
6. **window gives $\mathbf{0.827369}$ to $\mathbf{0.912949}$** (Computation 4.1): **the core.**
7. **the local slope is $1 - 1/\ln x$** (Observation 4.3): **not a power law.**
8. $\pi(10^6) = 78498$ (Computation 4.1): **matches the known value.**
9. **$\mathbf{0.99988751}$ at $n = 10000$** (Computation 5.1): **a limit, not a value.**
10. **$\mathbf{0.89780266}$ at $n = 10$** (Observation 5.4): **the far side.**
11. **the separator is what moves it** (Observation 6.1): **shape, group, window, n.**

Closing: **When one writes that A is proportional to B, the standing of the constant in front is rarely asked about.A geometric constant ignores size and answers to shape——a sphere's $S/V^{2/3}$ is $\mathbf{4.835976}$, agreeing to 12 digits with**

$(36\pi)^{1/3}$ assembled without the area and volume formulas. A cube stays at $\mathbf{6.000000}$ from $a = 1$ to $a = 7.3$, which is an identity rather than a measured agreement. And yet sphere and cube differ, so the constant is fixed again whenever the geometry changes and is not universal.Dimensional analysis fixes no constant——the form $T \propto \sqrt{L/g}$ follows from dimensions while the number does not, and 2π appears only on solving the equation of motion. That 2π is itself conditional: $\mathbf{6.283186}$ at small amplitude becomes $\mathbf{7.416299}$ at 90 degrees, a factor of $\mathbf{1.1803}$. It moves because the amplitude is itself dimensionless and supplies a second group, the very case Paper 117 excluded by requiring exactly one.A fitted value moves with its window——fitting a power law to the prime counting function gives $\mathbf{0.827369}$ on $[10^2, 10^3]$ and $\mathbf{0.912949}$ on $[10^5, 10^6]$, all below one, so the fit looks successful while the value refuses to settle. The truth is $x/\ln x$, whose local slope $1 - 1/\ln x$ runs from $\mathbf{0.855235}$ to $\mathbf{0.927618}$, so the fits were averaging a moving quantity. A fitted value therefore obliges one to state the window, and the test is easy: split it and fit both halves.A limit is not reached at finite size——the Catalan coefficient is $\mathbf{0.89780266}$ of $1/\sqrt{\pi}$ at $n = 10$ and still $\mathbf{0.99988751}$ at $n = 10000$, which is the definition of this standing rather than an error. This is where it parts from a fitted value: both disagree at finite size, but only a limit has a destination.The separator is what must be changed to move the constant——shape, the value of a second group, the window, or n. Under the criterion of Paper 300, a geometric constant carries a shape and a fitted value carries a window, and quoting the number without its belongings erases the standing. Paper 144 read an exponent as the signature of a conservation law, which works when the standing is geometric or asymptotic and fails when it is fitted.To be honest——Paper 117 divided the ways an exponent is chosen; this paper divides the standing of the coefficient, so the two fourfold splits differ in content. No claim is made that four exhausts the list, since exhaustion requires first fixing how to exhaust. The ubiquity of power laws and the merits of fitting methods were not discussed, the prime counting function served only as a specimen with no claim in number theory, and the next term for Catalan numbers was not treated. What can be said is that four constants of different standing stand in front of the same symbol, and each answers to a different change, and no further.

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