Taking Dead Time Out of the Loop Demands a Model in Its Place

— with a proportional gain of **2** the phase margin vanishes at a dead time of **1.209200** seconds, while a Smith predictor needs a model dead time of only **0.0617** of the true one, failing solely below **0.0925** seconds —

[Paper 532]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— *From thinking to syncing.* —

Abstract

Dead time costs a control loop its phase margin and eventually its stability.What this paper shows is that the Smith predictor removes dead time from the characteristic equation only in exchange for a model, and that the accuracy demanded of that model is far weaker than expected.No new theorem or law is claimed. First, the limit on dead time follows in closed form——with a proportional gain of 2 the crossover frequency is 1.7320508 and the phase margin vanishes at 1.209200 seconds (Section 2). **Second, there is margin short of that——halving the dead time leaves a phase margin of 60.0000 degrees, the far side of the boundary** (Section 2). **Third, beyond the limit the oscillation grows——at $L = 1.5$ seconds an amplitude of 34.723 in the middle window becomes 1804.7 at the end, a growth of 51.9721** (Section 3). **Fourth, with an exact model the same dead time is stable——growth of 1.0000 and a final error of 1.83×10^{-14}, settling where a delay-free loop settles** (Section 3). **Fifth, and this is the core.The accuracy demanded of the model is far weaker than expected——a model dead time of 1.60 or 0.10 times the true one is still stable, and the boundary lies at 0.0617, that is 0.0925 seconds** (Section 4). **Sixth, the separator is whether a model exists, not whether it is accurate——set the model's dead time to 0 and the predictor collapses to plain feedback, with growth back at 51.9721** (Section 5). **Keywords**: Smith predictor, dead time, phase margin, feedback control, model mismatch.

Scope of this paper (scope note): **No new theorem or law is claimed——the Smith predictor, phase margin and the stability of delay systems are all standard.No controller is designed——a single proportional gain is examined; PI control and settling-time optimisation are not entered.Disturbance rejection is not treated——the Smith predictor helps with setpoint tracking, and rejection needs a separate argument. Only tracking is examined here.Gain error is not treated——only the dead time was varied as model mismatch; errors in time constant or gain are a different account.Unstable plants are not covered——the plant here is a stable first-order lag, and it is known that the Smith predictor does not serve unstable plants.Numerical error is not used as evidence——results are at a step of 10^{-3} seconds, and the boundary value depends on that step.It is not said that dead time disappears——it leaves the characteristic equation only; the output still reaches the setpoint L later.Relation to earlier papers**: **Paper 350 showed that dead time eats the stability margin without touching the amplitude, and its scope note declined to enter Smith compensation and predictive control——this paper takes that seat and puts numbers on what removal costs.Paper 367 showed that pushing sensitivity down somewhere raises it elsewhere——the predictor is no exception: margin returns and dependence on a model is incurred.Paper 289 showed that one number sets the limit of forecasting——the predictor's interior is a forecast, and that limit applies directly.Paper 426 showed that look-ahead survives both a split sample and a high t——the "future values" a predictor uses are manufactured by the model, not observed, and confusing the two is look-ahead.Paper 300 showed that sharing a root is decidable——"no margin" and "no model" are separate roots.**What is added is **putting the zero-margin dead time at $\mathbf{1.209200}$ seconds and checking it by inversion, measuring a growth factor of $\mathbf{51.9721}$ past the limit, measuring $\mathbf{1.0000}$ with an exact model, bisecting the accuracy boundary to $\mathbf{0.0617}$, and placing the separator on the existence of a model**.

1. Notation and the Question

Setting: **the plant is a first-order lag $1/(s+1)$ under proportional gain K, with dead time L between input and output,**

$$\frac{dy}{dt} = -y + u(t-L), \qquad u = K(r-y)$$

The Smith predictor: **holding a model $1/(s+1)$ and a model dead time L_m, it replaces the feedback signal by**

$$y(t) + y_m(t) - y_m(t-L_m)$$

so that with an exact model $y(t) = y_m(t-L_m)$ and the feedback becomes $y_m(t)$, a signal without dead time.

> **Observation 1.1**
>
> **the dead time has not disappeared.It only leaves the characteristic equation——the output still reaches the setpoint L later.**

> **Observation 1.2**
>
> **a model is required in its place.Paper 350 showed that dead time eats the stability margin without touching the amplitude, and placed this predictor outside its scope——what the thing outside demands is the question here.**

Question: **how accurate must the model be?**

2. The Limit on Dead Time

Computation 2.1: **with $K = 2$, the gain crossover frequency is $\omega_c = \sqrt{K^2 - 1}$ and the phase margin is $\pi - \arctan \omega_c - \omega_c L$.**

quantity	value
crossover frequency ω_c	**1.7320508**
L at which the phase margin vanishes	**1.209200** seconds
phase margin there (checked by inversion)	**0.000** $\times$ 10^{0} radians
phase margin at half that L	**60.0000** degrees

> **Observation 2.2**
>
> **the inversion is a check.Feeding the recovered L back returns exactly zero margin——confirming separately that this is a solution.**

> **Observation 2.3**
>
> **the far side was examined too.Half that dead time leaves 60.0000 degrees——it is not that any dead time is fatal; the claim is that a limit exists.**

Observation 2.4

amplitude plays no part.ω_c does not depend on L, and L enters only the phase term——the claim of Paper 350 is visible in the formula itself.

3. Where the Predictor Works

Computation 3.1: **at $L = 1.5$ seconds, past the limit of 1.209200, integrating to 120 seconds at a step of 10^{-3} and comparing the amplitude of the error in a middle window of ten seconds against a final window of ten.**

case	middle amplitude	final amplitude	growth
no predictor	**34.723**	**1804.7**	**51.9721**
predictor with an exact model	**1.83×10^{-14}**	**1.83×10^{-14}**	**1.0000**

Observation 3.2

the same dead time, and one grows while the other does not.51.9721 against 1.0000——plant, dead time and gain are identical; only the feedback signal differs.

Observation 3.3

judging by "did it diverge" fails.Testing whether the magnitude exceeds 10^6 failed, because the unstable pole at $L = 1.5$ has a small real part and sixty seconds is not enough——comparing amplitudes in two windows is surer than extending the simulation.

Observation 3.4

the steady state is the same.The final error with the predictor is 1.83×10^{-14} against 1.84×10^{-14} for zero dead time——the steady-state offset of proportional control does not depend on dead time; only stability does.

4. How Accurate the Model Must Be

Computation 4.1: **holding the true dead time at 1.5 seconds and varying only the model's.**

model over true	model dead time (seconds)	growth
1.60	2.400	**0.0371**
1.30	1.950	**0.0000**
1.00	1.500	**1.0000**
0.70	1.050	**0.0000**
0.40	0.600	**0.0000**
0.20	0.300	**0.0012**
0.10	0.150	**0.2537**
0.00	0.000	**51.9721**

> **Observation 4.2**
>
> **this is the core.A model dead time of 1.60 or 0.10 times the true one is still stable——the prediction before writing was that a twenty per cent error would break it. That was wrong.**

Computation 4.3: **bisecting the boundary places it between 0.06165 and 0.06167 of the true dead time, that is where the model's dead time falls below 0.0925 seconds.**

> **Observation 4.4**
>
> **both sides were examined.Just below the boundary it grows, just above it does not——examining one side only would have hidden the boundary altogether.**

Observation 4.5

the reason is legible in the feedback signal.Even with a short model dead time, the term $y_m(t) - y_m(t - L_m)$ subtracts what is still on its way——the subtraction need not be exact; getting the sign right already buys margin. Existence, not accuracy, is what tells.

Observation 4.6

only the dead time was varied.Errors in time constant or gain are a different account——the figure 0.0617 must not be read as “a rough model will do”.

5. The Separator

Observation 5.1

the separator is whether a model exists, not whether it is accurate.Set the model dead time to 0 and the feedback returns to plain y, with growth back at 51.9721——it switches on the presence of the predictor, not on a continuous decay of accuracy.

Observation 5.2

as Paper 367 has it, push down somewhere and it rises elsewhere.Phase margin returns while dependence on a model is born——the price of the recovered margin is the duty of maintaining a model.

Observation 5.3

the limit of Paper 289 applies directly.The predictor’s interior is a forecast——if the plant is hard to forecast, no model can be had. What matters is forecastability, not the length of the delay.

Observation 5.4

the fence of Paper 426 is needed too.The “future values” the predictor uses are manufactured by the model ——mistake them for observations and it becomes look-ahead. Model output and measurement must be kept in separate places.

Observation 5.5

separate roots under Paper 300.“No margin” carries a frequency; “no model” carries none——the same instability, different repairs.

Observation 5.6

what is not said.Neither disturbance rejection nor unstable plants are treated——it is known that the Smith predictor does not serve unstable plants, and the plant here is a stable first-order lag.

6. Roll Call and Closing

1. **crossover frequency $\mathbf{1.7320508}$** (Computation 2.1): **at $K = 2$.**
2. **zero margin at $\mathbf{1.209200}$ seconds** (Computation 2.1): **checked by inversion.**
3. **$\mathbf{60.0000}$ degrees at half** (Observation 2.3): **the far side.**
4. **growth $\mathbf{51.9721}$ without a predictor** (Computation 3.1): **at $L = 1.5$.**
5. **growth $\mathbf{1.0000}$ with one** (Computation 3.1): **same dead time.**
6. **judge by growth, not magnitude** (Observation 3.3): **10^6 was never reached.**
7. **the steady state is unchanged** (Observation 3.4): **only stability changes.**
8. **stable at $\mathbf{1.60}$ and $\mathbf{0.10}$ times** (Computation 4.1): **the core.**
9. **the boundary is $\mathbf{0.0617}$** (Computation 4.3): **that is $\mathbf{0.0925}$ seconds.**
10. **the prediction was wrong** (Observation 4.2): **twenty per cent does not break it.**
11. **the separator is existence** (Observation 5.1): **not accuracy.**

Closing: **Dead time costs a control loop its phase margin and eventually its stability.The limit follows in closed form ——with a proportional gain of 2 the crossover frequency is $\mathbf{1.7320508}$ and the phase margin vanishes at**

1.209200 seconds, a value that returns exactly zero margin when fed back. Half that leaves 60.0000 degrees, so it is not that any dead time is fatal; a limit exists. Since ω_c does not depend on L and L enters only the phase, the claim of Paper 350 is visible in the formula.The predictor changes the outcome at the same dead time——at $L = 1.5$ seconds the amplitude without a predictor runs from 34.723 to 1804.7, a growth of 51.9721. With an exact model the growth is 1.0000 and the final error 1.83×10^{-14}, settling where a delay-free loop settles. Plant, dead time and gain are identical, and only the feedback signal differs. Judging by whether the magnitude exceeded 10^6 failed, since the unstable pole has a small real part; comparing two windows is surer than a longer run.The accuracy demanded is far weaker than expected——a model dead time of 1.60 or 0.10 times the true one is still stable, and bisection places the boundary between 0.06165 and 0.06167, where the model's dead time falls below 0.0925 seconds. The prediction before writing, that a twenty per cent error would break it, was wrong. The reason is legible in the feedback: $y_m(t) - y_m(t - L_m)$ subtracts what is still on its way, and getting the sign right already buys margin. Only the dead time was varied, however, so the figure must not be read as licence for a rough model.The separator is whether a model exists——set the model dead time to 0 and feedback returns to plain y with growth back at 51.9721, so it switches on presence rather than on a decay of accuracy. As Paper 367 has it, the recovered margin is paid for with a duty to maintain a model. The limit of Paper 289 applies, since the predictor's interior is a forecast and an unforecastable plant admits no model. The fence of Paper 426 is needed too: the future values are manufactured, and mistaking them for observations is look-ahead. Under Paper 300, "no margin" carries a frequency and "no model" carries none, so the same instability has different repairs.To be honest——the dead time has not disappeared; it merely leaves the characteristic equation, and the output still arrives L later. No controller was designed, only one proportional gain examined, with neither PI control nor settling-time optimisation entered. Disturbance rejection was not treated, only setpoint tracking. Model mismatch was varied in dead time alone, unstable plants were excluded, and the step of 10^{-3} seconds sets the precision of the boundary. What can be said is that margin is recovered at the price of a model, and that the price is paid in the model's existence rather than its accuracy, and no further.

References

1. Astrom, K. J., & Hagglund, T. (2006). *Advanced PID Control*. ISA.
2. Bode, H. W. (1945). *Network Analysis and Feedback Amplifier Design*. Van Nostrand.
3. Palmor, Z. J. (1996). *Time-Delay Compensation: Smith Predictor and Its Modifications*. In W. S. Levine (Ed.), The Control Handbook. CRC Press.
4. Smith, O. J. M. (1957). *Closer Control of Loops with Dead Time*. Chemical Engineering Progress.
5. Yamagishi, Y. (2026). *One Number Sets the Limit of Forecasting* [Paper 289]. Zenodo. 10.5281/zenodo.22171367
6. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914

7. Yamagishi, Y. (2026). *Dead Time Eats the Stability Margin Without Touching the Amplitude* [Paper 350]. Zenodo. 10.5281/zenodo.22695565

8. Yamagishi, Y. (2026). *Push the Sensitivity Down Somewhere and It Must Rise Somewhere Else* [Paper 367]. Zenodo. 10.5281/zenodo.22697709

9. Yamagishi, Y. (2026). *Look-Ahead Survives Both a Split Sample and a High* t [Paper 426]. Zenodo. 10.5281/zenodo.22700184

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)