The Multiple-Comparison Correction and the Truncated Inclusion-Exclusion Bound Are One Inequality

— With 20 independent tests, the chance of at least one error is $\mathbf{0.641514}$ uncorrected and $\mathbf{0.048830}$ once the level is set to α/k — the two are the first term and the general term of one man's inequality — [Paper 533]

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September 2026

— *From thinking to syncing.* —

Abstract

The multiple-comparison correction and the inequality that brackets a truncated inclusion-exclusion are taught as separate matters.What this paper shows is that they are one inequality, and that the correction uses only its first term.No new theorem or law is claimed. First, without correction the probability climbs fast——twenty independent tests at level 0.05 give a chance of at least one error of $\mathbf{0.641514}$, and a hundred give $\mathbf{0.994079}$ (Section 2). **Second, setting the level to α/k contains it——$\mathbf{0.048830}$ at $k = 20$, below 0.05** (Section 2). **Third, the correction follows from one line, that a union is at most the sum——$k \cdot (\alpha/k) = \alpha$, using neither independence nor normality** (Section 2). **Fourth, and this is the core.The continuation of that same inequality is the truncated inclusion-exclusion——against an exact $\mathbf{0.8859739369}$, one term gives $\mathbf{1.3954082362}$, two give $\mathbf{0.8101316015}$ and three give $\mathbf{0.8882566015}$, alternating** (Section 3). **Fifth, the correction is conservative because it stops at the first term——the exact level under independence is $\mathbf{0.00256138}$ against $\alpha/k = \mathbf{0.00250000}$, stricter by $\mathbf{2.4552}$ per cent** (Section 4). **Sixth, the separator is where the series was cut——not whether independence was assumed** (Section 4). **Keywords**: multiple comparisons, Bonferroni correction, Boole's inequality, inclusion-exclusion principle, type I error.

Scope of this paper (scope note): **No new theorem or law is claimed——Boole's inequality, the Bonferroni inequalities and the Bonferroni correction are all standard.No testing procedure is recommended——which correction to use is not the subject, and the procedures of Holm and of Benjamini and Hochberg are not entered.The false discovery rate is not treated——only the family-wise error rate is examined.Correlated tests are not treated——the numbers are for independent tests, and the degree of conservatism under correlation is a separate matter.Historical attribution is not adjudicated——whether Boole or Bonferroni came first is not entered.It is not said that correcting makes an analysis right——holding a level and retaining power are different things.Relation to earlier papers**: **Paper 317 showed that a significance level is a promise per test, not per study——this paper shows that the correction is merely the first term of an inequality.Paper 360 showed that cutting an exact formula short brackets the answer——that bracketing and this correction are the same inequality, and the two papers had not cited each other.Paper 125 showed that there are three Hopfs——this runs the other way, showing that two settings share one Bonferroni. A census of proper names can divide or unite.Paper 300 showed that sharing a root is decidable——the verdict here is reached under the criterion of a correspondence that preserves identity.**What is added is **setting the two settings side by side as first and general term**, **putting the correction's effect at 0.641514 against 0.048830**, **measuring the alternating brackets to five terms**, **measuring the conservatism at 2.4552 per cent**, and **placing the separator on where the series was cut**.

1. Notation and the Question

Setting: **for events $A_1, \ldots, A_k$, the Bonferroni inequalities bracket the probability of the union by alternating sums of intersections,**

$$S_1 \;\geq\; P\Big(\bigcup A_i\Big) \;\geq\; S_1 - S_2\,, \qquad S_r = \sum_{|I|=r} P\Big(\bigcap_{i\in I} A_i\Big)$$

and using S_1 alone gives Boole's inequality.

Observation 1.1

one name appears in two settings.The multiple-comparison correction and the truncated inclusion-exclusion ——both are called Bonferroni.

Observation 1.2

a shared name settles nothing.As Paper 125 showed, one name can hide several people——here the question runs the other way: do two settings share one formula?

Question: are the correction and the bracketing the same inequality?

2. The Correction Uses Only the First Term

Computation 2.1: computing the chance of at least one error across k independent tests at level $\alpha = 0.05$.

k	uncorrected	level set to α/k
1	0.050000	0.050000
5	0.226219	0.049010
20	**0.641514**	**0.048830**
100	**0.994079**	**0.048782**

> **Observation 2.2**
>
> uncorrected, it approaches one.$\mathbf{0.994079}$ at $k = 100$——the situation Paper 317 described, now with a number.

> **Observation 2.3**
>
> corrected, it stays below 0.05 however large k is.0.048830 and 0.048782——it tends to $1 - e^{-\alpha}$ without exceeding α.

> **Observation 2.4**
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> the justification is one line.「和事象の確率は各確率の和以下」から $k \cdot (\alpha/k) = \alpha$——no independence, no normality, no shape of a test statistic. Only S_1.

3. The Bracketing Is Its Continuation

Computation 3.1: the probability that some face is missing when a six-sided die is rolled eight times, with inclusion-exclusion cut at r terms.

truncation	value	direction
one term (Boole)	**1.3954082362**	above
two terms	**0.8101316015**	below
three terms	**0.8882566015**	above
four terms	**0.8859703647**	below
five terms	**0.8859739369**	above
exact (six terms)	**0.8859739369**	—

Observation 3.2

this is the core.The one-term upper bound is 1.3954082362, above one——useless as a bound on a probability, and an upper bound nonetheless. That coarse first term is exactly what multiple comparison uses.

Observation 3.3

the direction alternates.Above, below, above, below——fixed by the parity of the truncation, which is what Paper 360 established.

Observation 3.4

an independent check was run.Two hundred thousand random trials give 0.886785 against the exact 0.885974——confirming the inclusion-exclusion formula by a method that does not use it.

Observation 3.5

three terms already close to 0.26 per cent.0.8882566015 against 0.8859739369——the gap between one and two terms is wide, and improvement is abrupt from the third.

4. The Separator

Observation 4.1

the separator is where the series was cut.Multiple comparison stops at S_1; the bracketing uses S_2 and beyond——one formula, differing in how many terms are kept.

Computation 4.2: measuring the price of stopping. The exact level under independence is $\mathbf{0.00256138}$ against $\alpha/k = \mathbf{0.00250000}$.

Observation 4.3

the conservatism is only $\mathbf{2.4552}$ per cent.Bonferroni is often called too conservative, and for independent tests the price is this size——speaking of conservatism without the number overstates it.

Observation 4.4

the conservatism does not come from declining to assume independence.It comes from stopping at S_1——subtract S_2 and a lower bound appears, still without independence. The separator is truncation, not assumption.

Observation 4.5

the second criterion of Paper 300 is met.The map from one setting to the other is a relabelling of A_i as "an error on the ith test"——not agreement of numbers but a correspondence one can write.

Observation 4.6

the orientation is opposite to Paper 125.There, one name was divided into three people——here, two settings are united under one. A census of proper names runs both ways.

Observation 4.7

what is not said.No correction is recommended——holding a level and retaining power are different, and neither the false discovery rate nor correlated tests were treated.

5. Roll Call and Closing

1. **uncorrected 0.641514 at $k = 20$** (Computation 2.1): **0.994079 at $k = 100$.**
2. **corrected 0.048830** (Computation 2.1): **below 0.05.**
3. **one line of justification** (Observation 2.4): **no independence, no normality.**
4. **one-term bound 1.3954082362** (Computation 3.1): **above one.**
5. **the direction alternates** (Observation 3.3): **by parity.**
6. **0.886785 by simulation** (Observation 3.4): **an independent check.**
7. **three terms give 0.26 per cent** (Observation 3.5): **abrupt improvement.**
8. **exact level 0.00256138** (Computation 4.2): **against 0.00250000.**
9. **conservatism 2.4552 per cent** (Observation 4.3): **the core.**
10. **the cause is truncation** (Observation 4.4): **not assumption.**
11. **opposite to Paper 125** (Observation 4.6): **uniting rather than dividing.**

Closing: **The multiple-comparison correction and the inequality that brackets a truncated inclusion-exclusion are taught as separate matters.The correction uses only the first term——twenty independent tests at level 0.05 give 0.641514 and a hundred give 0.994079, while setting the level to α/k holds these at 0.048830 and 0.048782. The justification is the single line 「和事象の確率は各確率の和以下」から $k \cdot (\alpha/k) = \alpha$, with no independence, no normality and no shape of a test statistic. The situation named in Paper 317 now carries the number 0.994079.The bracketing is that same inequality continued——the probability that a face is missing from eight rolls of a die is 0.8859739369, and cutting inclusion-exclusion at one term gives 1.3954082362, at two 0.8101316015, at three 0.8882566015, alternating. The one-term bound exceeds one and is useless as a bound on a probability, yet it is an upper bound, and it is exactly what multiple comparison uses. The alternation is fixed by parity, as Paper 360 established. The formula itself was checked against 200000 random trials giving 0.886785.The separator is where the series was cut——the exact level under independence is 0.00256138 against $\alpha/k = 0.00250000$, so the conservatism is only 2.4552 per cent, and speaking of it without the number overstates it. Nor does the conservatism come from declining to assume independence: it comes from stopping at S_1, since subtracting S_2 gives a lower bound that still needs no independence. Under the second criterion of Paper 300 the correspondence can be written down, relabelling A_i as an error on the ith test, so this is not agreement of numbers. Paper 125 divided one**

name into three people; this unites two settings under one, and a census of proper names runs both ways.To be honest——no correction is recommended here. Holding a level and retaining power are different things, the procedures of Holm and of Benjamini and Hochberg were not entered, and the false discovery rate was not treated. The numbers are for independent tests, and conservatism under correlation is a separate matter, as is the question of whether Boole or Bonferroni came first. What can be said is that two settings are the first and general terms of one inequality, and that what divides them is truncation rather than assumption, and no further.

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