Whether a Formula Was Borrowed or Derived Shows Outside the Working Range

— Fitts's original $\log_2(2D/W)$ and the Shannon form $\log_2(D/W+1)$ coincide at $D/W=1$, but at $D/W=0.25$ the original returns -1.000000 — a channel capacity can never be negative, since $S/N \geq 0$ — [Paper 536]

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— From thinking to syncing. —

Abstract

Fitts's law is often said to be derived from Shannon's channel capacity.What this paper shows is that the original form violates a property every capacity must have, so that borrowing rather than derivation can be established with numbers.No new theorem or law is claimed. First, the two forms meet at exactly one point——at $D/W=1$ both return 1.000000 (Section 2). **Second, and this is the core.The original goes negative——0.000000 at $D/W=0.5$ and -1.000000 at $D/W=0.25$, values impossible for an amount of information** (Section 2). **Third, a capacity cannot go negative——$C=B\log_2(1+S/N)$ is at least zero because $S/N \geq 0$, reaching exactly 0 at $S/N=0$** (Section 3). **Fourth, so the original does not satisfy the form of a capacity——the same $\log_2$ appears in both, and the properties they must satisfy differ** (Section 3). **Fifth, the gap does not vanish at large ratios——1.249317 at $D/W=10$, 1.148034 at 100, and still 1.075246 at 10000** (Section 4). **Sixth, the separator is testing outside the working range——inside it the two look alike; outside it only one breaks** (Section 5). **Keywords**: Fitts' law, Shannon-Hartley theorem, information theory, human-computer interaction, extrapolation.

Scope of this paper (scope note): **No new theorem or law is claimed——Fitts's law, the Shannon--Hartley formula and the later Shannon-form rewriting are all standard.The validity of Fitts's law is not discussed——how well it describes human movement is not the subject.No experimental data are treated——no goodness of fit and no values for the coefficients a and b are given.Historical intent is not inferred——what Fitts had in mind is not at issue; only the properties the formula satisfies.No interpretation of information theory is offered——whether human movement constitutes a channel is not entered.The rewriting is not called an improvement——it is shown only that the Shannon form does not go negative; which to use is not stated.No claim is made that small D/W is a practical regime——a target wider than the starting distance is an unusual design, and a property of a formula is still a property of the formula.Relation to earlier papers**: **Paper 345 showed that one and the same $\log_2$ counts a ratio in one law and items in the other——this paper views the same pair along a different axis, derivation against borrowing, comparing properties rather than what is counted.Paper 304 showed that capacity is a number attainable only at infinite length ——unattainable or not, a capacity does not go negative, and that property serves as the test here.Paper 300 showed that sharing a root is decidable——the two are separate roots under the criterion of route.**What is added is **setting the two formulas side by side beyond the working range, showing the original returns $\mathbf{-1.000000}$ at $D/W = 0.25$, confirming that capacity bottoms out at zero, measuring a residual ratio of $\mathbf{1.075246}$ even at $D/W = 10000$, and placing the separator on testing outside the range**.

1. Notation and the Question

Setting: **Fitts's law writes the time to reach a target of width W at distance D as**

$$MT = a + b \cdot ID, \qquad ID = \log_2\left(\frac{2D}{W}\right)$$

and a later convention rewrites the index as $ID = \log_2(D/W + 1)$.

Shannon's capacity: **the channel capacity is $C = B\log_2(1 + S/N)$.**

> **Observation 1.1**
>
> **the forms resemble each other.Both put a ratio inside a $\log_2$——Fitts himself cited Shannon's theorem, and the law has been read as coming from it.**

> **Observation 1.2**
>
> **resemblance is not derivation.Paper 345 showed that one $\log_2$ can count different things——here the comparison is not of what is counted but of what properties the formula must satisfy.**

Question: **can borrowing be separated from derivation with numbers?**

2. Going Outside the Range

Computation 2.1: **comparing the two indices over a wide range of D/W.**

D/W	original $\log_2(2D/W)$	Shannon form $\log_2(D/W+1)$
0.25	$\mathbf{-1.000000}$	**0.321928**
0.50	**0.000000**	**0.584963**
1.00	**1.000000**	**1.000000**
2.00	2.000000	1.584963
10.00	4.321928	3.459432
100.00	7.643856	6.658211

> **Observation 2.2**
>
> **they meet at one point only.Both give $\mathbf{1.000000}$ at $D/W = 1$——away from it they part in either direction.**

> **Observation 2.3**
>
> **this is the core.The original returns $\mathbf{0.000000}$ at $D/W = 0.5$ and $\mathbf{-1.000000}$ at $D/W = 0.25$——a negative difficulty, impossible as an amount of information.**

Observation 2.4

the Shannon form stays positive.$\mathbf{0.321928}$ even at $D/W = 0.25$——as long as $D/W > 0$, $\log_2(1 + D/W) > 0$ and the sign never turns.

Observation 2.5

the negative region is rare in experiments.It requires a target wider than the starting distance——which is why practice never found it. Without leaving the range the two cannot be told apart.

3. A Capacity Cannot Be Negative

Observation 3.1

$C = B\log_2(1 + S/N)$ is non-negative by construction.A signal-to-noise ratio cannot be negative, so $1 + S/N \geq 1$——it reaches exactly $\mathbf{0}$ at $S/N = 0$ and goes no lower.

Observation 3.2

this comes from the definition.Capacity is the logarithm of a number of distinguishable codewords——a count is at least one, so its logarithm is at least zero. There is no state of negative capacity.

Observation 3.3

so the original does not satisfy the form of a capacity.A formula returning -1.000000 is the capacity of no channel——the same $\log_2$, different obligations.

Observation 3.4

this fits alongside the fence of Paper 304.There, capacity is attainable only at infinite length——attainability and non-negativity are different properties, and being unattainable does not make it negative.

Observation 3.5

separate roots under the route criterion of Paper 300.Capacity comes from counting codewords——the original index puts a ratio of distance to width on a logarithmic scale. The roads are not shared.

4. How Far the Gap Persists

Computation 4.1: **measuring the ratio of the two indices as D/W grows.**

D/W	original over Shannon form
2	**1.261860**
10	**1.249317**
100	**1.148034**
10000	**1.075246**

Observation 4.2

the ratio shrinks monotonically.From $\mathbf{1.261860}$ to $\mathbf{1.075246}$——at large D/W the two formulas approach each other.

Observation 4.3

and never vanishes.Still $\mathbf{7.5}$ per cent apart at $D/W = 10000$——a constant offset of $\log_2 2 = 1$ remains, diluted in relative terms as the logarithm grows but never zero.

Observation 4.4

fitting absorbs it.The a and b of $MT = a + b \cdot ID$ are free——a absorbs the constant offset and b the proportional one, so goodness of fit alone cannot separate the two.

5. The Separator

Observation 5.1

the separator is testing outside the working range.Inside it the two are alike and the coefficients absorb the difference——outside it, only one breaks.

Observation 5.2

the manner of breaking is the claim.That no state of negative information exists is a property on the information-theoretic side——a formula violating it was not derived from information theory.

Observation 5.3

this is not a complaint about borrowing.Borrowing a form to build a useful law is not in itself an error——the error is in calling what was borrowed a derivation.

Observation 5.4

the axis differs from Paper 345.That paper divided what the two logarithms count——this one divides them by whether a required property is violated. Counting different things and lacking a derivational link are separate claims.

Observation 5.5

what is not said.The validity of Fitts's law is not discussed——how well it describes movement is not the subject, and no data or fitted coefficients appear.

6. Roll Call and Closing

1. **both give $\mathbf{1.000000}$ at $D/W = 1$** (Computation 2.1): **one meeting point.**
2. **original gives $\mathbf{0.000000}$ at 0.5** (Computation 2.1): **difficulty vanishes.**

3. $\mathbf{-1.000000}$ **at** $\mathbf{0.25}$ (Observation 2.3): **the core.**
4. **Shannon form gives** $\mathbf{0.321928}$ (Observation 2.4): **positive throughout.**
5. **capacity is** $\mathbf{0}$ **at** $S/N = 0$ (Observation 3.1): **and no lower.**
6. **no negative capacity exists** (Observation 3.2): **it is a logarithm of a count.**
7. **ratio** $\mathbf{1.249317}$ **at** $D/W = 10$ (Computation 4.1): $\mathbf{1.148034}$ **at** 100**.**
8. $\mathbf{1.075246}$ **at** 10000 (Observation 4.3): **it does not vanish.**
9. **coefficients absorb it** (Observation 4.4): **fit cannot separate them.**
10. **the separator is outside the range** (Observation 5.1): **inside they are alike.**
11. **borrowing is not the error** (Observation 5.3): **calling it derivation is.**

Closing: **Fitts's law is often said to be derived from Shannon's channel capacity.Outside the working range the two part——the original** $\log_2(2D/W)$ **and the Shannon form** $\log_2(D/W + 1)$ **meet only at** $D/W = 1$**, where both return** $\mathbf{1.000000}$**. The original returns** $\mathbf{0.000000}$ **at** $D/W = 0.5$ **and** $\mathbf{-1.000000}$ **at** $D/W = 0.25$**, a negative difficulty. The Shannon form gives** $\mathbf{0.321928}$ **even there, and never changes sign for** $D/W > 0$**. The negative region needs a target wider than the starting distance, which is why practice never found it: without leaving the range the two cannot be told apart.A capacity cannot be negative——**$C = B\log_2(1 + S/N)$ **is non-negative because** $S/N \geq 0$**, reaching exactly** $\mathbf{0}$ **at** $S/N = 0$**. This follows from the definition: capacity is the logarithm of a count of distinguishable codewords, a count is at least one, and there is no state of negative capacity. A formula returning** -1.000000 **is therefore the capacity of no channel, whatever logarithm it uses. Paper 304 showed capacity is attainable only at infinite length, and attainability is a different property from sign: being unattainable does not make it negative.The gap persists——the ratio is** $\mathbf{1.261860}$ **at** $D/W = 2$**,** $\mathbf{1.249317}$ **at** 10**,** $\mathbf{1.148034}$ **at** 100 **and still** $\mathbf{1.075246}$ **at** 10000**. It shrinks monotonically and never reaches zero, since a constant offset of** $\log_2 2 = 1$ **remains. But** $MT = a + b \cdot ID$ **leaves** a **and** b **free, so the constant offset goes into** a **and the proportional one into** b**, and goodness of fit alone cannot separate them.The separator is testing outside the working range——inside it the two are alike and absorbed by coefficients; outside it only one breaks, and the manner of breaking is the claim. That no state of negative information exists belongs to information theory, and a formula violating it was not derived from information theory. Under the route criterion of Paper 300, capacity comes from counting codewords while the original index puts a ratio on a logarithmic scale, and those roads are not shared. Paper 345 divided what the two logarithms count; this divides them by a violated property, which is a different axis.To be honest——this is not a complaint about borrowing. Building a useful law on a borrowed form is not an error; calling the borrowing a derivation is. The validity of Fitts's law was not discussed, no experimental data or fitted coefficients appear, historical intent was not inferred, and whether human movement constitutes a channel was not entered. The rewriting was not called an improvement. What can be said is that the original violates a property every capacity must have, and that this shows in numbers once one leaves the working range, and no further.**

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