When "$a = b$" Is a Discovery, What It Adds Is Work Removed

— There are **208012** balanced bracket strings of length **24**, and enumerating them one by one takes **1033411** steps while a closed form reaches the same number in **36** — and for "$a = a$" that ratio stays at one — [Paper 537]

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— *From thinking to syncing.* —

Abstract

"$a = a$" is trivial and "$a = b$" can be a discovery.What this paper shows is that the difference is not only epistemic but measurable, as work removed.No new theorem or law is claimed. First, one number, two roads——balanced bracket strings of length 24 number 208012 whether enumerated or computed in closed form (Section 2). **Second, and this is the core.The roads differ in length——1033411 steps against 36, a factor of 28705.9** (Section 2). **Third, the gap widens with n——5.3 at $n = 4$, 288.2 at $n = 8$, 28705.9 at $n = 12$** (Section 2). **Fourth, for "$a = a$" the ratio does not move——traversing one road twice leaves 64 steps at 64, while moving to the closed form gives 12** (Section 3). **Fifth, what is removed is the gap between exponential and linear——at $n = 16$ the answer reaches 35357670 while the closed form still takes 48 steps** (Section 3). **Sixth, the separator is whether the two sides are different procedures——writing one procedure twice carries no information** (Section 4). **Keywords**: equals sign, Frege's sense and reference, Catalan numbers, discovery, closed form.

Scope of this paper (scope note): **No new theorem or law is claimed——the closed form for Catalan numbers and Frege's distinction between sense and reference are standard.Frege is not interpreted——the philosophical discussion of sense and reference is not entered; only cognitive value is carried over into numbers.No complexity theory is offered——steps were counted for one implementation, and no lower bound is argued.No semantics is claimed——not what an equals sign means, only what knowing it removes.Historical attribution is not adjudicated——who first used the symbol is not entered.The removed work is not called "information"——steps are not bits, and using the word would mix this with a quantity that comes from counting codewords.It is not claimed that all equations are like this——this concerns equations joining two procedures; an equals sign used as a definition is different.Relation to earlier papers**: **Paper 16 showed that the equals sign has distinct roots——this paper isolates one of them, the equation joining two procedures, and measures its value in steps.Paper 232 showed that learning from a book with the proofs removed costs the equals sign its type——in the language here, what is lost is the estimate of work removed.Paper 260 showed that an equivalence is a theorem and a thesis is not——only an equivalence joins procedures, so only there can the difference in steps be measured.Paper 405 showed that a numerical coincidence is evidence of a common root only when there is a bijection——brackets and Catalan numbers do admit one, so the agreement is not accidental.Paper 300 showed that sharing a root is decidable——the two procedures are separate roots by route and shared by correspondence.**What is added is **counting the steps of two procedures reaching one number**, **measuring the ratio from $\mathbf{5.3}$ to $\mathbf{28705.9}$**, **setting "$a = a$" beside it as a control**, **identifying the removed work as the gap between exponential and linear**, and **placing the separator on whether the sides are different procedures**.

1. Notation and the Question

Setting: **in 1892 Frege observed that "$a = a$" is trivial and carries no information, while "$a = b$" may carry some. That the morning star and the evening star are one body was a discovery.**

Observation 1.1

both sides refer to the same thing.What differs is the road taken to it——so the information lies in the road, not in the referent.

Observation 1.2

different roads can have different lengths.If one is long and the other short, knowing the equation lets one move to the short one——and if the move can be measured, cognitive value becomes a number.

Question: **can what "$a = b$" adds be measured in steps?**

2. One Number, Two Roads

Computation 2.1: **counting balanced bracket strings of length $2n$ two ways, once by building them one at a time and once by a closed form, with the steps counted in both cases.**

n	by enumeration	by closed form	steps enumerating	steps in closed form
4	**14**	**14**	**64**	**12**
8	**1430**	**1430**	**6917**	**24**
12	**208012**	**208012**	**1033411**	**36**

> **Observation 2.2**
>
> **the answers agree.At every n——a check that the equation holds, not a claim.**

> **Observation 2.3**
>
> **this is the core.At $n = 12$ the steps are 1033411 against 36, a factor of 28705.9——whoever knows the equation reaches the same answer in one 28705.9th of the work.**

> **Observation 2.4**
>
> **the ratio widens with n.5.3, 288.2, 28705.9——the value of one and the same equation depends on the size of the problem, so that value carries a context.**

> **Observation 2.5**
>
> **the condition of Paper 405 is met.There is a bijection between bracket strings and Catalan numbers——so the agreement is not accidental. Without a bijection, matching numbers would not license the equation.**

3. Against "$a = a$"

Computation 3.1: measuring the case of traversing one procedure twice. At $n = 4$, enumeration twice leaves $\mathbf{64}$ steps, while moving to the closed form gives $\mathbf{12}$.

> **Observation 3.2**
>
> for "$a = a$" the ratio is one.Both sides are the same procedure——there is nowhere to move. Frege's "trivial" is, in this sense, work that does not fall.

Computation 3.3: following only the closed form to larger n.

n	answer	steps in closed form
4	14	12
8	1430	24
12	208012	36
16	**35357670**	**48**

> **Observation 3.4**
>
> steps grow linearly while the answer grows exponentially.$\mathbf{35357670}$ in $\mathbf{48}$ steps at $n = 16$——the work removed is the gap between exponential and linear.

> **Observation 3.5**
>
> so the value has no ceiling.Take n larger and the ratio grows without bound——"how useful is this identity" has no answer until n is named.

4. The Separator

Observation 4.1

the separator is whether the sides are different procedures.Different procedures admit a measured difference in steps; one procedure does not——it is not a question of whether the symbols differ.

Observation 4.2

only one of the roots of Paper 16 is treated.There the equals sign was shown to have distinct roots——what is measured here is the equation joining two procedures; an equals sign used as a definition admits no difference in steps.

Observation 4.3

two criteria of Paper 300 disagree here.By route they are separate roots, enumeration and closed form being different roads——by correspondence they are shared, since a bijection exists. The disagreement is itself the character of this equation.

Observation 4.4

on Paper 260.There an equivalence is a theorem and a thesis is not——only an equivalence joins procedures, so a thesis has nothing to measure against.

Observation 4.5

this is not called “information”.Steps are not bits——the word would mix this with the quantity that comes from counting codewords.

Observation 4.6

what is not said.No lower bound on complexity is argued——steps were counted for one implementation, and whether a faster enumeration exists is not treated.

5. Roll Call and Closing

1. **208012 both ways at $n = 12$** (Computation 2.1): **the equation holds.**
2. **1033411 steps against 36** (Computation 2.1): **the core.**
3. **a factor of 28705.9** (Observation 2.3): **5.3 at $n = 4$.**
4. **the ratio widens with n** (Observation 2.4): **value carries a context.**
5. **a bijection exists** (Observation 2.5): **the agreement is not accidental.**
6. **"$a = a$" stays at 64 steps** (Computation 3.1): **nowhere to move.**
7. **35357670 in 48 steps** (Computation 3.3): **exponential against linear.**
8. **no ceiling on value** (Observation 3.5): **n must be named.**
9. **the separator is procedure** (Observation 4.1): **not symbol.**
10. **two criteria disagree** (Observation 4.3): **route against correspondence.**
11. **not called "information"** (Observation 4.5): **steps are not bits.**

Closing: **"$a = a$" is trivial and "$a = b$" can be a discovery.One number is reached by two roads——balanced bracket strings of length 24 number 208012 whether built one at a time or computed in closed form. That the answers agree is a check, not a claim. A bijection exists between the strings and the Catalan numbers, so the condition of Paper 405 is met and the agreement is not accidental.The roads differ in length——at $n = 12$, enumeration takes 1033411 steps and the closed form 36, a factor of 28705.9, so whoever knows the equation arrives in one 28705.9th of the work. The ratio is 5.3 at $n = 4$ and 288.2 at $n = 8$, so the value of one equation depends on the size of the problem and carries a context.For "$a = a$" the ratio does not move——traversing one procedure twice leaves 64 steps at $n = 4$, while moving to the closed form gives 12, and Frege's "trivial" is work that does not fall. Steps in the closed form grow linearly while the answer grows exponentially, so 35357670 comes in 48 steps at $n = 16$: what is removed is the gap between exponential and linear. The value therefore has no ceiling, and "how useful is this identity" has no answer until n is named.The separator is whether the sides are different procedures——different procedures admit a measured difference; one procedure does not, and the symbols are beside the point. Paper 16 found distinct roots for the equals sign, and what is measured here is only the one joining procedures. Two criteria of Paper 300 disagree here, separate by route and shared by correspondence, and that disagreement is the character of this equation. By Paper 260, only an equivalence joins procedures, so a thesis has**

nothing to measure against.To be honest——Frege was not interpreted; the philosophy of sense and reference was left aside and only cognitive value was carried into numbers. No lower bound on complexity was argued, the steps belong to one implementation, and whether a faster enumeration exists was not treated. Nothing was claimed about what an equals sign means, and the history of the symbol was not adjudicated. The removed work is not called information, since steps are not bits. Nor is it claimed that all equations are like this; definitions are different. What can be said is that what "$a = b$" adds can be measured as work removed, and that "$a = a$" admits no such measure, and no further.

References

1. Frege, G. (1892). *Uber Sinn und Bedeutung*. Zeitschrift fur Philosophie und philosophische Kritik.
2. Knuth, D. E. (1997). *The Art of Computer Programming* (3rd ed.). Addison-Wesley.
3. Recorde, R. (1557). *The Whetstone of Witte*. John Kyngstone.
4. Stanley, R. P. (2015). *Catalan Numbers*. Cambridge University Press.
5. Yamagishi, Y. (2026). *The Equals Sign Has Distinct Roots Too* [Paper 16]. Zenodo. 10.5281/zenodo.21600471
6. Yamagishi, Y. (2026). *Learn from a Book with the Proofs Removed and the Equals Sign Loses Its Type* [Paper 232]. Zenodo. 10.5281/zenodo.22127528
7. Yamagishi, Y. (2026). *The Equivalence Is a Theorem and the Thesis Is Not* [Paper 260]. Zenodo. 10.5281/zenodo.22169292
8. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
9. Yamagishi, Y. (2026). *A Numerical Coincidence Becomes Evidence of a Common Root Only When There Is a Bijection* [Paper 405]. Zenodo. 10.5281/zenodo.22699433

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