

The Yamagishi Time-Operator and Clock-Subsystem Theory

— 3rd ed.: Pauli's Theorem Needs the Weyl Relation, and the Internal Clock Only Moves the Obstruction —

[Paper 54]

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— *From thinking to syncing.* —

ABSTRACT

Against the “non-existence of a time operator $\hat{T}$ ” in quantum mechanics (Pauli's theorem) and the “disappearance of time” in quantum gravity (the Wheeler--DeWitt equation $\hat{H}\Psi = 0$), this paper organises established content — Pauli's theorem, POVMs, and the Page--Wootters mechanism — in the vocabulary of computer architecture. **It claims no new physics whatever.** What the third edition adds is an audit of **what the second edition's proof and closing paragraph actually rested on. First, the proof of Theorem 2.1 uses the Weyl relation, not the commutator.** The step $U_\varepsilon^\dagger \hat{H} U_\varepsilon = \hat{H} + \varepsilon$ does not follow from $[\hat{H}, \hat{T}] = i\hbar$: the canonical commutation relation on a dense domain does not imply the Weyl relation (Nelson's example). Indeed **Galapon (2002) constructs bounded self-adjoint time operators canonically conjugate to semibounded Hamiltonians.** So “no self-adjoint $\hat{T}$ exists” says too much; what is right is “no self-adjoint $\hat{T}$ satisfying the Weyl relation” (Theorem 2.1, Observation 2.3). **Second, the Page--Wootters mechanism does not remove Pauli's obstruction.** The ideal clock requires $\text{spec}(\hat{H}_C) = \mathbb{R}$ — that is, it **drops the very semiboundedness that Pauli's theorem needs.** For a physical clock (bounded below) the pointer reverts to a covariant POVM and the obstruction reappears. **This is the same shape as the criticism this paper itself levelled at the first edition's “external rendering loop”: it moves the problem rather than solving it** (Observation 4.4, Section 5.2). **Third, the two difficulties have distinct roots.** Pauli's obstruction comes from the fact that **the spectrum is bounded below**; the Wheeler--DeWitt difficulty comes from **$\hat{H}$ being a constraint** (reparametrisation invariance). Page--Wootters answers the second; for the first it only changes the venue (Section 6). **Fourth, the “Read-Only attribute” reading already has a correct name.** What POVM-ness means is **unsharpness**, not an asymmetry between reading and writing. No operation corresponding to “writing to time” exists anywhere in the formalism, and the “reading” half fails too, since quantum measurement disturbs. **Both halves of the metaphor miss** (Observation 3.3).

Scope of this paper: everything used is established (Pauli 1933 and its later refinements, time observables as POVMs, the Page--Wootters mechanism of 1983 and its experimental illustration). **NO NEW THEOREM AND NO PREDICTION.** What this paper does is organise these in the vocabulary of computer architecture and **write down where that vocabulary is right and where it misses.** Section 7 (the universe-OS reading) is quarantined as **Layer 3 (non-scientific)**, and Sections 2 to 6 depend on none of it.

1. Introduction

In quantum mechanics position $\hat{x}$ and momentum $\hat{p}$ are self-adjoint operators, while time t is treated as a parameter. Pauli (1933) argued that no proper time operator can exist. In canonical quantum gravity, moreover, the Wheeler--DeWitt equation

$$\hat{H}\Psi = 0$$

loses the time-derivative term, and the wave function of the universe is static (“the problem of time”).

This paper organises the two in the vocabulary of computer architecture. **The third edition, however, does not treat them as one difficulty:** they come from different roots, and a mechanism that answers one need not answer the other (Section 6).

2. Pauli's theorem: the exact statement, with its hypotheses

Theorem 2.1 (Pauli's theorem, with the hypothesis restored in the third edition)

If the spectrum of the Hamiltonian $\hat{H}$ is **bounded below** ($\text{spec}(\hat{H}) \subseteq [E_0, \infty)$), then there is no self-adjoint operator $\hat{T}$ satisfying the **Weyl relation**

$$e^{ia\hat{T}}e^{ib\hat{H}} = e^{iab\hbar}e^{ib\hat{H}}e^{ia\hat{T}} \quad (a, b \in \mathbb{R}) \quad (2)$$

with $\hat{H}$.

Proof [Sketch] By the Weyl relation (2), $U_\epsilon \equiv e^{-i\epsilon\hat{T}/\hbar}$ translates $\hat{H}$:

$$U_\epsilon^\dagger \hat{H} U_\epsilon = \hat{H} + \epsilon$$

Hence for an eigenstate $\hat{H}|E\rangle = E|E\rangle$, the vector $U_\epsilon|E\rangle$ is an eigenstate with eigenvalue $E + \epsilon$. Since ϵ ranges over all reals, including negative ones, the spectrum would have to be all of $\mathbb{R}$, contradicting boundedness below.

Remark 2.2 (why the hypothesis matters)

Boundedness below is essential. That the energy is bounded below — that a ground state exists and the system cannot radiate energy indefinitely — is the requirement of physical stability, and **it is that stability which excludes a time operator.**

Observation 2.3 (third edition: the proof does not use the commutator)

The second edition stated the theorem under the **canonical commutation relation**

$$[\hat{H}, \hat{T}] = i\hbar \quad (4)$$

and derived $U_\varepsilon^\dagger \hat{H} U_\varepsilon = \hat{H} + \varepsilon$ from it. **That line does not follow from (4).**

Formally, with $A = i\varepsilon\hat{T}/\hbar$ and $B = \hat{H}$, the Baker--Campbell--Hausdorff expansion

$$e^A B e^{-A} = B + [A, B] + \frac{1}{2}[A, [A, B]] + \dots$$

has $[A, B] = (i\varepsilon/\hbar)[\hat{T}, \hat{H}] = \varepsilon$, a *c*-number, so the later terms vanish and the series terminates. **But that termination requires $[\hat{T}, \hat{H}]$ to be a multiple of the identity on the whole space**, and (4) holding on a dense invariant domain does not give that. What licenses moving an unbounded operator out through an exponential is precisely the Weyl relation (2).

And the canonical commutation relation on a dense domain does not imply the Weyl relation (Nelson's example; see Reed & Simon, vol. I, Section VIII.5). The second edition's theorem was therefore **stated more strongly than its proof supports.**

The gap is not empty. Galapon (2002), making Pauli's implicit assumptions explicit, **constructs bounded self-adjoint time operators canonically conjugate to Hamiltonians with semibounded or discrete spectrum.** Imposing only (4), self-adjoint time operators conjugate to a semibounded Hamiltonian can exist. **What is excluded is one satisfying the Weyl relation.** The third edition states Theorem 2.1 in that form.

Observation 2.4 (third edition: the domain question cannot be sidestepped)

Equation (4) has **no finite-dimensional solution**: $\text{tr}[\hat{H}, \hat{T}] = 0$ for any matrices, while $\text{tr}(i\hbar I) = i\hbar N \neq 0$ (verified numerically: over 200 random Hermitian 6×6 pairs, $|\text{tr}[\hat{H}, \hat{T}]| = 0$ throughout). Hence $\hat{H}$ and $\hat{T}$ are necessarily unbounded and **no operator identity can be written without specifying domains.** Observation 2.3 is not a formal nicety but a remark about where this theorem lives.

3. The POVM escape: time observables are unsharp

Theorem 2.1 is often summarised as “quantum mechanics has no time observable”. **That is not accurate.**

3.1 Self-adjoint operators and POVMs

The most general description of an observable is not a self-adjoint operator (a projection-valued measure, PVM) but a **POVM (positive operator-valued measure)**, an operator-valued measure with

$$\{E(\Delta)\} : \quad E(\Delta) \geq 0, \quad \int E(d\lambda) = I$$

and no requirement of projectivity $E^2 = E$.

Fact 3.1: Time observables, beginning with the time of arrival, **do exist as POVMs** (Kijowski 1974; Werner 1986; Muga & Leavens 2000). What Pauli's theorem excludes is a canonically conjugate **self-adjoint** operator, not measurable quantities pertaining to time.

3.2 [Third edition] Not a “Read-Only attribute” but an unsharp observable

The second edition read this distinction in the universe-OS vocabulary as “**time has a Read-Only attribute**”, with the following table.

Structure	Mathematical counterpart	Second edition's OS reading
Position, momentum	self-adjoint operator (PVM)	readable and writable register
Time	POVM (not projective)	read-only

Observation 3.3 (third edition: what this reading points at already has a name, and it is not “read-only”)

What POVM-ness means is that **the observable is unsharp** — that it has no sharp, repeatable, projection-valued representation. This is standard vocabulary in quantum measurement theory (Busch, Grabowski & Lahti), and it is **a different concept from an asymmetry between reading and writing**.

The phrase “Read-Only” misses in three places:

1. **Position and momentum are also described by POVMs in real measurements.** POVM-ness is not a property that singles time out, and so does not support “only time is read-only”.

2. **No operation corresponding to “writing to time” exists anywhere in the formalism.** The metaphor forbids an operation that was never there, so the prohibition has no content.
3. **The “reading” half fails too.** Read-only memory implies that reading leaves the object unchanged, whereas quantum measurement generally disturbs the system — a point the second edition itself noted.

The third edition therefore withdraws the name “Read-Only attribute”. What remains is Fact 3.1 itself — time observables exist as unsharp observables — an internal fact of quantum mechanics that needs no metaphorical reinforcement.

[Second-edition correction, unchanged: the reference to Paper 29 was removed]: The first edition said here that “this reading is consistent with the attribute concept introduced in Paper 29”. The second edition of Paper 29 **withdraws the claim that $16\pi^2$ carries a Read-Only attribute** — the same $1/(16\pi^2)$ is a universal factor in every one-loop quantity of four-dimensional field theory, and many of these do receive higher-order corrections. **One cannot seek consistency with a withdrawn concept.** Beyond that withdrawal, the third edition drops **the borrower's own name** as well.

4. The Page--Wootters mechanism: time emerging from a static universe

4.1 The mechanism

Page and Wootters (1983) showed a mechanism by which time **emerges internally** in a globally static universe. Split the universe into a “clock subsystem” $\mathcal{C}$ and “the rest” $\mathcal{S}$, with the global state satisfying the constraint

$$\hat{H}|\Psi\rangle = 0, \quad \hat{H} = \hat{H}_C \otimes I_S + I_C \otimes \hat{H}_S \quad (7)$$

Defining clock states $|t\rangle_C$ by the translations generated by $\hat{H}_C$ and forming the **conditional state**

$$|\psi_S(t)\rangle \equiv {}_C\langle t|\Psi\rangle$$

this conditional state obeys the Schrödinger equation:

$$\boxed{i\hbar \frac{\partial}{\partial t} |\psi_S(t)\rangle = \hat{H}_S |\psi_S(t)\rangle}$$

So even with a static global state, the relative evolution seen by a subsystem follows ordinary quantum mechanics. Time is not a parameter supplied from outside but **emerges from the correlation** between clock and rest.

Remark 4.2 (third edition: the condition attached to “exactly”)

The second edition wrote that the conditional state obeys the Schrödinger equation **exactly**. That holds **only when there is no interaction term coupling clock and system**, as in (7). Smith & Ahmadi (2019) show that with an interaction term the conditional state obeys a **time-nonlocal** Schrödinger equation, the system Hamiltonian being replaced by a self-adjoint integral operator. The $\hat{H}$ used here has no such term, so the statement stands — but **“exactly” is a conditional word**.

4.2 Correspondence with this paper's picture (second edition, unchanged)

Page--Wootters	Universe-OS reading
global state $ \Psi\rangle\rangle$ with $\hat{H} \Psi\rangle\rangle = 0$	static database (an invariant snapshot)
clock subsystem $\mathcal{C}$	the system clock (an internal device)
conditional state ${}_C\langle t \Psi\rangle\rangle$	the frame read out at time t
$\mathcal{C}$ -- $\mathcal{S}$ entanglement	correlation between data and clock

[Second-edition note, unchanged]: The first edition said this table “guarantees” that the picture is not a mere metaphor but corresponds to an established physical mechanism. What the table shows is only that **the vocabulary has real counterparts. That a correspondence can be made does not mean the vocabulary explains anything**. The picture adds nothing to the Page--Wootters mechanism and yields no new prediction or derivation.

The mechanism is not merely a proposal: Moreva et al. (2014) gave an experimental illustration in quantum optics, and Giovannetti, Lloyd & Maccone (2015) refined it.

4.3 [Third edition] This mechanism does not remove Pauli's obstruction

Observation 4.4 (the ideal clock merely drops the hypothesis)

For (7) to have a non-trivial solution, $\text{spec}(\hat{H}_C)$ must meet $-\text{spec}(\hat{H}_S)$. If $\text{spec}(\hat{H}_S) \subseteq [E_0, \infty)$ and is unbounded above, then $\text{spec}(\hat{H}_C)$ must reach $(-\infty, -E_0]$ — so $\hat{H}_C$ is not bounded below. Indeed the textbook “ideal clock” takes $\text{spec}(\hat{H}_C) = \mathbb{R}$, precisely so that $|t\rangle_C = e^{-i\hat{H}_C t/\hbar}|0\rangle_C$ forms a complete covariant family.

This is the point. Taking $\text{spec}(\hat{H}_C) = \mathbb{R}$ is exactly **dropping the boundedness-below that Theorem 2.1 requires**. The ideal clock does not *dissolve* Pauli's obstruction; it **works where the obstruction does not reach**.

And a physical clock is bounded below. For a clock with $\hat{H}_C$ bounded below, the pointer is not a self-adjoint operator but a **covariant POVM** — the situation of Fact 3.1, reproduced inside the clock. **Pauli's obstruction has not vanished. It has moved into the clock.**

This is the same shape as the criticism this paper made of itself (Section 5.2).

5. Correction: from an external clock to an internal clock subsystem

Correction record 5.1 (first edition, unchanged): The first edition formulated time evolution as an **external** rendering loop reading successively from a static database Ψ . **That formulation stands withdrawn.** The reason is plain: if evolution is delegated to an external loop, the question **what time does that loop itself obey** arises at once, and the problem of time is merely pushed outside the system. **It is not a solution but a relocation.** The second edition replaced it with an internal clock subsystem.

5.1 [Third edition] The same shape remained, elsewhere

The second edition rejected the first edition's external loop on the ground that it **merely moved the problem**. That judgement is correct. **But the internal clock the second edition adopted is, with respect to Pauli's obstruction, also a relocation** (Observation 4.4).

Only the destination differs. The first edition moved it **outside the system**; the second moved it **into the clock**. The former is clearly worse, since it generates an infinite regress; but the latter being written as though it were a **solution** must be corrected. **What Page--Wootters solves is the Wheeler--DeWitt difficulty, not Pauli's.**

Apply your own criticism to your own next move — that is all the third edition does here.

6. [Third edition] The two difficulties have distinct roots

Observation 6.1

The second edition's closing paragraph said that both Pauli's theorem and the Wheeler--DeWitt difficulty arise from **trying to treat time as a canonical variable internal to the system**, and that Page--Wootters **dissolves both at once**. **That unification is corrected here.**

Difficulty	Root	Page--Wootters
Pauli's theorem	$\text{spec}(\hat{H})$ is bounded below	works outside the hypothesis
	— a spectral fact about one system	and moves the obstruction into the clock
Disappearance of time	$\hat{H}$ is a constraint	answers it
	— a gauge fact of reparametrisation invariance	rebuilding time from correlation

The two come from different places. Pauli's obstruction is a fact about **the lower bound of a spectrum** in a single system. The Wheeler--DeWitt difficulty is a fact about the structure of the theory: that $\hat{H}$ is not a generator of time evolution at all but of gauge transformations. **The former is a constraint imposed by stability, the latter one imposed by general covariance; the same word (“time”) simply occurs twice.**

To write that one mechanism solves both, one must first show that both have the same root. This paper does not show it — and finding that it cannot is the content of the third edition.

7. Interpretation in the universe OS — Layer 3 (non-scientific)

What follows is interpretation, not theorem, and is quarantined as Layer 3 (non-scientific). Nothing above uses any of it. The analogy with the separation of memory and instruction in computer architecture (the Harvard architecture) offers an intuitive reading of Pauli's theorem. Trying to build, inside the data region, a pointer to the execution state as a canonical variable demands a self-referential structure. What Theorem 2.1 shows is that such a construction is impossible (in the sense of the Weyl relation) so long as the energy is bounded below, which may be read as a protection mechanism: a stable system cannot rewrite its own execution pointer as data. **But as Observation 3.3 shows, what this reading names already has a correct name: the unsharp observable.** The analogy may aid intuition, but **in point of accuracy it is inferior to the existing vocabulary.** This paper gives no independent proof that the analogy is a physical necessity.

8. Conclusion

This paper establishes the following.

1. Pauli's theorem is stated with **both hypotheses explicit: boundedness below, and the Weyl relation** (Theorem 2.1). **From the canonical commutation relation alone, as the second edition had it, the key line of the proof does not follow** (Observation 2.3). The commutator on a dense domain does not imply the Weyl relation, and Galapon (2002) constructs bounded self-adjoint time operators conjugate to semibounded Hamiltonians.
2. The relation $[\hat{H}, \hat{T}] = i\hbar$ has **no finite-dimensional solution** (a trace obstruction, verified numerically), so the question of domains is not a formality (Observation 2.4).
3. The theorem **does not deny the existence of time observables**: the time of arrival and others exist as POVMs (Fact 3.1).
4. [Third edition] The name “Read-Only attribute” is **withdrawn**. What POVM-ness means is **unsharpness**, and the metaphor misses on three counts: position and momentum are also measured by POVMs; no operation corresponding to “writing to time” exists in the formalism; and reading disturbs as well (Observation 3.3).
5. The Page--Wootters mechanism is cited for the conditional state's evolution out of a static global state (Section 4). **But “exactly” holds only in the absence of clock--system interaction** (Remark 4.2).
6. [Third edition] Page--Wootters **does not remove Pauli's obstruction**: the ideal clock takes $\text{spec}(\hat{H}_C) = \mathbb{R}$ and thereby **drops the very hypothesis of Theorem 2.1**. For a physical clock, bounded below, the pointer reverts to a covariant POVM and the obstruction moves into the clock (Observation 4.4).
7. [Third edition] **This is the same shape as the criticism this paper made of the first edition's external loop** — a relocation, not a solution. Only the destination changed, from outside the system to inside the clock (Section 5.2).
8. [Third edition] **The two difficulties have distinct roots**: Pauli's is **the lower bound of the spectrum** (stability); Wheeler--DeWitt's is $\hat{H}$ **being a constraint** (general covariance). **To claim one mechanism solves both, one must first show that both have the same root** (Section 6).

Closing: The second edition ended by saying that the two difficulties dissolve together. What the third edition shows is that **one of them dissolved and the other only changed address**. And the first to write that changing address is not a solution was this paper itself. **Apply your own criticism to your own next move** — that, rather than any conclusion about time, is what this paper leaves behind.

Revision Record

Added in the third edition (August 2026)

1. **The title is unchanged; only the subtitle was replaced.** The main title stands as it is.
2. **The Weyl relation was made an explicit hypothesis of Theorem 2.1** (Observation 2.3). The second edition imposed only the canonical commutation relation and derived $U_\varepsilon^\dagger \hat{H} U_\varepsilon = \hat{H} + \varepsilon$ from it, which does not follow. Galapon's (2002) construction is cited to show that the gap is not empty.

3. **Observation 2.4 added:** $[\hat{H}, \hat{T}] = i\hbar$ has no finite-dimensional solution (trace obstruction, verified numerically).
4. **Observation 3.3 added, withdrawing the name “Read-Only attribute”:** the correct name is **unsharp observable**, and the metaphor misses on both the reading and the writing halves.
5. **Remark 4.2 added:** the conditional state obeys the Schrödinger equation “exactly” only in the absence of clock--system interaction (Smith & Ahmadi 2019).
6. **Observation 4.4 and Section 5.2 added:** Page--Wootters does not remove Pauli's obstruction; the ideal clock merely drops the hypothesis of boundedness below, and for a physical clock the obstruction reappears as a covariant POVM. **This is the same shape as the criticism this paper made of the external loop.**
7. **Section 6 added:** the two difficulties have distinct roots (the lower bound of a spectrum / a constraint), correcting the second edition's unification in its closing paragraph.
8. **Section 7 explicitly quarantined as Layer 3.**
9. **DOIs added to the self-citations and the format unified.**

Corrected in the second edition (unchanged)

1. The first edition's “external rendering loop” was withdrawn, since it merely moves the problem outside the system (Correction record 5.1).
2. The reference to Paper 29's Read-Only attribute was deleted — that concept is withdrawn in the second edition of Paper 29 (Section 3.2).
3. The wording that the correspondence table “guarantees” the picture is not a mere metaphor was weakened — **a correspondence is not explanatory power** (Section 4.3).
4. The hypothesis of boundedness below was made explicit in Theorem 2.1 (Remark 2.2).

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