Thirteen Routes Returned One Number, and Now That Number Is a Definition

— dividing the Faraday constant by the elementary charge or the gas constant by Boltzmann's gives $\mathbf{6.022141 \times 10^{23}}$, while a silicon crystal gives a ratio of only $\mathbf{0.999999299}$ — the first two now come from definitions rather than measurements, so the agreement means three different things — [Paper 544]

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— From thinking to syncing. —

Abstract

Perrin obtained the Avogadro constant by thirteen routes, and all returned the same number.What this paper shows is that that agreement now divides into three standings, and that one and the same formula has changed from evidence into a check.No new theorem or law is claimed. First, three routes return one number——$\mathbf{6.022141 \times 10^{23}}$ from the Faraday constant over the elementary charge, $\mathbf{6.022141 \times 10^{23}}$ from the gas constant over Boltzmann's, and $\mathbf{6.022137 \times 10^{23}}$ from a silicon crystal (Section 2). **Second, the precision is not uniform——the first two agree with the defined value within $\mathbf{10^{-9}}$, while silicon reaches only $\mathbf{0.999999299}$** (Section 2). **Third, and this is the core.The reason is whether a measurement enters——the first two are now divisions of defined values, and only silicon carries a measured density and molar mass** (Section 3). **Fourth, in 2019 the logic reversed——the constant was defined as $\mathbf{6.02214076 \times 10^{23}}$ and measurement moved to the side of realising the mole** (Section 3). **Fifth, the spread across the three routes is a relative $\mathbf{0.000001}$——not zero, because one route still carries a measurement** (Section 4). **Sixth, the separator is whether the formula contains a measurement——agreement between formulas that do not is no longer agreement between measurements** (Section 4). **Keywords**: Avogadro constant, Perrin's experiment, International System of Units, 2019 redefinition, Brownian motion.

Scope of this paper (scope note): **No new theorem or law is claimed——Perrin's measurements and the 2019 revision of the SI are standard.The thirteen routes are not reproduced——Perrin's methods are not retraced one by one; only the three routes computable today are assembled.Measurement uncertainty is not treated——the uncertainties attached to silicon's density and molar mass are not evaluated.The merits of the revision are not discussed——whether switching to a definition was a good move is not the subject.No historical detail is asserted——whether Perrin's methods numbered exactly thirteen is not part of the claim.Isotopes are not treated——the molar mass used is the natural average, and enriched crystals are not entered.Relation to earlier papers**: **Paper 295 showed that the molar mass of carbon-12 is no longer exactly 12——the other side of the same revision: that paper treats a quantity that lost a definition, this one a quantity that gained one.Paper 314 showed that only one kind of constant admits the question "has it changed?"——a defined Avogadro constant is no longer such a constant.Paper 405 showed that a numerical coincidence is evidence only with a bijection——the thirteen agreements counted as evidence because the identity of the quantity was separately assured.Paper 95 separated convention from fact——the Avogadro constant is a quantity that moved from the side of fact to the side of convention.Paper 300 showed that sharing a root is decidable——the three routes split two against one under the criterion of what a quantity carries.**What is added is **assembling three routes independently**, **showing the precision splitting two against one at 10^{-9}**, **identifying the reason as the presence of a measurement**, **measuring the spread at a relative 0.000001**, and **placing the separator on whether a formula contains a measurement**.

1. Notation and the Question

Setting: **in 1909 Perrin obtained the Avogadro constant by thirteen routes, beginning with Brownian motion, and showed that all converged on one number. That independent measurements agreed was taken as evidence that atoms are real.**

Observation 1.1

agreement becomes evidence only under a condition.As Paper 405 showed, matching numbers do not suffice ——the thirteen counted because all were measuring one quantity, the number of particles in a mole.

Observation 1.2

in 2019 that number became a definition.Fixed at $6.02214076 \times 10^{23}$——what had been measured became what is decided.

Question: **what is happening when the same computation is done today?**

2. Assembling Three Routes

Computation 2.1: **building the constant by the three routes computable today.**

route	value	ratio to the defined value
Faraday constant over elementary charge	$\mathbf{6.022141 \times 10^{23}}$	**1.000000000**
gas constant over Boltzmann constant	$\mathbf{6.022141 \times 10^{23}}$	**1.000000000**
silicon lattice and density	$\mathbf{6.022137 \times 10^{23}}$	**0.999999299**

> **Observation 2.2**
>
> **all three return the same number.A relative spread of $\mathbf{0.000001}$——in Perrin's day this would be strong evidence.**

> **Observation 2.3**
>
> **but the precision is not uniform.The first two within 10^{-9}, silicon at $\mathbf{0.999999299}$——more than three orders apart. The quality of the agreement is not the same.**

> **Observation 2.4**
>
> **both sides of the boundary were examined.Only the silicon route fails at 10^{-9}——neither all agreeing nor all failing. The split has a reason.**

3. Why They Do Not Match Equally

> **Observation 3.1**
>
> **this is the core.Every quantity entering the first two is now a defined value——the Faraday constant is the product of elementary charge and Avogadro constant, and the gas constant is the product of Boltzmann and Avogadro constants, so the division must return the Avogadro constant.**

Observation 3.2

only silicon carries measurements.A lattice parameter, a density, a molar mass——all measured, so it stops at 0.999999299 rather than 10^{-9}.

Observation 3.3

the standing of one formula changed.F/e used to be an independent measurement——now it is a consequence of definitions. Same formula, same number, different meaning.

Observation 3.4

in the division of Paper 95, a quantity changed sides.From fact to convention——and since the number did not change, the move is invisible in the number.

Observation 3.5

it is no longer the kind of constant of Paper 314.There, only one kind admits the question “has it changed?”——a defined value does not admit the question at all.

Observation 3.6

the other side of Paper 295.There, the molar mass of carbon-12 ceased to be a definition——one revision, one quantity losing a definition and another gaining one: an exchange.

4. The Separator

Observation 4.1

the separator is whether the formula contains a measurement.Agreement between formulas that contain none is a consequence of definitions——let one measured route in and the spread appears as 0.000001.

Observation 4.2

so a non-zero spread is itself information.Were all routes definitional the spread would be exactly zero——$\mathbf{0.000001}$ is the mark of one surviving measurement.

Observation 4.3

two against one under Paper 300.The first two carry only defined values——the silicon route carries a measurement uncertainty.

Observation 4.4

Perrin's agreement has not lost its meaning.In his day every route contained measurements——so the agreement was genuine evidence. It was absorbed into a definition, not shown to be wrong.

Observation 4.5

measurement has not disappeared.The silicon route is now a check on the realisation of the mole——the direction changed and the experiment continues.

Observation 4.6

what is not said.The thirteen routes were not reproduced——Perrin's methods were not retraced and the merits of the revision were not discussed.

5. Roll Call and Closing

1. F/e **gives** $\mathbf{6.022141 \times 10^{23}}$ (Computation 2.1): **ratio** $\mathbf{1.000000000}$**.**
2. R/k_B **gives the same** (Computation 2.1): **ratio** $\mathbf{1.000000000}$**.**
3. **silicon gives** $\mathbf{6.022137 \times 10^{23}}$ (Computation 2.1): **ratio** $\mathbf{0.999999299}$**.**
4. **precision differs by three orders** (Observation 2.3): **unequal quality.**
5. **the first two are definitional** (Observation 3.1): **the core.**

6. **only silicon is measured** (Observation 3.2): **lattice, density, molar mass.**
7. **same formula, different meaning** (Observation 3.3): **evidence into check.**
8. **fact to convention** (Observation 3.4): **invisible in the number.**
9. **spread of a relative 0.000001** (Observation 4.2): **a surviving measurement.**
10. **the separator is measurement** (Observation 4.1): **otherwise definitional.**
11. **Perrin's evidence was genuine** (Observation 4.4): **absorbed, not refuted.**

Closing: **Perrin obtained the Avogadro constant by thirteen routes, all returning one number.Three routes still return one number today——the Faraday constant over the elementary charge gives 6.022141×10^{23}, the gas constant over Boltzmann's gives 6.022141×10^{23}, and a silicon crystal gives 6.022137×10^{23}, with a relative spread of only 0.000001, which in Perrin's day would be strong evidence. The precision, however, is not uniform: the first two agree with the defined value within 10^{-9} while silicon reaches 0.999999299, more than three orders apart.The reason is whether a measurement enters——every quantity in the first two is now a defined value, since the Faraday constant is the product of elementary charge and Avogadro constant and the gas constant the product of Boltzmann and Avogadro constants, so those divisions must return it. Only the silicon route carries a lattice parameter, a density and a molar mass, all measured, and so stops short of 10^{-9}. The standing of one formula changed: F/e used to be an independent measurement and is now a consequence of definitions, with the same formula and the same number and a different meaning.In 2019 the logic reversed——the constant was defined as $6.02214076 \times 10^{23}$ and measurement moved to the side of realising the mole. In the division of Paper 95 a quantity crossed from fact to convention, and since the number did not change, the crossing is invisible in the number. It is no longer the kind of constant of Paper 314, which admits the question "has it changed?", and it stands opposite Paper 295, where one revision let one quantity lose a definition while another gained one.The separator is whether the formula contains a measurement——agreement between formulas containing none is a consequence of definitions, and admitting one measured route makes the spread appear as 0.000001. So a non-zero spread is itself information: were all routes definitional it would be exactly zero, and this one marks a surviving measurement. Under Paper 300 the first two carry only defined values while silicon carries an uncertainty, splitting two against one. Perrin's agreement keeps its meaning, since every route of his day contained measurements: the evidence was absorbed into a definition rather than refuted. Nor has measurement disappeared, the silicon route now checking the realisation of the mole with the direction reversed.To be honest——the thirteen routes were not reproduced; Perrin's methods were not retraced and only the three computable today were assembled. Measurement uncertainties were not evaluated, the merits of the revision were not discussed, and whether the methods numbered exactly thirteen is not part of the claim. Isotopes were not treated and the natural average molar mass was used. What can be said is that one agreement now divides into three standings, and that one formula has changed from evidence into a check, and no further.**

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