An Aperture and Its Complement Make the Same Pattern Only Away From the Forward Direction

— the intensities of a slit and of its complement converge away from the forward direction, while in the forward direction alone they do not agree, the ratio being **18.9055** — [Paper 546]

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— *From thinking to syncing.* —

Abstract

Babinet's principle says that an aperture and its complement produce the same diffraction pattern.What this paper shows is that this holds only away from the forward direction, and that why it holds and why it fails follow from one and the same identity.No new theorem or law is claimed. First, the two sum to the open screen——the field of a slit plus that of its complement equals the field of the open screen at every angle, to the 10^{-13} level (Section 2). **Second, the open screen's field is almost zero except forward——40.010000 at $u = 0$ and only 0.010000 from $u = 0.05$ onward** (Section 2). **Third, and this is the core.So away from the forward direction the complement's field is the slit's with the sign reversed——intensity is the squared modulus, so the two patterns coincide** (Section 3). **Fourth, the closeness is set by what remains of the open field——refining the grid from 0.0100 to 0.0020 improves the ratio from 0.99467820 to 0.99893196** (Section 3). **Fifth, the forward direction alone does not agree——the ratio there is 18.9055, an order apart** (Section 4). **Sixth, the separator is whether the open field is negligible——not the shape of the aperture but the angle being viewed** (Section 4). **Keywords**: Babinet's principle, Fraunhofer diffraction, complementary aperture, diffraction pattern, forward scattering.

Scope of this paper (scope note): **No new theorem or law is claimed——Babinet's principle and Fraunhofer diffraction are standard.Vector diffraction is not treated——the complement of a polarised field, exchanging conductor and aperture, needs a separate argument; only a scalar field is examined.Near fields are not treated——only the far field was computed.No experiment is proposed——how to block the forward beam in practice is not the subject.Only one dimension——two-dimensional apertures are not entered.Numerical limits are not used as claims——the agreement is imperfect because the aperture and the grid are finite, not because the principle fails.Relation to earlier papers**: **Paper 41 showed that fringes do not remember the exponent——fringe positions come from phase and are a separate root from amplitude, and this paper likewise separates amplitude from intensity.Paper 442 showed that white is made by a difference in refractive index rather than by a colour——there too what is seen is a difference; here the difference of two fields survives as the open field.Paper 300 showed that sharing a root is decidable——forward and off-forward are separate roots under the criterion of what a quantity carries.**What is added is **confirming the sum to the 10^{-13} level**, **showing the open field held at 0.010000 off axis**, **confirming that the closeness follows the residual by refining the grid**, **setting the forward ratio of 18.9055 beside it**, and **placing the separator on whether the open field is negligible**.

1. Notation and the Question

Setting: **on a one-dimensional screen, compare a slit of width 2 with its complement, which blocks exactly that region. The far-field amplitude is computed as a function of the angle $u = \sin\theta$.**

Babinet's principle: **the two far fields sum to the far field of an unobstructed screen.**

Observation 1.1

this is an identity of addition.The transmittances are t and $1 - t$, summing to 1——a consequence of linearity rather than a property of diffraction.

Observation 1.2

whether they make "the same pattern" is a separate matter.Summing to the open field is not the same as having equal intensities——the latter requires the open field to be small.

Question: **at which angles does it hold, and at which does it fail?**

2. They Sum to the Open Screen

Computation 2.1: the magnitudes of the three far fields by angle.

u	slit	complement	open	sum minus open
0.00	**2.010000**	**38.000000**	**40.010000**	3.98×10^{-13}
0.05	1.976772	1.966772	**0.010000**	4.56×10^{-16}
0.10	1.879063	1.869063	**0.010000**	2.87×10^{-15}
0.20	1.516724	1.506724	**0.010000**	1.80×10^{-15}
0.40	0.459630	0.449630	**0.010000**	1.51×10^{-15}

Observation 2.2

the addition holds at every angle.Differences at the 10^{-13} level——within rounding, a check that the computation is right.

Observation 2.3

the open field is large only forward.**40.010000** at $u = 0$ and **0.010000** elsewhere——more than four thousandfold. An open screen makes a sharp forward peak and almost nothing beside it.

Observation 2.4

that **0.010000** is the residue of a finite aperture and grid.Exactly the grid spacing of 0.0100——a quantity that should vanish in principle, left over at the size of the step.

3. Away From Forward, the Two Agree

> **Observation 3.1**
>
> **this is the core.Since the two sum to the open field and the open field is nearly zero off axis, the complement's field is the slit's with the sign reversed——intensity takes the squared modulus, so the sign disappears and the patterns coincide.**

Computation 3.2: **the ratio by angle.**

u	complement over slit
0.00	**18.9055**
0.05	**0.9949412482**
0.10	**0.9946781975**
0.20	**0.9934068414**
0.40	**0.9782433626**

> **Observation 3.3**
>
> **the agreement is not perfect.Off by 0.5 to 2 per cent——not because the principle fails but because the open field is not exactly zero.**

Computation 3.4: **refining the grid to reduce the residue.**

grid points	spacing	ratio at $u = 0.1$
4001	**0.0100**	**0.99467820**
20001	**0.0020**	**0.99893196**

Observation 3.5

a smaller residue gives better agreement.Dividing the spacing by five divides the departure from one by about five——confirming the residue as the cause, and not a limit of the principle.

Observation 3.6

amplitude and intensity are being kept apart.Amplitudes differ in sign; intensities agree——the same separation Paper 41 made between fringe position and amplitude.

4. The Separator

Observation 4.1

the separator is whether the open field is negligible.Decided by the viewing angle rather than the shape of the aperture——for one aperture it fails forward and holds elsewhere.

Observation 4.2

forward the ratio is 18.9055.An order apart——the far side of the boundary. Stating Babinet's principle without qualification fails here.

Observation 4.3

one identity explains both.Slit plus complement equals open——whether it holds or fails is whether that right-hand side is small or large.

Observation 4.4

separate roots under Paper 300.Agreement forward carries the open field as a belonging——agreement elsewhere does not. One question, different conditions.

Observation 4.5

the same shape as Paper 442.There white was a difference in refractive index rather than a colour——here too what is seen is a difference, the difference of two fields surviving as the open field.

Observation 4.6

what is not said.Vector diffraction is not entered——the polarised complement needs a separate argument, and neither near fields nor two-dimensional apertures were treated.

5. Roll Call and Closing

1. **the sum holds to 10^{-13}** (Computation 2.1): **a consequence of linearity.**
2. **open field $\mathbf{40.010000}$ at $u = 0$** (Computation 2.1): $\mathbf{0.010000}$ **elsewhere.**
3. **more than four thousandfold** (Observation 2.3): **a sharp forward peak.**
4. $\mathbf{0.010000}$ **is the grid residue** (Observation 2.4): **not a limit of the principle.**
5. **off axis only the sign differs** (Observation 3.1): **the core.**
6. **ratios such as $\mathbf{0.9949412482}$** (Computation 3.2): 0.5 **to** 2 **per cent.**
7. **refining gives $\mathbf{0.99893196}$** (Computation 3.4): **the residue falls.**
8. **the departure falls fivefold** (Observation 3.5): **the cause confirmed.**
9. **forward the ratio is $\mathbf{18.9055}$** (Observation 4.2): **the far side.**
10. **the separator is the open field** (Observation 4.1): **not the aperture.**
11. **one identity explains both** (Observation 4.3): **the size of the right-hand side.**

Closing: **Babinet's principle says that an aperture and its complement produce the same diffraction pattern.They sum to the open screen——the field of a slit plus that of its complement equals the field of an unobstructed screen at every angle, to the 10^{-13} level. The transmittances are t and $1 - t$ and sum to one, so this is a consequence of linearity rather than a property of diffraction. That open field is large only forward, $\mathbf{40.010000}$ at $u = 0$ and $\mathbf{0.010000}$ elsewhere, more than four thousandfold apart. Even the $\mathbf{0.010000}$ is only the residue of a finite aperture and grid, exactly the size of the step.So away from forward the two agree——with the open field nearly zero, the complement's field is the slit's with the sign reversed, and intensity takes the squared modulus, so the sign disappears. The ratios run from $\mathbf{0.9949412482}$ at $u = 0.05$ to $\mathbf{0.9782433626}$ at $u = 0.40$, off by 0.5 to 2 per cent. That is not the principle failing: refining the grid from $\mathbf{0.0100}$ to $\mathbf{0.0020}$ moves the ratio from $\mathbf{0.99467820}$ to**

0.99893196, and dividing the spacing by five divides the departure by about five. Keeping amplitude and intensity apart is what makes this legible, the separation Paper 41 made between fringe position and amplitude.The separator is whether the open field is negligible——decided by the viewing angle rather than the shape of the aperture, so one aperture fails forward and holds elsewhere. Forward the ratio is 18.9055, an order apart, and stating the principle without qualification fails there. One identity explains both, since slit plus complement equals open, and everything turns on whether that right-hand side is small or large. Under Paper 300, agreement forward carries the open field as a belonging while agreement elsewhere does not. As in Paper 442, what is seen is a difference: the difference of two fields survives as the open field.To be honest——vector diffraction was not entered; the complement of a polarised field, exchanging conductor and aperture, needs a separate argument. Near fields were not treated and only the far field computed. Only one dimension was examined, and two-dimensional apertures were not entered. How to block the forward beam in practice was not discussed, and the imperfect agreement was not used as a claim about the principle, since aperture and grid are finite. What can be said is that the range of validity is set by angle, and that its boundary is explained by the size of the open field, and no further.

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