# The Optimum Is Flat Around Itself, So It Need Not Be Held Exactly

— doubling or halving the order quantity raises the total cost only to **1.2500** times, and the range within five per cent of the minimum runs from **0.7298** to **1.3702** times the optimum — [Paper 547]

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*— From thinking to syncing. —*

**Abstract**

**The optimal order quantity has a closed form, and practice asks that it be held.What this paper shows is that the optimum is flat around itself, so that being off by a factor of two costs only a quarter more, and that this flatness follows from the condition defining the optimum.No new theorem or law is claimed. First, at the optimum the two costs are exactly equal——both 15811.3883, differing by $1.82 \times 10^{-12}$** (Section 2). **Second, this is the equality case of the arithmetic-geometric mean inequality——the product of the two costs does not depend on the order quantity, so their sum is least exactly when they are equal** (Section 2). **Third, and this is the core.The optimum is flat around itself——doubling the quantity raises the total only to 1.2500 times, and halving it gives the same 1.2500** (Section 3). **Fourth, the tolerance is wide——costs stay within five per cent from 2307.9687 to 4332.8144 units, that is from 0.7298 to 1.3702 times the optimum** (Section 3). **Fifth, it is not flat without limit——a factor of four gives 2.1250 times: the far side of the boundary** (Section 3). **Sixth, the separator is whether the cost sum is symmetric in the logarithm——that a factor of two and a half return the same value is what flatness is** (Section 4).
**Keywords**: economic order quantity, inventory management, AM-GM inequality, flatness of the optimum, Harris formula.

Scope of this paper (scope note): No new theorem or law is claimed——the economic order quantity and the inequality between means are standard.No inventory policy is recommended——which quantity to order is not stated; only what happens when the optimum is missed.Demand variability is not treated——demand is taken as constant, and safety stock under variability is a separate subject.Shortages and quantity discounts are excluded——neither stockout costs nor bulk discounts enter.Cost estimation is not discussed——how to estimate ordering and holding costs is not the subject; they are taken as given.It is not said that flatness excuses missing the optimum——the flatness belongs to cost, while storage space and capital act separately.Relation to earlier papers: Paper 486 showed that the "exactly half" of three fields is the vertex of a parabola——the equality of two costs here is the same shape, with an optimum announced as a coincidence of two quantities, though that one comes from a vertex and this one from a constant product.Paper 194 showed that the four means are one family with a single equality fence——the optimum here sits on that fence.Paper 300 showed that sharing a root is decidable——"the optimum is a coincidence of two quantities" and "the optimum is flat" are separate roots.What is added is confirming the two costs equal to $10^{-12}$, showing that a factor of two and a half both return **1.2500** to 12 digits, inverting for the five-per-cent range as **0.7298** to **1.3702**, placing a factor of four beside it as the far side, and placing the separator on symmetry in the logarithm.

## 1. Notation and the Question

Setting: with annual demand $D$, a cost $S$ per order and a cost $H$ to hold one unit for a year, ordering $Q$ at a time costs

$$C(Q) = \frac{DS}{Q} + \frac{HQ}{2}$$

per year, the first term for ordering and the second for holding.

**Observation 1.1**

the two terms move oppositely.Larger $Q$ means fewer orders and more stock——so a minimum lies somewhere.

**Observation 1.2**

their product does not depend on $Q$.$\frac{DS}{Q} \cdot \frac{HQ}{2} = \frac{DSH}{2}$——a sum to be minimised at constant product, settled by the equality case of the arithmetic-geometric mean inequality.

Question: how much does missing the optimum cost?

## 2. At the Optimum the Two Are Equal

**Computation 2.1**: taking $D = 10000$, $S = 5000$ and $H = 10$.

| quantity | value |
|---|---|
| optimum $Q^* = \sqrt{2DS/H}$ | **3162.2777** units |
| ordering cost there | **15811.3883** |
| holding cost there | **15811.3883** |
| difference | $\mathbf{1.82 \times 10^{-12}}$ |
| total | **31622.7766** |

**Observation 2.2**

**the two costs coincide exactly.The difference is at the rounding level——not a coincidence but a consequence of the constant product.**

**Observation 2.3**

**the value can be assembled independently.$\sqrt{DSH/2}$ returns the same 15811.3883——without passing through $Q^*$, so this is a check: disagreement would mean one of the formulas is wrong.**

**Observation 2.4**

**this sits on the fence of Paper 194.The arithmetic mean equals the geometric mean only when the two are equal——which is why the optimum announces itself as a coincidence of two quantities.**

**Observation 2.5**

**the same shape as Paper 486, with a different root.There the "exactly half" came from the vertex of a parabola——here the coincidence comes from a constant product. Two ways for a coincidence to announce an optimum.**

## 3. The Optimum Is Flat Around Itself

**Computation 3.1**: **moving the order quantity away from the optimum and measuring the total.**

| $Q/Q^*$ | total cost | ratio to the minimum |
|---|---|---|
| 0.50 | **39528.4708** | **1.2500** |
| 0.80 | 32413.3460 | 1.0250 |
| 1.00 | **31622.7766** | 1.0000 |
| 1.25 | 32413.3460 | 1.0250 |
| 2.00 | **39528.4708** | **1.2500** |
| 4.00 | **67198.4003** | **2.1250** |

> **Observation 3.2**
>
> this is the core.A factor of two costs $\mathbf{1.2500}$ times——off by double, and the cost rises by a quarter.

> **Observation 3.3**
>
> two and one half return the same value.Both $\mathbf{1.2500}$, agreeing to 12 digits——because $C(Q)/C(Q^*) = \tfrac{1}{2}(x + 1/x)$ with $x = Q/Q^*$, so $x$ and $1/x$ agree. Flatness is symmetry in the logarithm.

> **Observation 3.4**
>
> 0.80 and 1.25 likewise agree.Both 1.0250——off by a fifth, and the cost rises by two and a half per cent.

**Computation 3.5**: **bisecting for the range within five per cent of the minimum.**

| quantity | value |
|---|---|
| lower | **2307.9687** units (**0.7298** of the optimum) |
| upper | **4332.8144** units (**1.3702** of the optimum) |

> **Observation 3.6**
>
> **the inversion is a check.Both ends return exactly 1.05 times, within $10^{-6}$——confirming separately that these are solutions.**

> **Observation 3.7**
>
> **it is not flat without limit.A factor of four gives 2.1250——the far side of the boundary: flatness belongs to the neighbourhood.**

## 4. The Separator

> **Observation 4.1**
>
> **the separator is symmetry in the logarithm.A form in which $x$ and $1/x$ agree is flat——a cost with a heavy penalty on one side only, such as a stockout, loses the flatness.**

**Computation 4.2**: **how the optimum moves with demand.**

| demand | optimum | ratio |
|---|---|---|
| 1 | **3162.2777** | 1.0000 |
| 4 | **6324.5553** | **2.0000** |
| 9 | **9486.8330** | **3.0000** |

**Observation 4.3**

**four times the demand moves the optimum only twofold.A square root, to 12 digits——so misjudging demand by a factor of two moves the optimum by only 1.41. The flatness works twice.**

**Observation 4.4**

**separate roots under Paper 300.“The optimum is a coincidence of two quantities” carries a constant product ——“the optimum is flat” carries a second derivative. Two facts about one point, neither implying the other.**

**Observation 4.5**

**what is not said.Flatness does not excuse missing the optimum——it belongs to cost, while storage space and capital act separately.**

## 5. Roll Call and Closing

1. **optimum $\mathbf{3162.2777}$ units** (Computation 2.1): $\sqrt{2DS/H}$.
2. **both costs $\mathbf{15811.3883}$** (Computation 2.1): **differing by $\mathbf{1.82 \times 10^{-12}}$.**
3. $\sqrt{DSH/2}$ **returns the same** (Observation 2.3): **an independent assembly.**
4. **double costs $\mathbf{1.2500}$** (Computation 3.1): **the core.**
5. **half costs the same $\mathbf{1.2500}$** (Observation 3.3): **symmetric in the logarithm.**
6. **0.80 and 1.25 both give 1.0250** (Observation 3.4): **a fifth costs 2.5 per cent.**
7. **five per cent from $\mathbf{0.7298}$ to $\mathbf{1.3702}$** (Computation 3.5): **a wide range.**
8. **both ends return 1.05** (Observation 3.6): **the inversion checks.**
9. **four times gives $\mathbf{2.1250}$** (Observation 3.7): **the far side.**
10. **four times the demand doubles it** (Computation 4.2): **a square root.**
11. **the separator is logarithmic symmetry** (Observation 4.1): **a one-sided penalty removes it.**

**Closing**: **The optimal order quantity has a closed form, and practice asks that it be held.At the optimum the two costs are exactly equal——with $D = 10000$, $S = 5000$ and $H = 10$ the optimum is $\mathbf{3162.2777}$ units, where ordering and holding both cost $\mathbf{15811.3883}$, differing by $\mathbf{1.82 \times 10^{-12}}$. This is no coincidence: their product does**

**not depend on $Q$, and a sum at constant product is least when the terms are equal. That $\sqrt{DSH/2}$ returns the same value without passing through $Q^*$ makes it a check. The optimum sits on the equality fence of Paper 194, and shares the shape of Paper 486, though that coincidence came from a vertex and this one from a constant product.And the optimum is flat around itself——doubling the quantity raises the total only to $\mathbf{1.2500}$ times, so being off by double costs a quarter. Halving gives the same $\mathbf{1.2500}$ to 12 digits, since the ratio is $\frac{1}{2}(x + 1/x)$ and $x$ agrees with $1/x$: flatness is symmetry in the logarithm. 0.80 and 1.25 likewise both give 1.0250, so a fifth costs two and a half per cent. Costs stay within five per cent from $\mathbf{2307.9687}$ to $\mathbf{4332.8144}$ units, and both ends return exactly 1.05 when fed back. It is not flat without limit, however: a factor of four gives $\mathbf{2.1250}$.The flatness works twice——four times the demand moves the optimum only to $\mathbf{2.0000}$ times, so misjudging demand by a factor of two shifts the optimum by 1.41, and a shift of 1.41 still costs under five per cent.The separator is symmetry in the logarithm——a form in which $x$ and $1/x$ agree is flat, and a one-sided penalty such as a stockout removes it. Under Paper 300, "the optimum is a coincidence of two quantities" carries a constant product while "the optimum is flat" carries a second derivative: two facts about one point, neither implying the other.To be honest——no inventory policy is recommended; only what happens when the optimum is missed was examined. Demand was taken as constant, so safety stock under variability is a separate subject, and neither stockout costs nor quantity discounts were entered. How to estimate the costs was not discussed. Flatness does not excuse missing the optimum, since it belongs to cost while storage and capital act separately. What can be said is that the optimum announces itself as a coincidence of two quantities, and that it is flat around itself with logarithmic symmetry, and no further.**

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