Each Further Nine Costs More Stock Than the Last

— raising the no-stockout probability from **90** to **99** per cent takes **41.7919** units of safety stock, and from **99** to **99.99** still more, **55.7067** — quartering the lead time halves the safety stock exactly — [Paper 548]

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— From thinking to syncing. —

Abstract

Safety stock follows once a no-stockout probability is chosen.What this paper shows is that each further nine in that probability costs more stock than the last, while the same quantity responds to lead time only as a square root.No new theorem or law is claimed. First, the quantiles can be checked independently——$z(0.90) = \mathbf{1.2816}$, $z(0.99) = \mathbf{2.3263}$ **and** $z(0.9999) = \mathbf{3.7190}$ **agree with tabulated values to** 10^{-4} (Section 2). **Second, and this is the core.Each further nine costs more**——**90 to 99 takes 41.7919 units, 99 to 99.99 takes 55.7067, a ratio of 1.3330** (Section 2). **Third, the increments accelerate but the stock does not explode**——**from 51.2621 units at 90 per cent to 148.7607 at 99.99, within a factor of three** (Section 2). **Fourth, lead time enters only as a square root**——**quartering it halves the stock exactly to 12 digits, and halving it gives only 0.7071** (Section 3). **Fifth, the same** z **stands the other way up in Paper 383**——**there it is** $z\sigma/\sqrt{n}$**, falling as policies are added; here** $z\sigma\sqrt{L}$**, rising as the wait lengthens** (Section 4). **Sixth, the separator is whether what** z **multiplies sits in the denominator or the numerator**——**the same quantile means the opposite thing depending on where it is placed** (Section 4). **Keywords**: safety stock, service level, normal quantile, inventory management, stockout.

Scope of this paper (scope note): **No new theorem or law is claimed——normal quantiles and the safety-stock formula are standard.No inventory policy is recommended——which level to choose is not stated; only what happens as the level rises.Demand is assumed normal——heavy-tailed demand changes the quantiles themselves, and that case is not treated.Lead-time variability is excluded——lead time is taken as a constant; a fluctuating lead time gives a different formula.Stockout costs are not discussed——balancing stock against shortage is not the subject.It is not said whether a further nine is worth buying——only that the increments accelerate; where to stop is not stated.Relation to earlier papers**: **Paper 383 showed that what sets an insurance loading is the number of policies rather than the accuracy of the probability——the same z appears, but there as $z\sigma/\sqrt{n}$, falling with n, and here as $z\sigma\sqrt{L}$, rising with L: denominator against numerator.Paper 317 showed that a significance level is a promise per test——a no-stockout probability is likewise a promise per replenishment, not per year, and the same misreading occurs.Paper 300 showed that sharing a root is decidable——"the increments accelerate" and "lead time enters as a square root" are separate roots.**What is added is **checking the quantiles against independently known values**, **measuring the increments as $\mathbf{41.7919}$ and $\mathbf{55.7067}$ for a ratio of $\mathbf{1.3330}$**, **placing beside them the fact that the stock stays within a factor of three**, **confirming to 12 digits that quartering the lead time halves the stock**, and **placing the separator on where what z multiplies sits**.

1. Notation and the Question

Setting: **with a daily demand standard deviation σ and a lead time of L days, holding the no-stockout probability at p requires**

$$S(p, L) = z(p)\,\sigma\sqrt{L}$$

where $z(p)$ is the p quantile of the standard normal distribution.

> **Observation 1.1**
>
> **p is pressed against a ceiling.It cannot exceed one, so the room left is counted in nines——0.9, 0.99, 0.999, one nine at a time.**

Question: **what does one more nine cost?The same each time, or not?**

2. Each Further Nine Costs More

Computation 2.1: **taking $\sigma = 20$ and $L = 4$ days.**

no-stockout probability	$z(p)$	safety stock
0.9000	**1.2816**	**51.2621** units
0.9900	**2.3263**	**93.0539** units
0.9990	**3.0902**	**123.6093** units
0.9999	**3.7190**	**148.7607** units

> **Observation 2.2**
>
> **the quantiles check independently.$z(0.90) = 1.2816$, $z(0.99) = 2.3263$ and $z(0.9999) = 3.7190$ are tabulated values, matched to 10^{-4}——the implementation is confirmed against something outside itself, and would fail if wrong.**

Computation 2.3: **the increment per nine.**

step	extra safety stock
90 to **99** per cent (one nine)	**41.7919** units
99 to **99.99** per cent (two nines)	**55.7067** units
ratio	**1.3330**

> **Observation 2.4**
>
> **this is the core.Two nines cost 1.3330 times what one nine cost——the same operation grows dearer each time it is repeated.**

> **Observation 2.5**
>
> **the ratio lies above 1.3 and below 1.4.Measured at 1.3330——had "it doubles" been written first, it would have been wrong. Thresholds are measured, then bracketed.**

Observation 2.6

the stock itself does not explode.From 51.2621 to 148.7607 units, within a factor of three——what accelerates is the increment, not the level. This rests on the thin normal tail, and a heavy tail would break the contrast.

Observation 2.7

the misreading of Paper 317 recurs.Ninety-nine per cent is a promise per replenishment, not per year——with many replenishments a year, the chance of no stockout all year is lower. The same shape as a significance level being per test.

3. Lead Time Enters Only as a Square Root

Computation 3.1: **holding the level at 99 per cent and shortening the lead time.**

lead time	safety stock	ratio
1.00	**93.0539** units	1.0000
0.50	**65.7991** units	**0.7071**
0.25	**46.5270** units	**0.5000**

Observation 3.2

quartering halves it exactly.0.5000 to 12 digits——a square root.

Observation 3.3

halving does not halve.Only 0.7071——the return on halving a lead time is smaller than it looks.

Observation 3.4

two opposite habits in one formula.The level accelerates while the lead time dulls——the two variables behave in contrary ways.

4. The Separator

Observation 4.1

the same z stands the other way up in Paper 383.There the loading is $z\sigma/\sqrt{n}$, falling as policies n are added ——here the safety stock is $z\sigma\sqrt{L}$, rising as the lead time L lengthens. The quantile is the same $\mathbf{2.3263}$, but denominator and numerator reverse its meaning.

Observation 4.2

the separator is where what z multiplies sits.In the denominator: more of them makes safety cheaper——in the numerator: waiting longer makes safety dearer. The formulas look nearly alike, and only the direction is reversed.

Observation 4.3

separate roots under Paper 300.“The increments accelerate” carries the convexity of $z(p)$——“lead time enters as a square root” carries variance growing with time. Neither yields the other.

Observation 4.4

what is not said.Whether a further nine is worth buying is not stated——only that the increments accelerate; where to stop needs the cost of a shortage.

5. Roll Call and Closing

1. $z(0.90) = \mathbf{1.2816}$ (Computation 2.1): **tabulated, to 10^{-4}.**

2. $z(0.99) = \mathbf{2.3263}$ (Computation 2.1): **likewise.**
3. $z(0.9999) = \mathbf{3.7190}$ (Computation 2.1): **likewise.**
4. **one nine costs $\mathbf{41.7919}$ units** (Computation 2.3): **90 to 99.**
5. **two nines cost $\mathbf{55.7067}$** (Computation 2.3): **99 to 99.99.**
6. **ratio $\mathbf{1.3330}$** (Computation 2.3): **the core.**
7. **stock from $\mathbf{51.2621}$ to $\mathbf{148.7607}$** (Computation 2.1): **within a factor of three.**
8. **quartering gives $\mathbf{0.5000}$** (Computation 3.1): **a square root, to 12 digits.**
9. **halving gives $\mathbf{0.7071}$** (Computation 3.1): **a small return.**
10. **denominator against numerator with Paper 383** (Observation 4.1): **the same z reversed.**
11. **the separator is that placement** (Observation 4.2): **below or above the line.**

Closing: **Safety stock follows once a no-stockout probability is chosen.Each further nine in that probability costs more stock than the last——with $\sigma = 20$ and $L = 4$ days, going from 90 to 99 per cent takes $\mathbf{41.7919}$ units, while going from 99 to 99.99 takes still more, $\mathbf{55.7067}$, a ratio of $\mathbf{1.3330}$. The same operation grows dearer each time it is repeated. The quantiles used, $z(0.90) = \mathbf{1.2816}$, $z(0.99) = \mathbf{2.3263}$ and $z(0.9999) = \mathbf{3.7190}$, match tabulated values to 10^{-4}, so the implementation is confirmed against something outside itself. Had the ratio been guessed at "double" beforehand it would have been wrong: thresholds are measured, then bracketed.What accelerates is the increment, not the stock——which runs from $\mathbf{51.2621}$ to $\mathbf{148.7607}$ units, within a factor of three, resting on the thin normal tail; a heavy tail would break the contrast.Within the same formula, lead time behaves in the contrary way——quartering it halves the stock exactly to 12 digits, and halving it gives only $\mathbf{0.7071}$. Entering as a square root, it repays the effort of halving a lead time less than it appears to.The separator is whether what z multiplies sits in the denominator or the numerator——the loading of Paper 383 is $z\sigma/\sqrt{n}$ and falls as policies are added, while the safety stock here is $z\sigma\sqrt{L}$ and rises as the lead time lengthens. The quantile is the same $\mathbf{2.3263}$, and its placement decides between "more of them makes safety cheaper" and "waiting longer makes safety dearer". Under Paper 300 the two facts here carry different things: the convexity of $z(p)$, and variance growing with time.And the misreading of Paper 317 recurs——ninety-nine per cent is a promise per replenishment, not per year.To be honest——no inventory policy is recommended. Demand was assumed normal, and heavy-tailed demand changes the quantiles themselves. Lead time was taken as a constant, so a fluctuating one gives a different formula, and stockout costs were left out. Whether a further nine is worth buying is not stated: the increments accelerate, and that is as far as this goes.**

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