The Time Inside Changes Almost Sixfold, and the Product Does Not Move

— changing the service-time distribution moves the time in the system by a factor of **5.8189**, yet in all three distributions "number inside = arrival rate × time inside" holds to within 10^{-9} — the same product applies to red blood cells — [Paper 549]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
September 2026

— From thinking to syncing. —

Abstract

Queueing averages cannot be computed until a distribution is fixed.What this paper shows is that the product linking the number inside to the time inside holds without fixing one, and that it applies to systems that are not queues at all.No new theorem or law is claimed. First, and this is the core.The product holds across three distributions——deterministic, exponential and heavy-tailed alike agree to within 10^{-9} (Section 3). **Second, the time inside itself changes greatly——from 2.983790 to 17.362248, a factor of 5.8189: the law is not indifferent because nothing moved** (Section 3). **Third, it applies where there is no queue——a red-cell production rate of 2.0×10^{6} per second times a lifespan of 120 days gives 2.0736×10^{13} cells** (Section 2). **Fourth, that is 0.8294 of the accepted count——the order of magnitude agrees, but this is not a match: it is off by 17 per cent** (Section 2). **Fifth, under Paper 405 this agreement is not evidence of a common root——with no bijection, 0.8294 is a consistency check, not a derivation** (Section 2). **Sixth, the separator is that nothing is created or destroyed inside——only for contents that neither divide nor vanish is the product free of the distribution** (Section 4). **Keywords**: Little's law, time in system, queueing, red blood cell lifespan, average inventory.

Scope of this paper (scope note): **No new theorem or law is claimed——Little's law is standard and was proved in 1961 .Queue design is not discussed——how many servers to place is not stated.No closed form for the time inside is derived ——each W is a measured simulation value, not a formula.No physiological claim is made——the production rate and lifespan are representative literature values, with no individual or pathological variation; the $\mathbf{0.8294}$ is a consistency check, not a discovery about physiology.Non-stationary cases are excluded——start-up and transients are not the subject.It is not said that distributions are unnecessary——only the relation among the three is free of them; knowing two of the three still requires one.Relation to earlier papers**: **Paper 469 showed that what ten per cent more traffic costs depends on where you already are——its subject is how W grows nonlinearly with utilisation, while the subject here is that the product is free of the distribution: that one asks what is inside W, this one uses a relation that need not ask.Paper 308 showed that the gap between the buses you board is twice the average——an example where the observed quantity and the time average disagree, whereas this paper treats one where they do not.Paper 405 showed that a numerical coincidence is evidence of a common root only when there is a bijection——applied here to the red-cell figure of $\mathbf{0.8294}$.Paper 300 showed that sharing a root is decidable——"free of the distribution" and "W changes with the distribution" are separate roots.**What is added is **confirming the product to 10^{-9} across three distributions**, **placing beside it the $\mathbf{5.8189}$-fold movement of W in the same experiment**, **applying it to a non-queue and obtaining $\mathbf{0.8294}$**, **demoting that agreement under Paper 405**, and **placing the separator on nothing being created or destroyed inside**.

1. Notation and the Question

Setting: **things enter a region, stay a while and leave.Write λ for the arrivals per unit time, W for the mean time one of them stays, and L for the mean number inside at an instant.In steady state**

$$L = \lambda W$$

which is Little's law.

Observation 1.1

no distribution appears in this formula.Neither the pattern of arrivals nor the shape of service times is mentioned——queueing averages usually require a distribution, and this relation does not.

> **Observation 1.2**
>
> **there is no server in it either.It need not be a queue at all——any region things enter, stay in and leave will do.**

Question: **is it really free of the distribution?And what happens when it is applied to something that is not a queue?**

2. Applied Where There Is No Queue

Computation 2.1: **treating red blood cells as the region: the marrow makes them, the blood holds them, the spleen destroys them.**

quantity	value
production rate λ	2.0×10^{6} per second
lifespan W	120 days $= 1.0368 \times 10^{7}$ s
prediction $L = \lambda W$	$\mathbf{2.0736 \times 10^{13}}$ cells
accepted count	2.5×10^{13} cells
ratio	$\mathbf{0.8294}$

> **Observation 2.2**
>
> **the order of magnitude agrees.Both are 10^{13}——two of the three quantities gave the third.**

> **Observation 2.3**
>
> **it is not a match, however.The ratio lies above 0.8 and below 0.9 (measured $\mathbf{0.8294}$)——off by 17 per cent. Setting a threshold of "within ten per cent" first would have failed.**

Observation 2.4

under Paper 405 this agreement is demoted.A numerical coincidence is evidence of a common root only when there is a bijection——both the production rate and the lifespan are broad representative values, and their product landing near an accepted count is not a discovery about physiology. It is a consistency check.

Observation 2.5

it is still useful.When two of the three are easy to measure and one is hard, the third follows——for red cells the count and the lifespan can be measured while the production rate cannot, and the formula supplies the hard one.

3. It Does Not Break Across Three Distributions

Computation 3.1: **arrivals held Poisson at rate 0.8, only the service-time distribution varied, each with mean 1.0.**

service time	L	λ	W	λW
deterministic (always 1.0)	**2.381815**	0.798252	**2.983790**	**2.381815**
exponential	**3.971286**	0.798388	**4.974128**	**3.971286**
two-point (heavy tail)	**13.861879**	0.798392	**17.362248**	**13.861879**

Observation 3.2

this is the core.All three agree to within 10^{-9}——L was accumulated as a time average and W as a per-item average, by entirely separate routes, and they still agree.

Observation 3.3

the law is not indifferent because nothing moved.W itself ran from 2.983790 to 17.362248, a factor of 5.8189——the distribution certainly acts. What it does not act on is the relation among the three. Had W stayed put, the check could not have failed.

Observation 3.4

L moves by as much.From 2.381815 to 13.861879——L and W move together, leaving only the ratio.

Observation 3.5

Paper 469 asks in a different place.There the subject was how W grows when utilisation rises by a tenth——what is inside W. Here the relation need not ask. The same region, seen once from within and once from without.

Observation 3.6

and Paper 308 differs.There the observed gap was twice the time average——a quantity whose answer depends on where one stands. Here two separate accountings, one by time and one by item, agree. A disagreeing example and an agreeing one, side by side.

4. The Separator

Observation 4.1

the separator is that nothing is created or destroyed inside.Only when one thing entering leaves as one thing is the product free of the distribution——a region whose contents divide, or vanish, breaks it.

Observation 4.2

that is why red cells work.One cell made is one cell destroyed——had they divided in the blood it would not apply. The reason is neither that they are biological nor that they resemble a queue, but this one point.

Observation 4.3

a second separator is stationarity.It does not hold during start-up——it cannot be applied to red cells just after a haemorrhage.

Observation 4.4

separate roots under Paper 300.“Free of the distribution” carries a conservation law——“W changes with the distribution” carries queueing dynamics. Two facts from one experiment, neither yielding the other.

Observation 4.5

what is not said.Distributions are not declared unnecessary——only the relation among the three is free of them; knowing two of the three still requires one.

5. Roll Call and Closing

1. **deterministic $L = 2.381815$** (Computation 3.1): **matching λW.**
2. **exponential $L = 3.971286$** (Computation 3.1): **likewise.**
3. **heavy-tailed $L = 13.861879$** (Computation 3.1): **likewise, to 10^{-9}.**
4. **W from 2.983790 to 17.362248** (Observation 3.3): **a factor of 5.8189.**
5. **red-cell prediction 2.0736×10^{13}** (Computation 2.1): **rate times lifespan.**
6. **ratio to the accepted count 0.8294** (Computation 2.1): **the order agrees.**
7. **above 0.8 and below 0.9** (Observation 2.3): **measured, then bracketed.**
8. **demoted under Paper 405** (Observation 2.4): **a check, not a derivation.**
9. **the separator is conservation inside** (Observation 4.1): **division breaks it.**
10. **a second separator is stationarity** (Observation 4.3): **not for transients.**

Closing: **Queueing averages cannot be computed until a distribution is fixed.Yet the product linking the number inside to the time inside holds without fixing one——holding arrivals Poisson and varying only the service-time distribution across deterministic, exponential and heavy-tailed, all three satisfied “number inside = arrival rate × time inside” to within 10^{-9}. L was accumulated as a time average and W as a per-item average, by entirely separate routes, and they still agreed. The law is not indifferent because nothing moved: W itself ran from 2.983790 to 17.362248, a factor of 5.8189, so the distribution certainly acts. What it does not act on is the relation among the three, and had W stayed put the check could not have failed.And the relation applies where there is no queue——a red-cell production rate of 2.0×10^{6} per second times a lifespan of 120 days gives 2.0736×10^{13} cells, which is 0.8294 of the accepted 2.5×10^{13}. The order of magnitude agrees but the numbers do not, being 17 per cent apart; a threshold of “within ten per cent” set beforehand would have failed. Under Paper 405, with no bijection available,**

that 0.8294 is not evidence of a common root: both inputs are broad representative values, and this is a consistency check rather than a discovery about physiology. It remains useful, since when two of the three are easy to measure and one is hard, the formula supplies the hard one.The separator is that nothing is created or destroyed inside——only when one thing entering leaves as one thing is the product free of the distribution, and a region whose contents divide or vanish breaks it. Red cells work not because they are biological nor because they resemble a queue, but because one cell made is one cell destroyed. A second separator is stationarity: it cannot be applied just after a haemorrhage.Placed among the earlier papers——Paper 469 asked how W grows when utilisation rises by a tenth, which is a question about what is inside W; this paper uses a relation that need not ask, seeing the same region from without rather than within. Paper 308 gave a quantity whose answer depends on where one stands, while here two separate accountings agree: a disagreeing example and an agreeing one, side by side. Under Paper 300, "free of the distribution" carries a conservation law and "W changes with the distribution" carries queueing dynamics.To be honest——queue design was not discussed, and each W is a measured simulation value rather than a formula. The red-cell inputs are representative literature values with no individual or pathological variation, and non-stationary cases were excluded. Distributions are not declared unnecessary either: only the relation among the three is free of them, and knowing two of the three still requires one.

References

1. Franco, R. S. (2012). *Measurement of Red Cell Lifespan and Aging*. Transfusion Medicine and Hemotherapy.
2. Kleinrock, L. (1975). *Queueing Systems, Volume 1: Theory*. Wiley.
3. Little, J. D. C. (1961). *A Proof for the Queuing Formula $L = \lambda W$*. Operations Research.
4. Little, J. D. C., & Graves, S. C. (2008). *Little's Law*. In Building Intuition. Springer.
5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
6. Yamagishi, Y. (2026). *The Gap between the Buses You Board Is Twice the Average* [Paper 308]. Zenodo. 10.5281/zenodo.22172305
7. Yamagishi, Y. (2026). *A Numerical Coincidence Becomes Evidence of a Common Root Only When There Is a Bijection* [Paper 405]. Zenodo. 10.5281/zenodo.22699433
8. Yamagishi, Y. (2026). *What Ten Per Cent More Traffic Costs Depends on Where You Already Are* [Paper 469]. Zenodo. 10.5281/zenodo.22733401

— From thinking to syncing. —

Yuuki YAMAGISHI (山岸 勇輝)

© 2026 Yamagishi Complexity Torus Lab, Japan · Licensed under Creative Commons Attribution 4.0 International (CC BY 4.0)