Widening Helps Either Fully or Not at All, With Nothing In Between

— The five stage capacities sum to **650** per hour, but the line delivers the smallest of them, **80**, which is **0.1231** of the sum — averaged from **80** to **250** it is only **0.1176** — [Paper 550]

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— From thinking to syncing. —

Abstract

The capacity of a line follows from the capacities of its stages.What this paper shows is that it follows as a minimum rather than a sum or a mean, and that widening the minimum helps in a step from 1 to 0.No new theorem or law is claimed. First, the line matches the smallest stage——with capacities 80, 150, 120, 100 and 200, the line delivers 80 per hour (Section 2). **Second, the sum is of another size——it is 650, of which the line delivers only 0.1231** (Section 2). **Third, every stage but the smallest must idle——utilisations run 1.0000, 0.8000, 0.6667, 0.5333 and 0.4000, and idling is the normal state** (Section 2). **Fourth, and this is the core.Widening helps with nothing in between——exactly 1.0000 from 80 to 100, exactly 0.0000 beyond, and no intermediate value** (Section 3). **Fifth, the average is small ——widening all the way from 80 to 250 averages an effectiveness of 0.1176** (Section 3). **Sixth, the separator is whether every item passes through every stage——where they can be routed around, the sum is the right answer** (Section 4). **Keywords**: bottleneck, theory of constraints, serial production line, minimum capacity, factory physics.

Scope of this paper (scope note): No new theorem or law is claimed——that a serial line runs at its smallest stage is standard.No factory improvement is recommended——where to invest is not stated; only what happens when a stage is widened.Variability and buffers are excluded——each capacity is a constant, and with fluctuation the line falls below the minimum.Changeovers and breakdowns are not treated——availability is set to one.Costs are not discussed——what widening costs is not the subject.It is not said that an idle stage is waste——in a serial line, idling anywhere but the minimum is unavoidable.Relation to earlier papers: Paper 477 showed that while the wrong input is being added, the limit is somewhere else——that paper is about locating the limit, and this one about the shape of the help once it is located: place there, step here.Paper 194 showed that the four means are one family with a single equality fence——the minimum is the far end of that family, the power mean as the exponent goes to $-\infty$, and the line's capacity falls there.Paper 300 showed that sharing a root is decidable——"the line is the minimum" and "the help is a step" are separate roots.What is added is measuring the ratio to the sum as **0.1231**, listing five utilisations to put a number on idling being normal, confirming the switch from **1.0000** to **0.0000** to 12 digits on both sides of the boundary, measuring the average as **0.1176**, and placing the separator on whether every item passes through every stage.

1. Notation and the Question

Setting: five stages in series, every item passing through all of them in turn.Writing c_i for the capacity of stage i in items per hour, the line delivers

$$C = \min_i c_i$$

Observation 1.1

no sum is taken.Five capacities are brought together and not added——one is chosen and the rest discarded.

Observation 1.2

nor is it a mean.It is none of the four means of Paper 194 but sits at the end of that family——the minimum is the power mean with the exponent sent to $-\infty$. The line's capacity falls to the very outside of the family.

Question: how much does the line gain when the narrowest stage is widened?

2. The Line Is the Minimum

Computation 2.1: **five stage capacities.**

stage	capacity (per hour)	utilisation
machining	80	**1.0000**
painting	100	0.8000
assembly	120	0.6667
inspection	150	0.5333
packing	200	**0.4000**
sum	650	—
line capacity	**80**	—
ratio to the sum	**0.1231**	—

> **Observation 2.2**
>
> **the line matches the smallest.At 80——another size entirely from the sum of 650, a ratio of 0.1231.**

> **Observation 2.3**
>
> **only the smallest stage is fully used.The other four fall below one——packing reaches only 0.4000, standing idle three-fifths of the time.**

> **Observation 2.4**
>
> **idling is the normal state.Raising packing's utilisation adds nothing to the line——in a serial line, idling anywhere but the minimum is unavoidable, and driving every utilisation up produces only inventory.**

> **Observation 2.5**
>
> the sum does mean something elsewhere.Were the five in parallel, with each item needing only one of them, the line would deliver 650——the same five numbers give 80 in series and 650 in parallel: a difference of more than 8.1-fold produced by the arrangement alone.

3. The Help Has Nothing In Between

Computation 3.1: widening machining alone.

machining capacity	line capacity	effectiveness over the interval
80	80	—
90	90	**1.0000**
100	100	**1.0000**
120	100	**0.0000**
150	100	**0.0000**
250	100	**0.0000**

> **Observation 3.2**
>
> this is the core.The effectiveness is either **1.0000** or **0.0000**, never between——both confirmed to 12 digits, on either side of the boundary.

> **Observation 3.3**
>
> the switch is at the second narrowest stage.It falls to zero the moment painting's 100 is passed——the limit has moved from machining to painting, and no further widening of machining gets past painting.

Observation 3.4

the average is small.Widening by 170, from 80 to 250, gains the line only 20, an average effectiveness of 0.1176——above 0.1 and below 0.2; “about thirty per cent” set beforehand would have been wrong.

Observation 3.5

Paper 477 asks in a different place.There the subject was telling where the limit is——here it is the shape of the help at a limit already located: place there, step here.

Observation 3.6

a consequence of the step.There is no interval where widening half helps——estimating an improvement linearly understates it before the boundary and overstates it after. The error runs both ways.

4. The Separator

Observation 4.1

the separator is whether every item passes through every stage.If it must, the minimum; if it can be routed around, the sum——the same five numbers give 80 and 650. What decides is the arrangement, not the capacities.

Observation 4.2

a second separator is whether the capacities are constants.With fluctuating stages the line falls below the minimum——matching the minimum exactly requires no fluctuation and no starvation. Only that ideal side is examined here.

Observation 4.3

separate roots under Paper 300.“The line is the minimum” carries seriality——“the help is a step” carries the minimum moving to a different stage. With one stage only, the first survives and no step appears.

Observation 4.4

what is not said.An idle stage is not called waste——in a serial line idling is unavoidable, and removing it produces inventory.

5. Roll Call and Closing

1. **line capacity 80 per hour** (Computation 2.1): **the minimum.**
2. **sum 650** (Computation 2.1): **a ratio of 0.1231.**
3. **machining utilisation 1.0000** (Computation 2.1): **only the minimum fills.**
4. **packing utilisation 0.4000** (Computation 2.1): **idle three-fifths.**
5. **effectiveness 1.0000 from 80 to 100** (Computation 3.1): **to 12 digits.**
6. **effectiveness 0.0000 beyond 100** (Computation 3.1): **the far side, to 12 digits.**
7. **average effectiveness 0.1176** (Observation 3.4): **measured, then bracketed.**
8. **in parallel, 650** (Observation 2.5): **arrangement alone, over 8.1-fold.**
9. **the separator is passage through every stage** (Observation 4.1): **minimum or sum.**
10. **a second separator is constancy** (Observation 4.2): **fluctuation drops it lower.**

Closing: **The capacity of a line follows from the capacities of its stages.It follows, however, as a minimum rather than a sum or a mean——five stages of 80, 100, 120, 150 and 200 sum to 650, yet in series the line delivers 80 per hour, only 0.1231 of the sum. It is none of the four means of Paper 194 but the end of that family, the power mean with the exponent sent to $-\infty$. Consequently only the smallest stage reaches a utilisation of 1.0000, while packing sits at 0.4000 and stands idle three-fifths of the time. In a serial line this is unavoidable, and driving every utilisation up produces only inventory.And widening the narrowest stage helps with nothing in between——the effectiveness is exactly 1.0000 from 80 to 100 and exactly 0.0000 once painting’s 100 is passed, taking no intermediate value, both confirmed to 12 digits from either side of the boundary. The limit has moved from machining to painting, so widening by 170 all the way to 250 gains the line only 20, an average effectiveness of 0.1176. “About thirty per cent” set beforehand would have been wrong. Because the help is a step, estimating an improvement linearly**

understates it before the boundary and overstates it after.The separator is whether every item passes through every stage——if it must, the minimum; if it can be routed around, the sum, so that the same five numbers give 80 and 650. What decides is the arrangement, not the capacities. A second separator is whether the capacities are constants, since fluctuating stages drop the line below the minimum. Under Paper 300, "the line is the minimum" carries seriality while "the help is a step" carries the minimum moving elsewhere: with one stage only, no step appears.Placed beside Paper 477——that paper is about telling where the limit is, and this one about the shape of the help at a limit already located: place there, step here.To be honest——no factory improvement is recommended; where to invest was not stated, only what happens when a stage is widened. Capacities were taken as constants, changeovers and breakdowns were excluded with availability set to one, and the cost of widening was not discussed. An idle stage is not called waste either, since in a serial line idling is unavoidable.

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