The Index Does Not Move While the Defect Rate Rises 359-Fold

— shifting the centre 0.5 off target leaves C_p at **1.3333** while the defect rate climbs **359.1607**-fold, and C_{pk}, which watches the offset, falls to exactly half, **0.6667** — [Paper 551]

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— From thinking to syncing. —

Abstract

Process capability is reported as an index, with 1.33 taken as the customary target.What this paper shows is that an index watching only the spread is entirely blind to a second kind of failure, and that the defect rate rises 359.1607-fold while it stays blind.No new theorem or law is claimed. First, C_p does not respond to the centre——on target or 0.5 off it, the value stays 1.3333, unmoved to 12 digits (Section 2). **Second, and this is the core.The defect rate rises 359.1607-fold meanwhile——from 63.3425 ppm to 22750.1329 ppm** (Section 2). **Third, the index that watches the offset falls to exactly half——C_{pk} from 1.3333 to 0.6667** (Section 2). **Fourth, the customary 1.33 means 66.0733 ppm——above 60 and below 70; the frequently quoted 63 ppm belongs to a centred process** (Section 3). **Fifth, keeping the index while tolerating the offset is expensive——the spread must be cut to 0.5000 of what it was** (Section 3). **Sixth, the separator is whether the index takes the smaller of the two margins——an index that takes a minimum will not move however far the other limit is widened** (Section 4). **Keywords**: process capability index, Cp and Cpk, defect rate, normal distribution, quality control.

Scope of this paper (scope note): **No new theorem or law is claimed——the definitions of C_p and C_{pk} and the normal defect rate are standard.No quality practice is recommended——which index to use is not stated; only what each responds to and what it ignores.The distribution is assumed normal——a heavy tail changes the ppm figures directly: the property of the index survives, but the numbers do not.Estimation from samples is not treated——μ and σ are given as true values, and estimating them from few samples is a separate subject.The process is taken as stationary ——a centre drifting over time is not included.The customary "1.5σ shift" is not adopted——the offset used here is assumed, not imported from an industry rule of thumb.Relation to earlier papers: Paper 317 showed that a significance level is a promise per test——$C_{pk} = 1.33$ is likewise a promise about one characteristic, not about the whole product, and the same misreading occurs.Paper 375 showed that what chooses the first principal component is not the data but the units——the same shape: a number computed from data is in fact governed by something else. C_p is named for capability and governed only by spread.Paper 550 showed that a line's capacity is its minimum stage and not the sum——C_{pk} likewise takes a minimum over the two margins, and the same insensitivity to the far side appears in a line of stages and in a pair of specification limits.Paper 300 showed that sharing a root is decidable——"C_p ignores the offset" and "the defect rate rises 359-fold" are separate roots.**What is added is **confirming the immobility of C_p to 12 digits, measuring the defect-rate ratio under the same conditions as **359.1607**, separating the **66.0733** ppm of a true 1.33 from the frequently quoted 63 ppm, inverting for the cost of holding the index off centre and obtaining **0.5000**, and **placing the separator on whether a minimum is taken.

1. Notation and the Question

Setting: with specification limits L and U, a process mean μ and a standard deviation σ, two indices are in use.

$$C_p = \frac{U - L}{6\sigma}, \qquad C_{pk} = \min\left(\frac{U - \mu}{3\sigma},\ \frac{\mu - L}{3\sigma}\right)$$

Observation 1.1

μ does not appear in C_p.Only the specification width and the spread decide it——the same value results wherever the process is aimed.

Observation 1.2

C_{pk} takes a minimum.Only the smaller of the two margins is seen——the same shape as the line capacity of Paper 550: the far margin may be widened without effect.

Question: how far does the actual harm move while C_p does not?

2. The Same C_p, and 359 Times the Defects

Computation 2.1: holding $L = 9$, $U = 11$ and $\sigma = 0.25$, moving only the centre.

centre μ	C_p	C_{pk}	defect rate
10.00	**1.3333**	**1.3333**	**63.3425** ppm
10.25	**1.3333**	**1.0000**	1350.1847 ppm
10.50	**1.3333**	**0.6667**	**22750.1329** ppm

> **Observation 2.2**
>
> C_p does not move to 12 digits.A shift of 0.5 leaves it at **1.3333**——exactly as defined, and that is the problem.

> **Observation 2.3**
>
> this is the core.Meanwhile the defect rate went from **63.3425** ppm to **22750.1329** ppm, a factor of **359.1607**——above 300 and below 400, measured before being bracketed.

> **Observation 2.4**
>
> C_{pk} falls to exactly half.From **1.3333** to **0.6667**——the index watching the offset does respond. Only C_p does not.

> **Observation 2.5**
>
> one side takes everything.At $\mu = 10.5$ the upper margin is 0.5 and the lower is 1.5——C_{pk} sees only the smaller, so that lower margin of 1.5 contributes nothing at all. The same shape as the idle stage of Paper 550 contributing nothing to the line.

> **Observation 2.6**
>
> **the shape of Paper 375.There the first principal component was chosen by the units rather than the data ——here the number named “process capability” is chosen by the spread alone rather than by capability. A broad name over a narrow content.**

3. What the Customary 1.33 Is Worth

Computation 3.1: **the defect rate at exactly $C_{pk} = 1.33$.**

quantity	value
one-sided probability	3.303665×10^{-5}
counting both sides	**66.0733** ppm

> **Observation 3.2**
>
> **above 60 and below 70.Measured at $\mathbf{66.0733}$——the frequently quoted 63 ppm belongs to a centred process at $C_{pk} = 1.3333$, and the gap between 1.33 and 1.3333 shows up here.**

> **Observation 3.3**
>
> **the misreading of Paper 317 recurs.$C_{pk} = 1.33$ is a promise about one characteristic——a product with twenty characteristics has a lower chance of meeting all of them. The same shape as a significance level being per test.**

Computation 3.4: **inverting for the spread that restores C_{pk} to $\mathbf{1.3333}$ with the centre left at 10.5.**

quantity	value
required σ	**0.125000** (from **0.25**)
ratio	**0.5000**

Observation 3.5

the inversion is a check.Feeding the value back restores C_{pk}——confirming separately that this is a solution.

Observation 3.6

tolerating the offset is expensive.Halving a spread is usually harder than moving an aim back by 0.5——two routes to the same index, at entirely different cost.

4. The Separator

Observation 4.1

the separator is whether the index takes the smaller of the two margins.Taking a minimum responds to the offset; watching the width alone does not——C_p and C_{pk} differ in that one point.

Observation 4.2

an index taking a minimum has a blindness of its own.Widening the far limit does not move C_{pk}——as widening an idle stage did not raise the line in Paper 550. A minimum discards whatever it did not select.

Observation 4.3

separate roots under Paper 300.“C_p ignores the offset” carries the absence of μ from its definition——“the defect rate rises 359-fold” carries the normal tail. A heavier tail changes the factor and leaves the immobility untouched.

Observation 4.4

what is not said.C_p is not condemned——where only the spread is in question it is the right index. The trouble is the breadth of the name, not the quantity.

5. Roll Call and Closing

1. C_p **stays** $\mathbf{1.3333}$ (Computation 2.1): **unmoved to** 12 **digits.**
2. $\mathbf{63.3425}$ **ppm** (Computation 2.1): **on target.**
3. $\mathbf{22750.1329}$ **ppm** (Computation 2.1): 0.5 **off.**
4. **a factor of** $\mathbf{359.1607}$ (Observation 2.3): **the core.**
5. C_{pk} **from** $\mathbf{1.3333}$ **to** $\mathbf{0.6667}$ (Computation 2.1): **exactly half.**
6. **a true** 1.33 **is** $\mathbf{66.0733}$ **ppm** (Computation 3.1): **not the quoted** 63**.**
7. **the required** σ **is** $\mathbf{0.5000}$ (Computation 3.4): **obtained by inversion.**
8. **feeding it back restores** C_{pk} (Observation 3.5): **the inversion checks.**
9. **the separator is taking a minimum** (Observation 4.1): **one point of difference.**
10. **a minimum discards the unselected side** (Observation 4.2): **the shape of Paper 550.**

Closing: **Process capability is reported as an index, with** 1.33 **taken as the customary target.An index watching only the spread, however, is entirely blind to a second kind of failure——holding** $L = 9, U = 11$ **and** $\sigma = 0.25$ **and shifted the centre by** 0.5 **leaves** C_p **at** $\mathbf{1.3333}$**, unmoved to** 12 **digits. Meanwhile the defect rate climbs from** $\mathbf{63.3425}$ **ppm to** $\mathbf{22750.1329}$ **ppm, a factor of** $\mathbf{359.1607}$**.** C_{pk}**, which watches the offset, falls from** $\mathbf{1.3333}$ **to exactly half,** $\mathbf{0.6667}$**, so only** C_p **fails to respond. At** $\mu = 10.5$ **the upper margin is** 0.5 **and the lower** 1.5**, and since** C_{pk} **sees only the smaller, that lower margin contributes nothing at all.The customary** 1.33 **is worth** $\mathbf{66.0733}$ **ppm ——the frequently quoted** 63 **ppm belongs to a centred process at** $C_{pk} = 1.3333$**, and the gap between** 1.33 **and** 1.3333 **shows up here. The misreading of Paper 317 recurs as well:** 1.33 **is a promise about one characteristic, and a product with twenty of them has a lower chance of meeting all.Tolerating the offset while keeping the index is expensive——restoring** C_{pk} **to** $\mathbf{1.3333}$ **with the centre left at** 10.5 **requires the spread cut to** $\mathbf{0.5000}$ **of what it was, and feeding that value back restores the index, which confirms it separately. Halving a spread is usually harder than moving an aim back by** 0.5**: two routes to the same index at entirely different cost.The separator is whether the index takes the smaller of the two margins——taking a minimum responds to the offset, watching the width alone does not, and** C_p **and** C_{pk} **differ in that one point. An index taking a minimum has a blindness of its own, since widening the far limit does not move** C_{pk}**, just as widening an idle stage did not raise the line in Paper 550: a minimum discards whatever it did not select. Under Paper 300 the two facts carry different things, the absence of** μ **from a definition and the normal tail, so a heavier tail changes the factor and leaves the immobility untouched.It is also the shape of Paper 375——there the first principal component was chosen by the units rather than the data; here the number named "process capability" is chosen by the spread alone. A broad name over a narrow content.To be honest——no quality practice is recommended. The distribution was assumed normal, so a heavy tail changes the ppm figures directly while leaving the property of the index intact.** μ **and** σ **were given as true values, so estimating them from few samples is a separate subject, a drifting centre was not included, and the customary "**1.5σ **shift" was**

not imported. Nor is C_p condemned: where only the spread is in question it is the right index, and the trouble is the breadth of the name.

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