Half the Charts That “Last 370 Points on Average” Alarm by 256

— A three-sigma chart runs a mean of **370.3983** points before a false alarm, but half of them alarm within **256.3938** — and the chance of alarming before the mean is reached is **0.632618**, which is $1 - 1/e$ — [Paper 554]

Yuuki YAMAGISHI (山岸 勇輝)
Yamagishi Complexity Torus Lab, Japan
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— *From thinking to syncing.* —

Abstract

The performance of a control chart is quoted as the mean number of points before a false alarm.What this paper shows is that the mean is not the typical value, that half of all charts alarm at 0.69 of it, and that this 0.69 is $\ln 2$ and belongs to the distribution rather than to the chart.No new theorem or law is claimed. First, the probability used can be checked independently——a point falling outside three sigma with probability 0.0026997961 matches the customary 0.0027 to 10^{-5} (Section 2). **Second, and this is the core.The mean is 370.3983 points and the median 256.3938——half alarm before that** (Section 2). **Third, the ratio is $\ln 2$——0.692211 against 0.693147, within 0.001: a property of the geometric distribution, not of the chart** (Section 2). **Fourth, the chance of alarming before the mean is $1 - 1/e$——0.632618 against 0.632121, so more than six in ten come early** (Section 3). **Fifth, false alarms are not rare even on a single chart——0.065353 over 25 points and 0.236884 over 100** (Section 3). **Sixth, the separator is whether the alarm probability is constant from point to point——without that the run length is not geometric, and neither $\ln 2$ nor $1 - 1/e$ appears** (Section 4). **Keywords**: control chart, average run length, three-sigma limits, geometric distribution, median.

Scope of this paper (scope note): **No new theorem or law is claimed——three-sigma limits, average run length and the geometric distribution are standard.No charting practice is recommended——where to place the limits is not stated.Run lengths under a shifted process are not treated——only false alarms on a stable process are measured; how long detection takes is a separate subject.The distribution is assumed normal——a heavy tail changes the 0.0027 itself.Correlation between successive points is excluded——the points are taken as independent, and autocorrelation makes the run length non-geometric.Supplementary run rules are not included——only a single point outside the limits is counted, and rules such as seven in a row would raise the false-alarm rate further.Three sigma is called neither loose nor tight——only the gap between the mean and the median is shown.Relation to earlier papers**: **Paper 308 showed that the gap between the buses you board is twice the average——both are of the form "the mean is not the typical", with different roots: that one is a sampling bias favouring long intervals, this one the skew of the geometric distribution itself, and the departures run in opposite directions.Paper 317 showed that a significance level is a promise per test——the 0.0027 is likewise a promise per point and not per chart, and 25 points make it $\mathbf{0.065353}$ .Paper 300 showed that sharing a root is decidable——"the median is $\ln 2$ of the mean" and "the chance of alarming by the mean is $1 - 1/e$" share one root.**What is added is **checking the three-sigma probability against an independently known value**, **measuring the mean and median as $\mathbf{370.3983}$ and $\mathbf{256.3938}$ and matching their ratio to $\ln 2$**, **inverting the median to confirm a probability of exactly 0.5**, **matching the chance of an early alarm to $1 - 1/e$**, and **placing the separator on constancy of the alarm probability**.

1. Notation and the Question

Setting: **with a stable process, write α for the probability that one plotted point falls outside the three-sigma limits.If the points are independent, the number L of points before the next excursion is geometric, with**

$$\Pr(L > m) = (1 - \alpha)^m, \qquad \mathbb{E}[L] = \frac{1}{\alpha}$$

and this mean is called the average run length.

Observation 1.1

performance is quoted through that mean."Three sigma lasts 370 points on average"——which assumes the mean is the typical value.

> **Observation 1.2**
>
> **the geometric distribution is skewed.Its most likely value is the first point——the likeliest outcome is an early alarm, and the mean is held up by a long tail.**

Question: **how far apart are the mean and the half-way case?**

2. A Mean of 370 and a Median of 256

Computation 2.1: **obtaining α from the normal distribution and then the mean and median.**

quantity	value
probability outside three sigma, α	**0.0026997961**
mean run length $1/\alpha$	**370.3983** points
median run length $\ln(1/2)/\ln(1-\alpha)$	**256.3938** points
median divided by mean	**0.692211**
$\ln 2$	0.693147

> **Observation 2.2**
>
> **α checks independently.It matches the customary 0.0027 to within 10^{-5}——a comparison with a value from outside, not with the implementation itself.**

> **Observation 2.3**
>
> **so does the mean.It matches the customary 370 to within one point——thus far the textbook figures are simply reproduced.**

Observation 2.4

this is the core.The median is 256.3938 points, fully 114 short of the mean——half of all charts alarm earlier than the number anyone told “370 on average” would picture.

Observation 2.5

the ratio is $\ln 2 . 0.692211$ agrees with 0.693147 to within 0.001——$\ln 2$ is fixed entirely outside this computation, so this is a check: disagreement would mean the geometric distribution is being handled wrongly.

Observation 2.6

it is not a property of the chart.The $\ln 2$ comes from the run length being geometric, not from three sigma or from normality——wherever the limits are placed, so long as α is small the median is 0.69 of the mean.

Observation 2.7

the inversion was checked.Feeding the median back gives a probability of exactly 0.5, to 10^{-12}——confirming separately that it solves the equation.

3. How Soon an Alarm Comes

Computation 3.1: the chance of at least one alarm within a given number of points.

points	chance of at least one alarm
25 (one chart)	**0.065353**
100	**0.236884**
370	**0.632222**
370.3983 (the mean itself)	**0.632618**
$1 - 1/e$	0.632121

Observation 3.2

the chance of alarming by the mean is $1 - 1/e$.0.632618 against 0.632121, within 0.001——again a comparison with a number fixed outside this work.

Observation 3.3

more than six in ten come early.Lasting to the mean is the minority outcome——so the word “average” does not describe what most charts do.

Observation 3.4

false alarms are not rare on a single chart.0.065353 over 25 points, so roughly one chart in 15 alarms on a perfectly good process——above 0.06 and below 0.07; over 100 points it is 0.236884, close to one in four.

Observation 3.5

the misreading of Paper 317 recurs.The 0.0027 is a promise per point, not per chart——the same shape as a significance level being per test.

Observation 3.6

Paper 308 has a different root.There a sampling bias favouring long intervals made the experienced value twice the average——here the skew of the geometric distribution makes the typical value smaller than the average. The same slogan, opposite directions.

4. The Separator

Observation 4.1

the separator is whether the alarm probability is constant from point to point.If it is, the run length is geometric and the median is $\ln 2$ of the mean——a process drifting gradually raises the alarm probability as it goes, the run length ceases to be geometric, and neither $\ln 2$ nor $1 - 1/e$ appears.

Observation 4.2

a second separator is independence between points.With autocorrelation, an alarm makes the next one likelier——and the median moves earlier still.

Observation 4.3

one root, under Paper 300.“The median is $\ln 2$ of the mean” and “the chance of alarming by the mean is $1 - 1/e$” both come from $(1 - \alpha)^m$ alone——the usual verdict in this body of work is separate roots, and here it is one. That a common root can be asserted is also a result of the test.

Observation 4.4

what is not said.Three sigma is called neither loose nor tight——only the gap is shown; where to place the limits needs the missed-detection side measured too.

5. Roll Call and Closing

1. $\alpha = \mathbf{0.0026997961}$ (Computation 2.1): **matching 0.0027 to 10^{-5}.**
2. **mean run length $\mathbf{370.3983}$** (Computation 2.1): **matching 370 within a point.**
3. **median run length $\mathbf{256.3938}$** (Computation 2.1): **the core.**
4. **ratio $\mathbf{0.692211}$** (Observation 2.5): **within 0.001 of $\ln 2$.**
5. **the inversion returns 0.5** (Observation 2.7): **to 10^{-12}.**
6. **alarming by the mean, $\mathbf{0.632618}$** (Computation 3.1): **within 0.001 of $1 - 1/e$.**
7. **25 points give $\mathbf{0.065353}$** (Computation 3.1): **not rare on one chart.**
8. **100 points give $\mathbf{0.236884}$** (Computation 3.1): **close to one in four.**
9. **the separator is a constant alarm probability** (Observation 4.1): **drift breaks it.**
10. **the two facts share a root** (Observation 4.3): **both from $(1 - \alpha)^m$.**

Closing: **The performance of a control chart is quoted as the mean number of points before a false alarm.That mean, however, is not the typical value——on a three-sigma chart a point falls outside with probability $\mathbf{0.0026997961}$ and the mean run length is $\mathbf{370.3983}$ points, but the median is $\mathbf{256.3938}$, fully 114 short. Half of all charts alarm earlier than the number anyone told "370 on average" would picture. The α used matches the customary 0.0027 to 10^{-5} and the mean matches 370 within a point, so thus far the textbook figures are simply reproduced.And the ratio of median to mean is $\ln 2$——$\mathbf{0.692211}$ against 0.693147, within 0.001, and since $\ln 2$ is fixed entirely outside this computation the agreement is a check. It comes from the run length being geometric rather than from three sigma or from normality, so wherever the limits are placed the median stays at 0.69 of the mean while α is small: a property of the distribution, not of the chart. Feeding the median back returns a probability of exactly 0.5, confirming it separately.The chance of alarming before the mean is reached is $1 - 1/e$——$\mathbf{0.632618}$ against 0.632121, within 0.001. With more than six in ten coming early, the word "average" does not describe what most charts do. Even one chart of 25 points alarms on a perfectly good process with probability $\mathbf{0.065353}$, roughly one chart in 15, and over 100 points it is $\mathbf{0.236884}$, close to one in four. The misreading of Paper 317 recurs, since the 0.0027 is a promise per point and not per chart.The separator is whether the alarm probability is constant from point to point——if it is, the run length is geometric and the median is $\ln 2$ of the mean; a process drifting gradually raises that probability as it goes, and neither $\ln 2$ nor $1 - 1/e$ appears. A second separator is independence between points, since autocorrelation moves the median earlier still. Under Paper 300 the two facts here share one root, both coming from $(1 - \alpha)^m$ alone. The usual verdict in this body of work is separate roots, and that a common root can be asserted is also a result of the test.Paper 308 has a different root——there a sampling bias favouring long intervals made the experienced value twice the average, while here the skew of the geometric distribution makes the typical value smaller than the average: the same slogan, running in opposite directions.To be honest——no charting practice is recommended, and where to place the limits was not stated. Only false alarms on a stable process were measured, so detection time is a separate subject. The distribution was assumed normal, which fixes the 0.0027; points were taken as independent;**

and supplementary rules such as seven in a row were left out, which would raise the false-alarm rate further. Three sigma is called neither loose nor tight: only the gap between mean and median is shown.

References

1. Feller, W. (1968). *An Introduction to Probability Theory and Its Applications, Volume 1* (3rd ed.). Wiley.
2. Montgomery, D. C. (2019). *Introduction to Statistical Quality Control* (8th ed.). Wiley.
3. Shewhart, W. A. (1931). *Economic Control of Quality of Manufactured Product*. Van Nostrand.
4. Woodall, W. H. (2000). *Controversies and Contradictions in Statistical Process Control*. Journal of Quality Technology.
5. Yamagishi, Y. (2026). *The Three Hundredth Census: Whether Two Things Share a Root Is Decidable* [Paper 300]. Zenodo. 10.5281/zenodo.22171914
6. Yamagishi, Y. (2026). *The Gap between the Buses You Board Is Twice the Average* [Paper 308]. Zenodo. 10.5281/zenodo.22172305
7. Yamagishi, Y. (2026). *A Significance Level Is a Promise per Test, Not per Study* [Paper 317]. Zenodo. 10.5281/zenodo.22208819

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Yuuki YAMAGISHI (山岸 勇輝)

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