Under Multiplication, Fixing a Stage That Is Not the Worst Still Helps

— with one stage down at 0.80, raising an unrelated 0.99 to 1.00 still gives **1.010101**, which is not zero — ask the same line about speed and the rule becomes a minimum, where a stage that is not the bottleneck returns exactly **0.0000** — [Paper 555]

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September 2026

— *From thinking to syncing.* —

Abstract

A line is judged by finding what holds it back.What this paper shows is that the manner of aggregation depends on the quantity asked for: yield multiplies, and under multiplication a stage that is not the worst still repays repair.No new theorem or law is claimed. First, multiplication works quietly——stages at 0.99 leave 0.366032 after a hundred of them, and halve after 68.967564 (Section 2). **Second, it is near $1/e$ but not equal to it——0.366032 against 0.367879, differing by 0.001847, and demoted under Paper 405** (Section 2). **Third, and this is the core.A stage that is not the worst still repays repair——with one stage at 0.80, raising an unrelated 0.99 to 1.00 gives 1.010101** (Section 3). **Fourth, repairing the worst helps far more, though——1.237500, for a ratio of 23.512500: large, and finite** (Section 3). **Fifth, asking the same line about speed gives a minimum——in Paper 550 that same stage returned exactly 0.0000: the difference between 1.010101 and nothing** (Section 4). **Sixth, the separator is whether a stage's failure discards the item or caps the rate——discarding gives a product, capping gives a minimum** (Section 4). **Keywords**: yield, serial process, law of the minimum, process improvement, semiconductor manufacturing.

Scope of this paper (scope note): **No new theorem or law is claimed——that yields multiply along a line, and that capacities take a minimum, are standard.No factory improvement is recommended——where to invest is not stated; only what happens when a stage is repaired.Stages are assumed independent——if a defect at one stage causes defects later, the product form fails.Rework and re-entry are excluded——an item once lost is taken as lost.The cost of improvement is not discussed——raising 0.99 to 0.995 and raising 0.80 to 0.99 must cost very different amounts, and that is not included.Neither "start with the worst" nor "anywhere will do" is asserted——only the ratio of the effects is measured, and an order requires costs.Relation to earlier papers**: **Paper 292 showed that only the logarithm adds——the $\mathbf{0.605006}$ obtained here from fifty stages at 0.99 is the same number as the 0.60500607 obtained there from fifty gains of ten per cent and fifty losses of ten per cent, both being 0.99 raised to the fiftieth; the same arithmetic serves the averaging of returns there and the allocation of improvement here.Paper 550 showed that a line's capacity is its minimum stage and that widening helps either fully or not at all——this paper is its counterpart: for one line, changing the quantity asked changes the aggregation from a minimum to a product, and the effect of repairing a non-binding stage from $\mathbf{0.0000}$ to $\mathbf{1.010101}$.Paper 405 showed that a numerical coincidence is evidence of a common root only when there is a bijection——applied here to the nearness of $\mathbf{0.366032}$ to $1/e$.Paper 300 showed that sharing a root is decidable——"the aggregation is a product" and "the effect never reaches zero" are separate roots.**What is added is **inverting for the halving length of $\mathbf{68.967564}$, checking the nearness to $1/e$ both for closeness and for inequality, measuring the non-binding effect as $\mathbf{1.010101}$, measuring the ratio of effects as $\mathbf{23.512500}$, and placing the separator on discarding against capping**.

1. Notation and the Question

Setting: **n stages in series, with a fraction y_i passing stage i as good, and nothing lost ever returning.The overall yield is**

$$Y = \prod_{i=1}^{n} y_i$$

Observation 1.1

this is not a minimum.The capacity treated in Paper 550 was $\min_i c_i$——one line is being looked at, and the manner of aggregation differs.

> **Observation 1.2**
>
> **multiplication is inconspicuous.With every stage at 0.99, every stage looks nearly perfect——the overall yield falls with no bad stage anywhere.**

Question: **which stage repays repair, and by how much?**

2. Multiplication Works Quietly

Computation 2.1: **every stage at 0.99, varying the number of stages.**

stages	overall yield
10	0.904382
50	**0.605006**
100	**0.366032**
200	0.133980

> **Observation 2.2**
>
> **every stage nearly perfect, and a third left after a hundred.$\mathbf{0.366032}$——a poor result with no culprit anywhere.**

> **Observation 2.3**
>
> **it halves after $\mathbf{68.967564}$ stages.Feeding that number back returns exactly 0.5, to 10^{-12}——confirmed by inversion.**

Observation 2.4

the 0.605006 at fifty stages is the number of Paper 292.There fifty gains of ten per cent and fifty losses of ten per cent gave 0.60500607——both are 0.99 raised to the fiftieth. The same arithmetic serves the averaging of returns there and the allocation of improvement here: one number, two questions.

Observation 2.5

it is near $1/e$.0.366032 against 0.367879, differing by 0.001847——close enough to tempt one into writing "a hundred stages leave $1/e$".

Observation 2.6

but it is not equal.The difference exceeds 10^{-4}——demoted under Paper 405. The hundredth power of 0.99 is $e^{-1.005033}$, not e^{-1}; the nearness comes from $\ln 0.99$ being near -0.01, which is an approximation and not an identity. The check demands both closeness and inequality.

3. Which Stage Repays Repair

Computation 3.1: **on a line of stages all at 0.99, comparing two ways to improve.**

improvement	overall yield	ratio
none	**0.366032**	1.000000
one stage to 0.995	0.367881	**1.005051**
every stage to 0.995	0.605770	**1.654964**

Observation 3.2

repairing one stage barely moves it.1.005051——against 1.654964 for all of them, a ratio of gains above a hundredfold. The more stages there are, the worse concentration pays.

Computation 3.3: **now a line with a single stage down at 0.80.**

improvement	overall yield	ratio
none (99 stages at 0.99 and one at 0.80)	**0.295784**	1.000000
raising the 0.80 to 0.99	**0.366032**	**1.237500**
raising an unrelated 0.99 to 1.00	**0.298771**	**1.010101**
ratio of the gains	—	**23.512500**

> **Observation 3.4**
>
> **this is the core.Repairing a stage that is not the worst gives 1.010101, which is not zero——"that one is not the culprit, so touching it is wasted" does not hold under multiplication.**

> **Observation 3.5**
>
> **the worst is still far better to repair.The gains stand at 23.512500 to one, above 20 and below 30——measured, then bracketed; "orders of magnitude" set beforehand would have been wrong.**

> **Observation 3.6**
>
> **position does not matter.A product does not care about order, so the same improvement placed elsewhere gives the same answer, to 12 digits——what decides the effect is the original value, not the position.**

4. The Separator

> **Observation 4.1**
>
> **the separator is whether a stage's failure discards the item or caps the rate.Discarding gives a product, capping gives a minimum——both are stages in series, aggregated differently.**

Observation 4.2

one line, and the aggregation follows the question.Ask how many come out per hour and the answer is a minimum; ask how many of those started come out good and the answer is a product——there is only one line, and two ways to build an answer about it.

Observation 4.3

that is the contrast with Paper 550.There, widening a stage that was not the constraint returned exactly 0.0000——here it returns 1.010101, which is not zero. A minimum discards whatever it did not select, while a product keeps every stage. The answer to "is touching an unrelated stage wasted?" reverses with the question asked.

Observation 4.4

separate roots under Paper 300."The aggregation is a product" carries items lost never returning——"the effect never reaches zero" carries each factor entering the result linearly. Allow rework and the first fails, and the product form goes with it.

Observation 4.5

what is not said.Neither "start with the worst" nor "anywhere will do"——only the ratio of effects was measured, and raising 0.99 to 0.995 must cost quite differently from raising 0.80 to 0.99. An order requires costs.

5. Roll Call and Closing

1. **a hundred stages leave 0.366032** (Computation 2.1): **each at 0.99.**
2. **halving after 68.967564 stages** (Observation 2.3): **confirmed by inversion.**
3. **the 0.605006 at fifty is Paper 292's number** (Observation 2.4): **a different question.**
4. **differing from $1/e$ by 0.001847** (Observation 2.5): **close.**
5. **but not equal** (Observation 2.6): **demoted under Paper 405; $e^{-1.005033}$.**

6. **one stage to 0.995 gives 1.005051** (Computation 3.1): **barely moving.**
7. **repairing the worst gives 1.237500** (Computation 3.3): **0.80 to 0.99.**
8. **a non-binding stage gives 1.010101** (Computation 3.3): **the core; not zero.**
9. **a ratio of 23.512500** (Observation 3.5): **measured, then bracketed.**
10. **in Paper 550 the same position gives 0.0000** (Observation 4.3): **the question reverses it.**
11. **the separator is discarding against capping** (Observation 4.1): **product or minimum.**

Closing: **A line is judged by finding what holds it back.Yield, however, aggregates by multiplication, so the result falls with no culprit anywhere——with every stage at a nearly perfect 0.99, a hundred of them leave an overall yield of 0.366032. It halves after 68.967564 stages, a value confirmed by feeding it back to return exactly 0.5. The hundred-stage figure lies near $1/e$, differing by 0.001847, but is not equal to it: the hundredth power of 0.99 is $e^{-1.005033}$ rather than e^{-1}, the nearness coming from $\ln 0.99$ being near -0.01. It is demoted under Paper 405, and the check demands both closeness and inequality. The 0.605006 at fifty stages is moreover the same number as the 0.60500607 of Paper 292, where fifty gains and fifty losses of ten per cent were compounded: both are 0.99 raised to the fiftieth, serving the averaging of returns there and the allocation of improvement here.And under multiplication, a stage that is not the worst still repays repair——on a line with one stage down at 0.80, raising it to 0.99 gives 1.237500, while raising an unrelated 0.99 to 1.00 still gives 1.010101, which is not zero. "That one is not the culprit, so touching it is wasted" does not hold here. The worst is still far better to repair, the gains standing at 23.512500 to one, above 20 and below 30 — measured before being bracketed, since "orders of magnitude" set beforehand would have been wrong. A product does not care about order, so what decides the effect is the original value and not the position.The separator is whether a stage's failure discards the item or caps the rate——discarding gives a product and capping gives a minimum, both from stages in series. For one and the same line, asking how many come out per hour gives a minimum while asking how many of those started come out good gives a product: one line, two ways to build an answer. That is the contrast with Paper 550, where widening a stage that was not the constraint returned exactly 0.0000 against the 1.010101 here. A minimum discards whatever it did not select while a product keeps every stage, so the answer to whether touching an unrelated stage is wasted reverses with the question asked. Under Paper 300, "the aggregation is a product" carries items lost never returning while "the effect never reaches zero" carries each factor entering linearly.To be honest——no factory improvement is recommended. Stages were assumed independent, so a defect at one stage causing defects later would break the product form, and rework was excluded with an item once lost taken as lost. The cost of improvement was not discussed, though raising 0.99 to 0.995 must cost quite differently from raising 0.80 to 0.99. Neither "start with the worst" nor "anywhere will do" is therefore asserted: only the ratio of the effects was measured.**

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