The Angle That Strengthens a Tooth Points Against the Angle That Smooths the Drive

— Raising the pressure angle from **14.5** to **30** degrees thickens the tooth root but drops the contact ratio from **1.919856** to **1.344019**, a factor of **1.428444** — and throughout, the gear ratio stays at two, unmoved to **12** digits — [Paper 557]

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— From thinking to syncing. —

Abstract

The pressure angle of a gear is chosen for strength.What this paper shows is that the same angle moves the smoothness of the drive the other way, and that within one gear pair the ratio and the smoothness are settled by quantities of entirely different kinds.No new theorem or law is claimed. First, and this is the core.Raising the pressure angle lowers the contact ratio——1.919856 at 14.5 degrees against 1.344019 at 30, a factor of 1.428444 (Section 2). **Second, more teeth raise it——1.463161 for a pair of 10 and 20, 1.784715 for 60 and 60** (Section 2). **Third, below one the drive is interrupted——cutting the tooth height reaches exactly one at 0.572468 modules, below which one tooth leaves before the next arrives** (Section 3). **Fourth, both sides of the boundary were examined ——one per cent below it falls short of one and one per cent above it exceeds one** (Section 3). **Fifth, the gear ratio does not move meanwhile——40 teeth against 20 give exactly two, unchanged to 12 digits** (Section 4). **Sixth, the separator is that the ratio is settled by integers——smoothness is settled by continuous quantities and the ratio by a count of teeth: two kinds of quantity in one machine** (Section 4). **Keywords**: gears, involute tooth profile, pressure angle, contact ratio, tooth strength.

Scope of this paper (scope note): **No new theorem or law is claimed——the involute tooth form and the contact-ratio formula are standard.No gear design is recommended——which pressure angle to choose is not stated; only what moves when it does.Root strength is not computed——that a larger pressure angle thickens the root is taken as given and the bending stress is not solved, so which choice is better is not settled here.Only spur gears are treated——helical gears add engagement along the tooth, which changes the shape of the conflict.Profile shift is excluded——the calculation of reduced tooth height carries no addendum modification.Lubrication, wear and noise are not treated——contact ratio is known to affect noise, and no noise was measured here.It is not said that a larger contact ratio is always better——what it costs is precisely the conflict of Section 2.Relation to earlier papers**: **Paper 556 showed that what carries most of an external load is not the bolt, and that more preload delays separation while leaving the bolt less margin——the pressure angle here has the same shape, one knob moving two good things in opposite directions, though that one concerns a division of load and this one a contest within a geometry.Paper 551 showed that one index watches only one of two failures——the same frame, with one quantity unable to serve two demands, appearing there in an index and here in a design knob.Paper 300 showed that sharing a root is decidable——"the pressure angle lowers the contact ratio" and "the gear ratio is an unmoving integer" are separate roots.**What is added is **sweeping the pressure angle over six settings and measuring the factor as $\mathbf{1.428444}$, bisecting for the tooth height at which the contact ratio is one, $\mathbf{0.572468}$ modules, and confirming it by inversion, checking that the sign changes on both sides of that boundary, confirming that zero tooth height returns zero contact length**, and **placing the separator on the ratio being settled by integers**.

1. Notation and the Question

Setting: **two spur gears of module m, pressure angle α and tooth counts z_1 and z_2 mesh, with contact ratio**

$$\varepsilon = \frac{\sqrt{r_{a1}^2 - r_{b1}^2} + \sqrt{r_{a2}^2 - r_{b2}^2} - a\sin\alpha}{\pi m \cos\alpha}$$

whose numerator is the length over which contact persists and whose denominator is the tooth pitch on the base circle.

Observation 1.1

ε is the average number of teeth in mesh.Above one, the next tooth engages before the last one leaves——below one, there are moments with nothing in mesh.

> **Observation 1.2**
>
> **the pressure angle has been chosen for strength.The move from 14.5 to 20 degrees was made to thicken the tooth root——yet α stands in both the numerator and the denominator.**

Question: when it is moved towards strength, which way does smoothness go?

2. Raising the Pressure Angle Lowers the Contact Ratio

Computation 2.1: tooth counts 20 and 40, module 2, varying only the pressure angle.

pressure angle	contact ratio
14.5 degrees	**1.919856**
17.5 degrees	1.750404
20.0 degrees	**1.635186**
22.5 degrees	1.539636
25.0 degrees	1.460768
30.0 degrees	**1.344019**

> **Observation 2.2**
>
> **this is the core.It falls monotonically, and 14.5 against 30 degrees gives 1.428444——above 1.4 and below 1.5, measured before being bracketed.**

> **Observation 2.3**
>
> **strength and smoothness point opposite ways.A larger pressure angle thickens the root and thins the number of teeth in mesh——one knob moving two good things against each other. Which to take is not settled here.**

> **Observation 2.4**
>
> **the shape of Paper 556.There, more preload delayed separation and left the bolt less margin——one knob, two opposed goods; that one a division of load, this one a contest within a geometry.**

Computation 2.5: **varying the tooth counts at 20 degrees.**

teeth	contact ratio
10 and 20	**1.463161**
12 and 24	1.511122
20 and 20	1.556838
20 and 40	1.635186
60 and 60	**1.784715**

> **Observation 2.6**
>
> **more teeth raise it.From 1.463161 to 1.784715——small gears are at a disadvantage in smoothness, and what a larger pressure angle costs can be bought back with teeth.**

3. Below One, the Drive Is Interrupted

Computation 3.1: **tooth counts 20 and 40 at 20 degrees, cutting the tooth height.**

tooth height	contact ratio
1.0 module	**1.635186**
0.8 module	1.346530
0.6 module	1.043104
0.5 module	**0.884820**
0.3 module	0.552264

Computation 3.2: **bisecting for the height at which the ratio is exactly one gives 0.572468 modules.**

> **Observation 3.3**
>
> **the inversion is a check.Feeding that value back returns exactly one, within 10^{-9}——confirming separately that it solves the equation.**

> **Observation 3.4**
>
> **both sides were examined.One per cent below falls short of one, one per cent above exceeds it——showing one side would not show that this is the boundary.**

> **Observation 3.5**
>
> **the far extreme was checked too.Tooth height approaching zero returns zero contact length——the cheapest confirmation that the formula is intact.**

> **Observation 3.6**
>
> **one is not a graded target but a break.The gap between 1.05 and 0.95 is not ten per cent of anything——above it something is always in mesh, below it moments of nothing arrive periodically. A difference of kind, not of amount.**

4. The Separator

Computation 4.1: **meanwhile the gear ratio: 40 teeth against 20 gives exactly 2.000000000000.**

> **Observation 4.2**
>
> **it does not move to 12 digits.Sweeping the pressure angle from 14.5 to 30 degrees, or halving the tooth height, leaves it at two——because the ratio of rotation is settled not by the geometry but by an integer count of teeth.**

> **Observation 4.3**
>
> **the separator is that the ratio is settled by integers.Smoothness follows continuous quantities, angle and height; the ratio follows a count——two kinds of quantity within one pair of gears, which is why there is no slip as in a belt, and why smoothness may nevertheless degrade continuously.**

> **Observation 4.4**
>
> **separate roots under Paper 300.“The pressure angle lowers the contact ratio” carries the involute geometry ——“the gear ratio is an unmoving integer” carries the countability of teeth. Change the tooth form and the second survives; make the teeth a continuum and only the second is lost.**

> **Observation 4.5**
>
> **what is not said.A larger contact ratio is not called better——what it costs is the conflict of Section 2, and with the root bending stress unsolved, which choice wins is not settled here.**

5. Roll Call and Closing

1. **1.919856 at 14.5 degrees** (Computation 2.1): **the smoothest.**
2. **1.344019 at 30 degrees** (Computation 2.1): **the core.**
3. **a factor of 1.428444** (Observation 2.2): **measured, then bracketed.**
4. **1.463161 for 10 and 20** (Computation 2.5): **small gears lose.**

5. **1.784715 for 60 and 60** (Computation 2.5): **buy it back with teeth.**
6. **the boundary at 0.572468 modules** (Computation 3.2): **by bisection.**
7. **feeding it back returns one** (Observation 3.3): **the inversion checks.**
8. **the sign changes on both sides** (Observation 3.4): **a genuine boundary.**
9. **gear ratio 2.000000000000** (Computation 4.1): **unmoved to 12 digits.**
10. **the separator is integrality** (Observation 4.3): **two kinds of quantity.**

Closing: **The pressure angle of a gear is chosen for strength.Yet the same angle moves the smoothness of the drive the other way——with tooth counts 20 and 40 at module 2, sweeping only the pressure angle takes the contact ratio monotonically from 1.919856 at 14.5 degrees to 1.344019 at 30, a factor of 1.428444. That lies above 1.4 and below 1.5, measured before being bracketed. A larger angle thickens the tooth root and thins the number of teeth in mesh: one knob moving two good things against each other, and with the root bending stress unsolved, which to take is not settled here. It can be bought back with teeth, since 10 and 20 give 1.463161 while 60 and 60 give 1.784715, so small gears are at a disadvantage in smoothness.And below a contact ratio of one, the drive itself is interrupted ——cutting the tooth height reaches exactly one at 0.572468 modules, a value confirmed by feeding it back. One per cent below falls short and one per cent above exceeds, so the boundary is shown from both sides, and a vanishing tooth height returns a vanishing contact length. This one is not a graded target but a break: above it something is always in mesh, below it moments of nothing arrive periodically. A difference of kind, not of amount.The separator is that the ratio is settled by integers——sweeping the pressure angle from 14.5 to 30 degrees, or halving the tooth height, leaves the ratio of 40 teeth to 20 at 2.000000000000, unmoved to 12 digits. Smoothness follows continuous quantities and the ratio follows a count: two kinds of quantity within one pair of gears, which is why there is no slip as in a belt and why smoothness may still degrade continuously. Under Paper 300, “the pressure angle lowers the contact ratio” carries the involute geometry while “the gear ratio is an unmoving integer” carries the countability of teeth.It is also the shape of Paper 556——there more preload delayed separation and left the bolt less margin: one knob, two opposed goods, that one a division of load and this one a contest within a geometry. Paper 551 gave the same frame in an index rather than a knob, one quantity unable to serve two demands.To be honest——no gear design is recommended. The root bending stress was not solved, so which choice is better is not settled, and the thickening of the root was taken as given. Only spur gears were treated, helical ones adding engagement along the tooth and changing the shape of the conflict; profile shift was excluded, and lubrication, wear and noise were not measured. Nor is a larger contact ratio called better: what it costs is precisely the conflict shown here.**

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