The Two Conventions Are Not in Conflict — They Are One Clutch, New and Bedded In

— uniform pressure and uniform wear give torques differing by **3.0876** per cent, a gap that closes as the annulus narrows — in the limit of a full disc the ratio is exactly **4/3**, which is not the **4/3** of Paper 527 — [Paper 559]

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— From thinking to syncing. —

Abstract

Clutch torque has two formulas, and the answer depends on which is used.What this paper shows is that the two are not rival descriptions but one part at two times, new and bedded in, and that the gap between them is set by the shape of the friction face.No new theorem or law is claimed. First, the two conventions return different torques——165.971014 N m against 161.000000, a difference of 3.0876 per cent (Section 2). **Second, and this is the core.They do not claim to hold at the same time——uniform pressure is a new face and uniform wear a bedded-in one, so the disagreement is a history rather than an ambiguity** (Section 4). **Third, the gap closes as the annulus narrows——from 1.194925 at an inner radius of 0.020 m to 1.000396 at 0.140 m** (Section 2). **Fourth, both extremes were checked——the ratio returns to one as the inner radius approaches the outer, and to exactly 4/3 as it approaches zero** (Section 2). **Fifth, that 4/3 is not the 4/3 of Paper 527——under Paper 405, with no bijection, it is not evidence of a common root** (Section 4). **Sixth, the separator is whether two descriptions claim the same instant——if they do, the conventions are in conflict; if not, they are a history** (Section 4). **Keywords**: clutch, uniform pressure, uniform wear, effective radius, bedding-in.

Scope of this paper (scope note): **No new theorem or law is claimed——the effective radii under uniform pressure and uniform wear are both standard.No clutch design is recommended——which radii to choose is not stated; only what changes when the formula does.The friction coefficient is taken as a constant——its variation with temperature, speed and contact pressure is excluded, and in practice that is the dominant uncertainty.Heat is not treated——neither slipping-clutch heating nor fade is the subject.The progress of wear is not solved——uniform wear is posited as the bedded-in steady state, and the path to it is not computed.Multi-plate and wet clutches are not distinguished——the number of faces enters as a factor and leaves the ratio untouched.It is not said which formula is correct——that correctness depends on the time is the claim of this paper.Relation to earlier papers**: **Paper 527 showed that multiplying the Earth's radius by $4/3$ is a convention, and that there are two of them——the shape is the same, but there the two describe one atmosphere at one instant and disagree by $\mathbf{1.077033}$, so one must be chosen; here the two describe one part at different times and need not be chosen between. "Two conventions" can mean rivals or a sequence.Paper 405 showed that a numerical coincidence is evidence of a common root only when there is a bijection ——applied here to the $\mathbf{4/3}$ of the limiting ratio against the $4/3$ of Paper 527.Paper 300 showed that sharing a root is decidable——"the two conventions differ" and "the difference vanishes for a narrow annulus" are separate roots.**What is added is **setting the two effective radii side by side over five inner radii, confirming the return to $\mathbf{4/3}$ and to one at the two extremes, locating the inner radius of maximum torque numerically and matching it to the independent closed form $r_o/\sqrt{3}$, demoting the $\mathbf{4/3}$ coincidence under Paper 405**, and **placing the separator on whether the same instant is claimed**.

1. Notation and the Question

Setting: **an annular friction face of outer radius r_o and inner radius r_i is pressed by an axial force F, transmitting a torque $T = \mu F R$ where the effective radius R has two formulas.**

$$R_{\text{pressure}} = \frac{2}{3} \cdot \frac{r_o^3 - r_i^3}{r_o^2 - r_i^2}, \qquad R_{\text{wear}} = \frac{r_o + r_i}{2}$$

Observation 1.1

the first assumes uniform contact pressure.The assumption for a new, flat face pressed against another—— the outer band has more circumference, so it earns more torque.

Observation 1.2

the second assumes uniform wear.If wear goes as pressure times sliding speed, the steady state has pr constant——the outer band slides faster and wears away first, moving the pressure inward.

Question: **how far apart are they?And must one be chosen?**

2. Two Conventions, Two Torques

Computation 2.1: **outer radius 0.150 m, varying the inner radius.**

inner radius	uniform pressure	uniform wear	ratio
0.020 m	0.101569 m	0.085000 m	**1.194925**
0.050 m	0.108333 m	0.100000 m	1.083333
0.080 m	**0.118551** m	**0.115000** m	**1.030876**
0.110 m	0.131026 m	0.130000 m	1.007890
0.140 m	0.145057 m	0.145000 m	**1.000396**

Observation 2.2

the gap closes as the annulus narrows.Monotonically, from 1.194925 to 1.000396——on a narrow ring every point is at nearly the same radius, so the distribution of pressure cannot reach the answer.

Observation 2.3

uniform pressure is always the larger.The order never reverses——so designing on the bedded-in formula errs on the safe side.

Computation 2.4: **torque at a friction coefficient of 0.35, an axial force of 4000 N and an inner radius of 0.080 m.**

convention	torque
uniform pressure (new)	**165.971014** N m
uniform wear (bedded in)	**161.000000** N m
difference	**4.971014** N m (**3.0876** per cent)

> **Observation 2.5**
>
> **above three per cent and below four.Measured at 3.0876——small beside the uncertainty in the friction coefficient, yet it arises from the choice of formula alone and not from any error of measurement.**

Computation 2.6: **the two extremes.**

limit	ratio
inner radius approaching the outer (a thin ring)	**1.000000**
inner radius approaching zero (a full disc)	**1.333333**

> **Observation 2.7**
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> **both ends check independently.One for the thin ring and $4/3$ for the disc, the latter solvable by hand——either failing would mean the implementation is wrong. Both sides of the boundary are in view.**

3. Where to Put the Inner Radius

Computation 3.1: **holding the maximum contact pressure fixed, searching numerically for the inner radius that maximises torque.**

quantity	value
numerical search	**0.086602** m (**0.577350** of the outer radius)
independent closed form $r_o/\sqrt{3}$	0.086603 m (0.577350)

Observation 3.2

search and closed form agree.To within 10^{-5}——the closed form comes from differentiating $r(r_o^2 - r^2)$ quite apart from the search, so this is a comparison.

Observation 3.3

the two sides of the peak were checked.Torque falls at 0.9 and at 1.1 times the optimum——a single point is not called a maximum.

Observation 3.4

keeping the inside or extending the outside both lose.Too small an inner radius lets the slow-sliding centre consume area——too large a one leaves too little area at all. The $1/\sqrt{3}$ is that balance.

4. The Separator

Observation 4.1

this is the core.The two formulas do not claim to hold at the same time——uniform pressure is a new face and uniform wear a bedded-in one, and one clutch travels from the first to the second as it is used. The disagreement is a history, not an ambiguity.

Observation 4.2

the separator is whether two descriptions claim the same instant.If they do, one is wrong, or at least a choice is forced——if they do not, both are right, and merely right in turn.

Observation 4.3

that is where Paper 527 differs.Its optical and radio conventions describe one atmosphere at one instant and disagree by 1.077033, so a choice is forced——here no choice is needed. “Two conventions” can mean rivals or a sequence, and the same shape is settled in different ways.

Observation 4.4

the $4/3$ is not evidence of a common root.The limiting ratio 1.333333 is the same number as the $4/3$ of Paper 527——but under Paper 405 there is no bijection between them: that one is a rounded convention on an atmospheric refractivity gradient, this one a ratio out of an area integral over a disc. One number, two roots.

Observation 4.5

separate roots under Paper 300.“The two conventions differ” carries two possible pressure distributions ——“the difference vanishes for a narrow annulus” carries the width in radius. On a thin ring the first still holds while the second erases the gap.

Observation 4.6

what is not said.Neither formula is called correct——that correctness depends on the time is the claim here; and with the friction coefficient held constant, the dominant practical uncertainty lies elsewhere.

5. Roll Call and Closing

1. **uniform pressure 0.118551 m** (Computation 2.1): **inner radius 0.080 m.**
2. **uniform wear 0.115000 m** (Computation 2.1): **likewise.**
3. **ratio from 1.194925 to 1.000396** (Computation 2.1): **closing with the annulus.**
4. **torque 165.971014 against 161.000000 N m** (Computation 2.4): **3.0876 per cent.**
5. **thin-ring limit 1.000000** (Computation 2.6): **one side.**
6. **disc limit 1.333333** (Computation 2.6): **a $4/3$ solvable by hand.**

7. **optimum inner radius 0.086602 m** (Computation 3.1): **by search.**
8. **matching $r_o/\sqrt{3}$** (Observation 3.2): **an independent comparison.**
9. **the separator is claiming the same instant** (Observation 4.2): **rivalry or history.**
10. **the $4/3$ differs in root from Paper 527** (Observation 4.4): **under Paper 405.**

Closing: **Clutch torque has two formulas, and the answer depends on which is used.Uniform pressure gives an effective radius of 0.118551 m and uniform wear 0.115000 m——at a friction coefficient of 0.35 and an axial force of 4000 N the torques are 165.971014 and 161.000000 N m, differing by 4.971014 N m, or 3.0876 per cent. That is small beside the uncertainty in the friction coefficient, yet it comes from the choice of formula alone and not from any error of measurement. The gap closes as the annulus narrows, running monotonically from 1.194925 at an inner radius of 0.020 m to 1.000396 at 0.140 m, since on a narrow ring every point is at nearly the same radius. Both extremes check: the ratio returns to 1.000000 for a thin ring and to 1.333333 for a full disc, the latter solvable by hand. Uniform pressure is always the larger and the order never reverses, so designing on the bedded-in formula errs safely.Yet the two are not in conflict——uniform pressure is a new face and uniform wear a bedded-in one, and one clutch travels from the first to the second as it is used, so the disagreement is a history rather than an ambiguity. The separator is therefore whether two descriptions claim the same instant: if they do a choice is forced, and if they do not both are right, merely right in turn.That is where Paper 527 differs——its optical and radio conventions describe one atmosphere at one instant and disagree by 1.077033, so there a choice is forced. "Two conventions" can mean rivals or a sequence. The limiting 1.333333 here is the same number as that paper's $4/3$, but under Paper 405 there is no bijection between them: one is a rounded convention on an atmospheric refractivity gradient and the other a ratio out of an area integral over a disc. Under Paper 300 as well, "the two conventions differ" carries two possible pressure distributions while "the difference vanishes for a narrow annulus" carries the width in radius.The placement of the inner radius was measured too——holding the maximum contact pressure fixed, torque is greatest with the inner radius at 0.577350 of the outer. The numerical search gave 0.086602 m against the independently differentiated closed form $r_o/\sqrt{3}$, agreeing within 10^{-5}, and torque falls on both sides of the peak. Too small an inner radius lets the slow-sliding centre consume area, and too large a one leaves too little area at all.To be honest——no clutch design is recommended. The friction coefficient was held constant, excluding its variation with temperature, speed and pressure, which in practice dominates the uncertainty. Heat was not treated, and uniform wear was posited as the bedded-in steady state without solving the path to it. Multi-plate and wet clutches were not distinguished, the number of faces entering only as a factor. And neither formula is called correct: that correctness depends on the time is the claim here.**

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