

A Structural Study of the 4π Patch

— 2nd ed.: Of the Four Claims About the Yamagishi Patch, Two Have Been Overtaken by Their Own Author —

[Paper 56]

Yuuki YAMAGISHI (山岸 勇輝)

Yamagishi Complexity Torus Lab, Japan

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— *From thinking to syncing.* —

ABSTRACT

In its first edition this paper organised the structure of the Yamagishi patch ($G = 4\pi$, $\hbar = 1/4\pi$) into four claims: (A) the closure of the Planck quantities is not peculiar to 4π ; (B) the synchronisation equation $e = 4\pi v r$ makes charge consistent as a geometric quantity; (C) the $16\pi^2$ falls into two distinct roots; (D) four independent requirements all select $d = 3$. What the second edition does is **set those four against the papers of this corpus written afterwards**. The outcome is asymmetric: **(A) and (C) were generalised and sharpened by the later work, (B) was narrowed by it, and (D) survives untouched. First, (A) was a special case of a theorem in Paper 116.** The first edition said “geometric quantities carry a^0 , material ones a^{-1} ”. Paper 116 proved the general statement $\epsilon = \mu$ — the power of the patch constant **is the mass dimension** — half-integers included. The first edition's dichotomy is merely the case $\mu = 0$ versus $\mu = 1$ (Observation 2.4). **Second, the “problem” (B) set out to solve was not a problem.** The first edition called the half-integer power $a^{-1/2}$ arising from $q_P = \sqrt{\hbar c}$ something that “falls outside Theorem 1”. **It does not.** Gaussian charge has $[e] = M^{1/2} L^{3/2} T^{-1}$, that is $\mu = 1/2$, and by $\epsilon = \mu$ the value $a^{-1/2}$ is **correct**. And $e = 4\pi v r$ has $\mu = 0$ by construction, so this **did not resolve the half-integer; it substituted a quantity that never had one** (Observation 3.2). Further, the third edition of Paper 15 has narrowed $e = 4\pi v r$ to “**a restatement holding at one radius**”, and the first edition's use of it as a general representation exceeds that range (Observation 3.3). **Third, Family II in (C) was not a single root.** The third edition of Paper 46 showed that the loop factor's $16\pi^2$ is $2 \times (2\pi)^4 / \Omega_3$ — a **Jacobian times a Fourier convention divided by a geometry** — and that changing the convention turns $16\pi^2$ into 4 or $1/\pi^2$ (re-verified here). **The loop factor carries a convention and has a different standing from the characteristic-class normalisations** (Gauss--Bonnet, Chern, Hopf). The third edition of Paper 48 also requires correcting the table's entry “the source of 4π is the Pfaffian normalisation”: that normalisation is $(2\pi)^n \times (2^n n!)$, and it contains no $(4\pi)^n$ (Observation 4.4). **Fourth, (D) is untouched.** The generalisation to $n = d - 1$, the selection of $d = 3$ by four requirements, and the limitation that **this is a selection by consistency rather than a derivation**, all stand. Paper 114 later placed $n = d - 1$ on the side of “does not select the dimension”, which agrees with what the first edition had said of itself.

Scope of this paper: NO NEW PHYSICAL LAW IS CLAIMED. All external physics is standard (Bertrand 1873, Ehrenfest 1917, Hadamard 1923, Hopf 1931, BPST 1975, Wigner 1939). What the second edition adds is **an internal reconciliation**, and every correction comes from later papers of this corpus (15, 46, 48, 116).

1. Notation, assumptions, position

Definition 1.1 (the Yamagishi patch)

As a choice of natural units, set

$$G = 4\pi, \quad \hbar = \frac{1}{4\pi}$$

and more generally the **reciprocal patch** $G = a, \hbar = 1/a$ for a positive constant a (with $a = 4\pi$ a special case).

Here $4\pi = \int_{S^2} d\Omega$ is the solid angle of the unit sphere (Paper 01).

Position (throughout): This paper **claims no new physical law**. It shows what becomes structurally visible when known physics is rewritten under that patch, and **which agreements are genuine identities and which are numerical coincidences**. It avoids treating numerical agreement as identity, and keeps the discipline of not merging things of different provenance (Paper 48).

Added in the second edition: The first edition applied that discipline to the claims of others. **The second edition applies it to the first edition itself.**

2. Route A: the closure of the Planck quantities is not peculiar to 4π

Lemma 2.1 (cancellation in the reciprocal patch)

With $\hbar = 1/a, G = a$ one has $\hbar G = 1$ identically in a , and $\hbar/G = 1/a^2$.

Theorem 2.2 (closure and the dichotomy)

A Planck quantity whose dependence on $\hbar$ and G enters only through the product $\hbar G$ or the ratio $\hbar/G$ separates, under the reciprocal patch, into $c^N a^0$ (geometric) and $c^N a^{-1}$ (material). **The separation holds for every $a > 0$.**

Corollary 2.3 (the first edition's main conclusion, unchanged)

Closure and the dichotomy are consequences of the reciprocal patch $\hbar = 1/G$ and are not peculiar to $a = 4\pi$. The closure of the Planck quantities must therefore not be used as evidence for the specialness of 4π .

2.1 [Second edition] This theorem is generalised in Paper 116

Observation 2.4

Theorem 2.2 is a special case of the $\varepsilon = \mu$ proved in Paper 116. For any quantity of dimension $[X] = L^\lambda M^\mu T^\tau$, Paper 116 shows that under $G = a$, $\hbar = 1/a$,

$$X = a^{-\mu} c^N$$

that is, the power of the patch constant is the mass dimension itself.

The first edition's “geometric versus material” is nothing other than $\mu = 0$ versus $\mu = 1$:

Quantity	Dimension	μ	Power of a
l_P	L	0	a^0
t_P	T	0	a^0
m_P	M	1	a^{-1}
E_P	ML^2T^{-2}	1	a^{-1}
F_P	MLT^{-2}	1	a^{-1}
$\hbar$	ML^2T^{-1}	1	a^{-1}
G	$M^{-1}L^3T^{-2}$	-1	a^{+1}

The first edition found the dividing line; Paper 116 gave that line a name. And once named, it turned out to be **not a dichotomy but a continuous scale** — μ takes -1 and $1/2$ as well as 0 and 1 . That generalisation decisively changes the next section.

3. Route A, continued: the geometrisation of charge

The first edition observed that the Gaussian Planck charge $q_P = \sqrt{\hbar c}$ becomes $c^{1/2}a^{-1/2}$ under the reciprocal patch, so that **a half-integer power of a appears and falls outside Theorem 2.2**, and proposed to remove it through the synchronisation equation $e = 4\pi v r$.

3.1 [Second edition] That “problem” was not a problem

Observation 3.1 (the half-integer power is the correct value)

Gaussian charge has dimension $[e] = M^{1/2}L^{3/2}T^{-1}$, that is **mass dimension $\mu = 1/2$** . By $\varepsilon = \mu$ (Observation 2.4) the power of a must be $-1/2$. That q_P carries $a^{-1/2}$ is **not a defect of the patch but the half-integer mass dimension of charge**.

The first edition's phrase “falls outside Theorem 2.2” is therefore corrected. Nothing falls outside: **Theorem 2.2 had merely been stated for $\mu \in \{0, 1\}$** , and in Paper 116's form $\mu = 1/2$ is inside it.

Observation 3.2 ($e = 4\pi v r$ substitutes rather than resolves)

The synchronisation equation is built **from lengths and times alone** and so has $\mu = 0$ by construction. Its independence of a is therefore **a consequence of the definition, not a discovery**.

The half-integer power has not been removed. It appears to have been removed only because **a quantity with half-integer mass dimension was replaced by one without**. It is Gaussian charge that carries $\mu = 1/2$, not $4\pi v r$. **This is the same shape as Paper 45's verdict that “ $S = 1/(2\alpha)$ is a restatement of a definition”.**

Observation 3.3 (what the third edition of Paper 15 leaves standing)

The source of $e = 4\pi v r$, Paper 15, has been **substantially narrowed** in its third edition:

1. $\epsilon_0 = r$ is a **function of the configuration**, not a choice of units (a convention must be a number).
2. Read as a modification of the force law it gives $e^2/(4\pi r^3)$ — **an inverse cube**, for which no stable circular orbit exists.
3. The extension to a general principal quantum number was **withdrawn**.
4. **What remains is that $(4\pi v)r = e$ is a restatement holding at one radius**, and that in the unit system it implies, **charge has the dimension of area per second**.

The first edition used it as a **general representation of charge**, a reading wider than what Paper 15 now leaves standing; the second edition **returns it to that range**.

Corollary 3.4 (this section's conclusion rewritten)

The first edition's corollary that “under $m = 4\pi$ charge takes the form of the angular momentum mvr ” is **correct as a numerical restatement at one radius**. But it does not stand as the general claim that **charge is a geometric quantity**: the mass dimension of charge is $1/2$, and that does not change with the representation. **What changed was the quantity, not the dimension of the quantity**.

4. Route C: adjudicating the origins of $16\pi^2$

Definition 4.1 (criterion of common root, unchanged)

Two occurrences of $16\pi^2$ have a **common root** when there is an identity-preserving correspondence between the objects they normalise and **the source of the exponent 2 on the 4π is the same**. **Numerical agreement alone does not establish a common root** (Paper 48).

4.1 [Second edition] The corrected table

Tag	Occurrence	Source of the 2	Standing in the 2nd ed.
A	$(m=4\pi)^2$	algebraic square	convention $\times$ convention (Papers 46, 116)
B	loop factor $1/(4\pi)^{d/2}$	$d/2 = 4/2$	Jacobian $\times$ convention $\div$ geometry
C	$\text{Vol}(S_R^3)$ with $R=2$	the adjustment $R=2$	not admitted as evidence (Paper 45)
D	4D Gauss--Bonnet	dimension 4	$(2\pi)^n \times (2^n n!)$ (Paper 48)
E	second Chern number / instanton	dimension 4	characteristic-class normalisation
F	Hopf invariant	two forms	characteristic-class normalisation

Observation 4.2 (Family II is not a single root)

The first edition placed B, D, E and F together as “normalisations of characteristic classes and measures in four dimensions”, a **single root**. **That merger is too strong.**

As the third edition of Paper 46 showed, the loop factor's $16\pi^2$ decomposes as

$$16\pi^2 = 2 \times (2\pi)^4 / \Omega_3, \quad \Omega_3 = 2\pi^2$$

(re-verified here symbolically), where the **2** is the **Jacobian** of the change of variable $k^3 dk = \frac{1}{2} u du$, the $(2\pi)^4$ is the **convention** of where the 2π of the Fourier transform is placed, and $\Omega_3 = 2\pi^2$ is the surface area of the unit three-sphere, that is the **geometry**. And **changing the convention changes the number**: replacing $\int d^4 k / (2\pi)^4$ by $\int d^4 k / (2\pi)^2$ gives **4**, and $\int d^4 k$ gives $1/\pi^2$ (verified here). No physical amplitude changes, but **the number $16\pi^2$ does**.

D, E and F, by contrast, are characteristic-class normalisations and do not move with the Fourier convention. Hence **B has a different standing from D, E and F**, and Family II splits into at least two.

Observation 4.3 (Family I and Family II are distinct for a stronger reason)

The first edition called the agreement of A with Family II “a coincidence arising because $d/2 = 2$ at $d = 4$ ”. **That verdict is right, and the second edition supplies a stronger reason**: by the third edition of Paper 46, $2(2\pi)^4 / \Omega_3$ is **convention** $\times$ **geometry** $\times$ **Jacobian** while $(4\pi) \times (4\pi)$ is **convention** $\times$ **convention** (the patch constant may be any $a > 0$; Paper 116). **The compositions are entirely different.**

Observation 4.4 (correcting the attribution in row D)

The first edition's table gave the “source of 4π ” in row D as **the Pfaffian normalisation**. By the third edition of Paper 48, the Chern--Gauss--Bonnet normalisation is $(2\pi)^n \times (2^n n!)$, **one block**, and peeling 2^n off it to manufacture $(4\pi)^n$ was **cutting in the wrong place**. For $n = 1, 2, 3$ the block is $4\pi, 32\pi^2, 384\pi^3$ (verified here). **It contains no $(4\pi)^n$.**

4.2 Testing the bridge: is the atom's $16\pi^2$ a Hopf invariant? (unchanged)

Proposition 4.5 (the base S^2 is intrinsic)

In $E^2 = |\mathbf{p}|^2 c^2 + (4\pi c^2)^2$, fixing $m_0 = 4\pi$ and E makes $|\mathbf{p}|$ constant, so the momentum direction $\hat{\mathbf{p}}$ sweeps an S^2 . This corresponds to the base S^2 of the Hopf bundle and is forced by the physics.

Proposition 4.6 (the naive momentum S^3 is trivial)

The direction map from the one-point compactification S^3 of momentum space to the base S^2 is radial, and is the **trivial class** ($H = 0$) in $\pi_3(S^2) = \mathbb{Z}$. No Hopf structure ($H = 1$) arises.

Proposition 4.7 (independence of the Casimirs)

The state space of an electron of spin $1/2$ is a genuine Hopf bundle. But its $16\pi^2$ comes from spin, while A's comes from the mass patch. Mass and spin are **independent Casimirs** of the Poincaré group,

$$P^2 = m^2 c^2, \quad W^2 = -m^2 c^2 s(s+1)$$

and neither follows from the other. **The two $16\pi^2$ therefore have distinct roots, and identifying them would confuse mass with spin.**

Corollary 4.8 (the verdict on the bridge, unchanged)

The two $16\pi^2$ inside the atom have **distinct roots**. Since a genuine Hopf $16\pi^2$ does exist in electron spin, the natural entrance for any future Hopf-based formulation is **spin and the Berry phase**, not the mass representation. **The second edition does not alter this verdict:** it is the part the first edition wrote correctly, as a **negative result**.

5. Route D: the selection $n = d - 1$ and $3 + 1$ dimensions (unchanged)

Theorem 5.1 (the d -dimensional generalisation)

In d dimensions with a spherically symmetric power density $\rho(r) = r^{-n}$, the shell mass rate $dm/dr = \Omega_{d-1} r^{d-1-n}$ is constant if and only if $n = d - 1$.

This holds for every d , so the uniqueness theorem by itself does not select $d = 3$. Paper 114 later sorted objects by whether they “know how many dimensions they are in” and placed $n = d - 1$ on the side that does not select — in agreement with what the first edition had said of itself (confirmed in the second edition).

Proposition 5.2 (the consistent selection of $3 + 1$)

For the flux-conserving force $F \propto 1/r^{d-1}$, each of the following requires $d = 3$:

1. **Orbital stability**: the only central forces for which all bound orbits close are $1/r^2$ and r (Bertrand's theorem); the flux force matches the first only at $d = 3$.
2. **Atomic stability**: an attractive potential $-1/r^{d-2}$ has a bound ground state only for $d \leq 3$; for $d \geq 4$ collapse to the centre is not avoided.
3. **Wave propagation**: distortionless propagation with a sharp wavefront (Huygens' principle) holds in odd spatial dimensions, the smallest being $d = 3$.
4. **Topological normalisation**: the loop measure comes out as $16\pi^2$ at spacetime $d = 4$, that is spatial $d = 3$.
The second edition notes that this fourth item contains a Fourier convention (Observation 4.2) — it cannot be counted with the same force as the other three.

Corollary 5.3 (what $d = 3$ means, unchanged)

$d = 3$ is not the dimension in which flux conservation is special (it holds in every d). It is the only dimension in which flux conservation coincides with orbital stability, atomic stability and wave propagation.

Position stated (unchanged): This section is not a derivation that space **must** be three-dimensional but a **selection by consistency** (the lineage from Ehrenfest). **The two must not be confused.**

On the distribution of special dimensions (unchanged): $d = 3$ is selected relative to a particular set of criteria. **Change the criteria and the special dimension changes** — the dimension maximising the volume of the unit ball is $d = 5$. **There is no single magic dimension**; special dimensions are distributed according to what is being optimised.

6. Conclusion

1. **(A) was generalised by Paper 116.** That the closure and the dichotomy follow from the reciprocal patch and are not peculiar to 4π **remains correct**. In addition, that dichotomy has been shown to be **the mass dimension μ itself**. The first edition found the dividing line; **a later paper gave it a name** (Observation 2.4).
2. **The problem (B) set out to solve was not a problem.** The $a^{-1/2}$ of q_P is **the mass dimension $\mu = 1/2$ of Gaussian charge**, and nothing falls outside (Observation 3.1). Since $e = 4\pi v r$ has $\mu = 0$ by construction, this is **a substitution of quantities, not a resolution of the half-integer** (Observation 3.2). The third edition of Paper 15 has moreover narrowed $e = 4\pi v r$ to **a restatement at one radius**, and the first edition's general reading exceeded that (Observation 3.3). **The general claim that charge is a geometric quantity is withdrawn** (Corollary 3.4).
3. **Family II in (C) was not a single root.** By the third edition of Paper 46 the loop factor's $16\pi^2$ is **Jacobian $\times$ Fourier convention $\div$ geometry**, and changing the convention gives $16\pi^2 \rightarrow 4 \rightarrow 1/\pi^2$ (verified here). **B has a different standing from the characteristic-class normalisations D, E, F** (Observation 4.2). The row-D attribution also requires correction by the third edition of Paper 48: the normalisation is $(2\pi)^n \times (2^n n!)$ and contains no $(4\pi)^n$ (Observation 4.4). **The distinctness of Family I from Family II, on the other hand, holds for a stronger reason — the compositions differ entirely** (Observation 4.3).
4. **(D) is untouched.** The generalisation to $n = d - 1$, the selection of $d = 3$ by four requirements, and the limitation that **this is selection by consistency and not derivation**, all stand. **The fourth requirement (topological normalisation) does contain a Fourier convention and cannot be counted with the same force as the other three** (Proposition 5.2). **The negative result from the independence of the Casimirs is likewise untouched** (Corollary 4.8).

Closing: The first edition set out to separate which agreements are genuine identities and which are numerical coincidences. **What the second edition did was turn that measure on the first edition.** Of four claims, two were generalised by their own author, one was narrowed by him, and one survived intact. **Turning one's own discipline on oneself is how one finds out what is left.**

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Yuuki YAMAGISHI (山岸 勇輝)

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