The Fastest Moment Is the First, and Waiting Longer Adds Almost Nothing

— The intermetallic layer of a solder joint thickens as the square root of time, so doubling it takes **4** times as long and tenfolding it **100** times — and the averaged rate is exactly **2** times the present one, a two that is not the two of Paper 308 — [Paper 560]

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— *From thinking to syncing.* —

Abstract

A solder joint will not hold without an intermetallic layer, and cracks with too much of one.What this paper shows is that when such a layer grows as the square root of time, the growth is heavily front-loaded, and the averaged rate stands at exactly twice the present one.No new theorem or law is claimed. First, here the distance to be crossed lengthens as it is crossed——Paper 445 treated the time to cross a fixed distance, whereas this layer extends the next atom's journey by exactly what it has already built (Section 2). **Second, and this is the core.The growth is heavily front-loaded——one per cent of the time reaches 10 per cent of the thickness, and 25 per cent reaches exactly half** (Section 3). **Third, the rate itself keeps falling——by a factor of 244.948974 between one minute and a thousand hours** (Section 3). **Fourth, the averaged rate is exactly twice the present one——two to 12 digits, and solvable by hand** (Section 3). **Fifth, that two is not the two of Paper 308——under Paper 405, with no bijection** (Section 4). **Sixth, the separator is whether the distance to be crossed is fixed or lengthens——fixed, and the question is Paper 445's, when it arrives; lengthening, and it becomes this paper's, how the growth is distributed** (Section 4). **Keywords**: soldering, intermetallic compound, diffusion-limited growth, parabolic law, layer growth.

Scope of this paper (scope note): No new theorem or law is claimed——that diffusion-limited layer growth follows the square root of time is standard.No soldering schedule is recommended——neither temperature nor dwell is stated.No growth constant is assigned——only ratios are treated; the constant varies by orders with temperature, so no absolute thickness appears.No optimal thickness is stated——that too little fails to bond and too much embrittles is taken as given, and neither boundary is measured.Composition and phases are not distinguished——how many copper-tin phases form is not treated, and the layer is counted as one.Mechanics are not treated——why a thick layer cracks is not the subject.No comparison is made across temperatures——the factor of **1621.110730** in Section 4 is a counterfactual, "were the growth constant unchanged", and not a prediction.Relation to earlier papers: Paper 249 showed that the four pi that catches molecules is the four pi that spreads a field, treating only the steady state of diffusion control——this paper takes the same diffusion control in its moving-boundary transient: the same words, different phases of time.Paper 445 showed that what the waiting is for is diffusion rather than reaction, measuring "a hundred times the time for ten times the distance"——"ten times the thickness takes a hundred times the time" is the same law turned over, and is not new here. That paper fixes a distance and asks for a time; this one fixes a time and asks how the growth is distributed. The division between diffusion and reaction control is also that paper's subject, so it is only named here.Paper 308 showed that the gap between the buses you board is twice the average——"the averaged rate is twice the present one" carries the same two, but that one arises from a sampling bias favouring long intervals and this one from differentiating a square root.Paper 405 showed that a numerical coincidence is evidence of a common root only when there is a bijection——applied here to those two twos.Paper 300 showed that sharing a root is decidable——"the growth is front-loaded" and "the rate ratio is two" share one root, the square root itself.What is added is confirming the correspondence of time and thickness ratios to 12 digits, showing that the fraction of thickness always exceeds the fraction of time, measuring the rate ratio as **244.948974**, solving the averaged-to-present ratio independently and matching it to two, and placing the separator on whether the distance crossed is fixed or lengthens.

1. Notation and the Question

Setting: consider the intermetallic layer forming between solder and copper.The thicker it grows, the further the next atom must travel through it.When transport limits the process, the thickness x follows

$$x(t) = A\sqrt{t}$$

and since A varies by orders with temperature, no numerical value is assigned to it here; only ratios are taken.

Observation 1.1

the layer is wanted and unwanted at once.Without it the solder does not wet and does not hold——with too much of it the joint turns brittle and cracks. A quantity that must be neither too small nor too large.

> **Observation 1.2**
>
> **the layer is its own barrier.What has formed slows what forms next——so the rate cannot be constant.**

Question: **how heavily front-loaded is it?**

2. Growth by the Square Root

Computation 2.1: **how a ratio of times becomes a ratio of thicknesses.**

time	thickness
4 times	**2.000000**
60 times	**7.745967**
100 times	**10.000000**
3600 times	**60.000000**
60000 times	**244.948974**

> **Observation 2.2**
>
> **the square is already in the corpus.Paper 445 showed "a hundred times the time for ten times the distance", measuring 2.7778 hours for 1 mm and 3.17 years for 100 mm——"tenfolding the thickness takes 100 times the time" is that same law turned over. That paper fixes a distance and asks for a time; this one fixes a time and asks for a thickness. What is added begins after this.**

> **Observation 2.3**
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> **the boundary was checked.Zero time gives zero thickness——the cheapest confirmation that the formula is intact.**

Observation 2.4

this much holds without assigning A.Taking ratios cancels it——the effect of time can be stated without knowing the temperature, which keeps an unknown out of the argument.

3. The Growth Is Front-Loaded

Computation 3.1: **the growth reached by a given moment, as a fraction of the whole.**

fraction of time elapsed	fraction of thickness reached
1 per cent	**10.00** per cent
10 per cent	**31.62** per cent
25 per cent	**50.00** per cent
50 per cent	**70.71** per cent

Observation 3.2

this is the core.One per cent of the time reaches ten per cent of the thickness, and twenty-five per cent reaches exactly half——both exact to 12 digits, and the fraction reached always exceeds the fraction elapsed.

Observation 3.3

so “cool it slowly” means something else here.Most of the layer forms in the first instant of heating——shortening the dwell afterwards removes only the small remainder.

Computation 3.4: **the fall of the rate itself: between one minute and a thousand hours the growth rate differs by 244.948974 (above 240 and below 250).**

Computation 3.5: **comparing the averaged rate with the present one.The averaged rate is $x(T)/T$ and the present rate $\dot{x}(T)$, and the ratio is exactly 2.**

Observation 3.6

this two is solvable by hand.With $x = A\sqrt{t}$, $x(T)/T = A/\sqrt{T}$ while $\dot{x}(T) = A/(2\sqrt{T})$——matching the computed $\mathbf{2.000000}$ to 12 digits: a comparison against an independently solvable value.

Observation 3.7

reporting the averaged rate overstates the present one twofold.Dividing “it advanced x in a thousand hours” by the hours gives twice the current rate——for this quantity, the average does not represent the present.

4. The Separator

Observation 4.1

the separator is whether the distance to be crossed is fixed or lengthens.Fixed, and the question is when it arrives, which Paper 445 answered——lengthening, and the question becomes how the growth is distributed over time. Both the front-loading and the twofold gap between the averaged and the present rate come from the distance extending itself. One square root, two different questions.

Observation 4.1b: a second separator is whether transport or reaction limits.Under reaction control the thickness grows in proportion to time and nothing written here survives——but that division is the subject of Paper 445, so it is only named here: ten kelvin more multiplies the reaction by 4.0553 and the diffusion by only 1.5220, which is that paper’s answer.

Observation 4.2

what Paper 249 excluded is the subject here.Its scope note states that reaction control is not treated, and it handles only the steady state of diffusion control——this paper handles the same diffusion control in its moving-boundary transient. In the steady state the concentration gradient is fixed; in the transient it slackens as the layer lengthens. The same words, different phases of time.

Observation 4.3

the two is not evidence of a common root.Paper 308's "the gap between the buses you board is twice the average" carries the same number——but under Paper 405 there is no bijection. That one arises from a sampling bias favouring long intervals and depends on the exponential distribution; this one arises from differentiating a square root and depends on no distribution at all.

Observation 4.4

within this paper, one root under Paper 300."The growth is front-loaded" and "the rate ratio is two" both come from the square root——change the exponent to one and both vanish.

Computation 4.5: **a counterfactual across temperatures.The 60 seconds of a reflow and five years (1.5768×10^{8} s) stand in a ratio of 2.628000×10^{6}, so were the growth constant unchanged the thicknesses would stand at $\mathbf{1621.110730}$.**

Observation 4.6

this is not a prediction.The growth constant varies by orders with temperature, so the figure cannot be used as it stands——it shows rather that the square-root law alone cannot compare across temperatures. Knowing how time acts leaves how temperature acts still to be supplied.

Observation 4.7

what is not said.No optimal thickness is stated——that too little fails to bond and too much embrittles is taken as given, and neither boundary is measured.

5. Roll Call and Closing

1. **four times the time doubles it** (Computation 2.1): **to 12 digits.**
2. **a hundred times tenfolds it** (Computation 2.1): **to 12 digits.**
3. **zero time, zero thickness** (Observation 2.3): **the boundary.**
4. **one per cent of time, 10 per cent of thickness** (Computation 3.1): **the core.**

5. **twenty-five per cent of time, exactly half** (Computation 3.1): **to 12 digits.**
6. **a rate ratio of 244.948974** (Computation 3.4): **measured, then bracketed.**
7. **the averaged rate is twice the present** (Computation 3.5): **solvable by hand.**
8. **a different root from Paper 308's two** (Observation 4.3): **under Paper 405.**
9. **the separator is a lengthening distance** (Observation 4.1): **fixed, and it is Paper 445's question.**
10. **the counterfactual 1621.110730** (Computation 4.5): **not a prediction.**

Closing: **A solder joint will not hold without an intermetallic layer, and cracks with too much of one.Because that layer is its own barrier, it grows as the square root of time——doubling the thickness takes 4 times as long and tenfolding it 100 times, both exact to 12 digits and confirmed by inversion. The square itself is already in the corpus: Paper 445 showed "a hundred times the time for ten times the distance", measuring 2.7778 hours for 1 mm and 3.17 years for 100 mm. That paper fixes a distance and asks for a time, this one fixes a time and asks for a thickness, and the law is the same turned over. Taking ratios cancels the growth constant, so the effect of time can be stated without knowing the temperature.And the growth is heavily front-loaded——one per cent of the time reaches 10 per cent of the thickness and twenty-five per cent reaches exactly half, with the fraction reached always exceeding the fraction elapsed. Most of the layer forms in the first instant of heating, so shortening the dwell afterwards removes only the small remainder. The rate itself keeps falling, differing by 244.948974 between one minute and a thousand hours.The averaged rate stands at exactly twice the present one——with $x = A\sqrt{t}$, $x(T)/T = A/\sqrt{T}$ while $\dot{x}(T) = A/(2\sqrt{T})$, matching the computed 2.000000 to 12 digits: a comparison against a value solvable by hand. So dividing "it advanced x in a thousand hours" by the hours overstates the current rate twofold. For this quantity the average does not represent the present.The separator is whether the distance to be crossed is fixed or lengthens——fixed, and the question is when it arrives, which Paper 445 answered; lengthening, and it becomes how the growth is distributed over time, from which both the front-loading and the twofold gap follow. A second separator is whether transport or reaction limits, since under reaction control nothing written here survives; but that division is the subject of Paper 445, so it is only named here, its answer being that ten kelvin more multiplies the reaction by 4.0553 and the diffusion by only 1.5220. Paper 249 handles only the steady state of diffusion control, while this paper handles the same diffusion control in its moving-boundary transient: in the steady state the gradient is fixed, in the transient it slackens as the layer lengthens.The two was treated with care——Paper 308's "twice the average" carries the same number, but under Paper 405 there is no bijection: that one comes from a sampling bias favouring long intervals and depends on the exponential distribution, this one from differentiating a square root and depends on no distribution. Within this paper, however, Paper 300 gives one root: "the growth is front-loaded" and "the rate ratio is two" both come from the square root, and changing the exponent to one erases both.To be honest——no soldering schedule is recommended. No growth constant was assigned and only ratios were treated, so no absolute thickness appears, and no optimal thickness is stated either. Composition and phases were not distinguished, the layer being counted as one, and the mechanics of why a thick layer cracks were not entered. No comparison was made across temperatures: the factor of 1621.110730 between a reflow of 60 seconds and five**

years is a counterfactual, since the growth constant varies by orders with temperature. That calculation shows rather that the square-root law alone cannot cross a temperature difference.

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