Doubling the Overlap Adds Less Than One Per Cent

— Shear in a lap joint is not uniform: at an overlap of **20** mm the peak is **2.698236** times the average, and at **40** mm it is **5.345468** — and what sets the ceiling is not the adhesive but the stretching of the adherends — [Paper 561]

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— *From thinking to syncing.* —

Abstract

When a bond is not strong enough, the natural move is to lengthen the overlap.What this paper shows is that the shear crowds to the ends so that the capacity saturates, and that the ceiling is set by the stretching of the adherends rather than by the adhesive.No new theorem or law is claimed. First, the shear is not uniform——the peak is 2.698236 times the average at an overlap of 20 mm and 5.345468 at 40 mm (Section 2). **Second, and this is the core.So doubling adds nothing——from 20 to 40 mm the capacity rises only to 1.009542** (Section 3). **Third, while short, doubling does work——from 5 to 10 mm gives 1.491669: the same word, two different acts** (Section 3). **Fourth, the ceiling has a closed form——$2/\omega = 7.483315$ mm equivalent, already exceeded by 99 per cent at 40 mm** (Section 3). **Fifth, a thicker adhesive layer lowers the peak——3.783587 at 0.1 mm falls to 1.534604 at 0.8 mm** (Section 4). **Sixth, the separator is whether the adherends stretch——were they rigid, the stress would be uniform and the capacity would grow with the overlap without limit** (Section 4). **Keywords**: adhesive joint, shear lag, Volkersen analysis, overlap length, stress concentration.

Scope of this paper (scope note): No new theorem or law is claimed——Volkersen's shear-lag analysis dates from 1938 .No bonding design is recommended——which overlap to choose is not stated.Peel stress is excluded——a single lap joint bends about its eccentricity and peels; only shear is examined here, which makes this coarser than Goland and Reissner.The adhesive is taken as linearly elastic——a yielding adhesive caps the peak, and the capacity then exceeds what is computed here.No failure criterion is chosen——capacity is given as a ratio under failure at peak stress; a fracture-mechanics criterion would give other numbers.Cure shrinkage, moisture and temperature are excluded——neither residual stress nor ageing is included.It is not said that a thick adhesive layer is better——the peak falls, but a thick layer admits more defects and resists peel less well, and neither was measured here.Relation to earlier papers: Paper 343 showed that the size of the hole is not in the stress concentration——here the reverse: the overlap length does enter, and its entering is exactly what creates the ceiling. There a conclusion was built on a quantity being absent, here on one being present.Paper 344 showed that strength is a function of volume rather than a constant of the material, setting quantities that carry size beside those that do not——this paper adds one to that row: on one bonded face, area tells and length stops telling.Paper 560 showed that square-root growth is front-loaded——both are of the form "adding stops repaying", but there because the layer becomes its own barrier and here because the adherends stretch and crowd the load to the ends.Paper 300 showed that sharing a root is decidable——"the shear is not uniform" and "the capacity saturates" share one root.What is added is measuring the peak ratio over five overlaps, contrasting the gains from doubling as **1.009542** and **1.491669**, matching the ceiling to the independent closed form $2/\omega$, sweeping the adhesive thickness over four settings for a **2.465513**-fold fall in the peak, and placing the separator on whether the adherends stretch.

1. Notation and the Question

Setting: two plates of thickness t and modulus E are lapped over a length L with an adhesive layer of thickness t_a and shear modulus G_a, the load passing from one plate through the adhesive to the other.In Volkersen's solution the ratio of peak to average shear is

$$\frac{\tau_{\max}}{\bar{\tau}} = u \coth u, \qquad u = \frac{\omega L}{2}, \qquad \omega = \sqrt{\frac{2G_a}{E\,t\,t_a}}$$

Observation 1.1

there is a reason to expect uniformity.Twice the bonded area, twice the strength, is a natural thought——it assumes every part of the bond is working equally.

> **Observation 1.2**
>
> but the plates stretch.The plate still carrying the load stretches, the one that has handed it over does not—— their difference is the shear in the adhesive, and it is greatest at the two ends where the handover happens.

Question: how much does lengthening the overlap add?

2. The Shear Is Not Uniform

Computation 2.1: aluminium plates ($E = 70$ GPa, 2 mm thick) bonded with a layer of $G_a = 1$ GPa and 0.2 mm thickness give $\omega = \mathbf{267.261242}$ per metre, a characteristic length $1/\omega = \mathbf{3.741657}$ mm.

overlap	peak over average	capacity (mm equivalent)
5 mm	**1.144561**	4.368487
10 mm	**1.534604**	6.516337
20 mm	**2.698236**	7.412250
40 mm	**5.345468**	7.482974
80 mm	**10.690450**	**7.483315**

> **Observation 2.2**
>
> the peak grows with the overlap.Nearly uniform at 5 mm (**1.144561**) and **10.690450** at 80 mm——the longer it is, the more the middle idles.

> **Observation 2.3**
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> the boundary was checked.An overlap approaching zero returns a peak of one——so the intuition that a short bond is uniform is correct. It ceases to be correct when the bond is lengthened.

> **Observation 2.4**
>
> the characteristic length is only 3.741657 mm.The handover is essentially complete within that distance of each end——the adhesive further in is applied but not working.

3. So Doubling Adds Nothing

Computation 3.1: the gain from doubling, measured at three places.

doubling the overlap	ratio of capacity
5 mm to 10 mm	1.491669
10 mm to 20 mm	1.137487
20 mm to 40 mm	1.009542
40 mm to 80 mm	1.000046

> **Observation 3.2**
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> this is the core.From 20 to 40 mm the capacity rises to 1.009542, under one per cent——above 1.00 and below 1.02, measured before being bracketed.

> **Observation 3.3**
>
> the same word covers two different acts.From 5 to 10 mm the gain is 1.491669——the return on one operation differs nearly fiftyfold with where it is performed, so “double the overlap” is not an instruction.

> **Observation 3.4**
>
> it never falls, though.Every ratio exceeds one——a ceiling, not a reversal. Lengthening is not called harmful.

Computation 3.5: the ceiling in closed form: as $L \to \infty$ the capacity approaches $2/\omega = 7.483315$ mm equivalent.

Observation 3.6

the numerical value meets the closed form.At $L = 1000$ mm they agree within 10^{-6}——the closed form follows from $u \coth u \to u$, so this is a comparison.

Observation 3.7

40 mm already exceeds 99 per cent of it.Beyond that, no addition reaches one per cent——bonded area is being added and capacity is not.

Observation 3.8

the shape of Paper 560.There the growth was front-loaded because the layer became its own barrier——here the added length does not work because the adherends stretch and crowd the load to the ends. Both stop repaying, for different reasons.

4. The Separator

Observation 4.1

the separator is whether the adherends stretch.Send E in $\omega = \sqrt{2G_a/(Ett_a)}$ to infinity and $\omega \to 0$, so the peak becomes one——the stress is then uniform and the capacity grows with the overlap without limit. What sets the ceiling is not the adhesive but the plates.

Observation 4.2

so “the adhesive is weak” will not do.With the same adhesive, thinner plates raise ω and sharpen the peak——part of a joint’s weakness comes from the stiffness of the adherends rather than from any property of the adhesive.

Computation 4.3: sweeping the adhesive thickness at a fixed overlap of 20 mm.

adhesive layer	ω	peak over average
0.1 mm	377.9645	**3.783587**
0.2 mm	267.2612	2.698236
0.4 mm	188.9822	1.978131
0.8 mm	133.6306	**1.534604**

Observation 4.4

thicker gives a lower peak.Eight times thicker lowers it **2.465513**-fold (above 2 and below 3)——the far side of the intuition that a bond should be thin. A thicker layer can deform, and so spreads the load over a longer stretch.

Observation 4.5

but “thicker is better” is not written.The peak falls, and a thick layer admits more defects and resists peel less well——neither was measured here. Only what was measured is claimed.

Observation 4.6

Paper 343 points the other way.There the size of the hole was shown not to enter the stress concentration, and the conclusion was built on that absence——here the overlap length does enter, and its entering creates the ceiling. It adds one more to the row Paper 344 set out, of quantities that carry size and quantities that do not: on one bonded face, area tells and length stops telling.

Observation 4.7

one root, under Paper 300.“The shear is not uniform” and “the capacity saturates” both come from $u \coth u$——make the peak one and the ceiling goes with it.

5. Roll Call and Closing

1. $\omega = 267.261242$ **per metre** (Computation 2.1): **a characteristic length of** 3.741657 **mm.**
2. **peak** 2.698236 **at** 20 **mm** (Computation 2.1): **the middle idles.**
3. **peak** 10.690450 **at** 80 **mm** (Computation 2.1): **longer means more crowded.**
4. **a vanishing overlap gives a peak of one** (Observation 2.3): **the boundary.**
5. 20 **to** 40 **mm gives** 1.009542 (Computation 3.1): **the core.**
6. 5 **to** 10 **mm gives** 1.491669 (Computation 3.1): **the same word, two acts.**
7. **the ceiling** $2/\omega = 7.483315$ **mm** (Computation 3.5): **matching the closed form.**
8. 40 **mm exceeds** 99 **per cent of it** (Observation 3.7): **area without capacity.**
9. **eight times thicker lowers the peak** 2.465513**-fold** (Computation 4.3): **the far side of intuition.**
10. **the separator is a stretching adherend** (Observation 4.1): **rigid plates keep it proportional.**

Closing: **When a bond is not strong enough, the natural move is to lengthen the overlap.Yet the shear in a lap joint is not uniform——aluminium plates bonded with a 0.2 mm layer have a characteristic length of only 3.741657 mm, so the peak stands at 2.698236 times the average over 20 mm, 5.345468 over 40 and 10.690450 over 80. The handover of load is essentially complete within that distance of each end, and the adhesive further in is applied but not working. An overlap approaching zero returns a peak of one, so the intuition that a short bond is uniform is correct, and ceases to be correct when the bond is lengthened.So doubling the overlap adds less than one per cent ——from 20 to 40 mm the capacity rises only to 1.009542. Doubling from 5 to 10 mm gives 1.491669, so the return on one operation differs nearly fiftyfold with where it is performed and “double the overlap” is not an instruction. Every ratio still exceeds one, however: a ceiling, not a reversal. That ceiling is $2/\omega = 7.483315$ mm equivalent, agreeing with the value at $L = 1000$ mm within 10^{-6}, and 40 mm already exceeds 99 per cent of it, so beyond that bonded area is added and capacity is not.The separator is whether the adherends stretch——send E to infinity and ω goes to zero, the peak becomes one, the stress is uniform and the capacity grows with the overlap without limit. What sets the ceiling is not the adhesive but the plates. So “the adhesive is weak” will not do: with the same adhesive, thinner plates raise ω and sharpen the peak. Thickening the adhesive instead lowers the peak, from 3.783587 at 0.1 mm to 1.534604 at 0.8 mm, a 2.465513-fold fall on the far side of the intuition that a bond should be thin. Even so “thicker is better” is not written, since a thick layer admits more defects and resists peel less well, and neither was measured.Placed among the earlier papers——Paper 343 showed that the size of the hole is not in the stress concentration, building a conclusion on an absence, while here the overlap length does enter and its entering creates the ceiling. It adds one more to the row Paper 344 set out: on one bonded face, area tells and length stops telling. It shares with Paper 560 the form “adding stops repaying”, though there because the layer became its own barrier and here because the adherends stretch. Under Paper 300, “the shear is not uniform” and “the capacity saturates” share one root in $u \coth u$.To be honest——no bonding design is recommended. Peel stress was excluded, and since a single lap joint bends about its eccentricity this is coarser than Goland and Reissner. The adhesive was taken as linearly elastic, so a yielding adhesive caps the peak and the capacity then exceeds what is computed here.**

No failure criterion was chosen beyond failure at peak stress, and cure shrinkage, moisture and temperature were left out.

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