One Exponent Is Cruel in One Direction and Generous in the Other

— Life goes as the **−11.764706** power of stress, so a ten per cent error in stress costs two thirds of the life, leaving **0.325857** — yet read the other way the same exponent is generous, a tenfold life costing a stress reduction of only **17.7757** per cent — [Paper 564]

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September 2026

— *From thinking to syncing.* —

Abstract

Fatigue life depends steeply on stress, so estimates of stress are urged to be careful.What this paper shows is that the same large exponent is cruel where stress rises and generous where it falls, and that the asymmetry lies in the counting rather than in the exponent.No new theorem or law is claimed. First, the exponent is near twelve——life goes as the −11.764706 power of stress, recovered as a log-log slope without reading the formula (Section 2). **Second, and this is the core.A ten per cent error in stress leaves a third of the life——0.325857** (Section 2). **Third, twenty per cent brings it near a tenth——0.117073** (Section 2). **Fourth, the other direction is generous——a tenfold life costs a stress reduction of 17.7757 per cent** (Section 3). **Fifth, a second decade does not cost twice as much——32.3917 per cent, less than twice 17.7757** (Section 3). **Sixth, the separator is which quantity is counted in proportions——count stress in per cent and it looks cruel; count life in decades and it looks generous. The exponent is one and the same** (Section 4). **Keywords**: fatigue, Basquin's law, S-N curve, fatigue life, stress amplitude.

Scope of this paper (scope note): **No new theorem or law is claimed——Basquin's relation and the representative value of its exponent are standard.No design life is provided——safety factors, scatter and the surface, size and notch factors of a real component are excluded; the numbers are to be read only as ratios.The exponent is a representative value ——$b = -0.085$ is typical for steel and moves with material and treatment; the conclusion here follows not from that value but from the exponent being large.The existence of an endurance limit is not discussed——ferrous alloys are said to show a knee and aluminium not; only the power-law range is examined, and nothing beyond the knee.Mean stress is excluded——fully reversed loading is assumed, and a tensile mean stress shortens life.Crack growth is not treated—— life is handled as one number, cycles to failure; its division into initiation and growth belongs to Paper 565.It is not said that lowering the stress is cheap——what an 18 per cent reduction costs to build is not included. What is cheap is the exponent, not the construction.Relation to earlier papers**: **Paper 262 showed that there are three ways to break and strength does not decide which, stating in its scope note that fatigue and time dependence are not treated and that only a single application of load is examined——this paper takes the seat it left. There the limits were buckling, yield and fracture under one loading; here the limit itself becomes a function of the number of cycles.Paper 558 showed that the same ten per cent gives 1.4641 or 0.7513 depending on which dimension——there three exponents lived in one formula and the answer changed with which was moved; here there is only one exponent, but it is so large that the answer changes with which way it is read.Paper 300 showed that sharing a root is decidable——"cruel upward" and "generous downward" share one root, the exponent itself.**What is added is **recovering the exponent as a log-log slope without the formula**, **measuring the life at 1.1 times the stress as 0.325857 and bracketing it**, **inverting for the 17.7757 per cent that buys a decade and confirming the return of exactly tenfold**, **showing that two decades cost less than twice one**, and **placing the separator on which quantity is counted in proportions**.

1. Notation and the Question

Setting: **between a cyclic stress amplitude σ and the cycles to failure N, take Basquin's relation**

$$\sigma \propto N^{\,b}, \qquad b = -0.085$$

with b a representative value for steel, so that inversely $N \propto \sigma^{1/b}$.

> **Observation 1.1**
>
> **a small b and a large $1/b$ are one fact.$b = -0.085$ says that stress falls only slowly——and says at the same time that life moves violently. One number, read two ways.**

> **Observation 1.2**
>
> **the working phrase leans towards stress.“Estimate the stress carefully”——spoken from the side of life, the view should be different.**

Question: **how differently does one exponent look when read both ways?**

2. Upward It Is Cruel

Computation 2.1: **recovering the exponent numerically: $1/b = -11.764706$, and a numerical log-log slope returns the same within 10^{-6}.**

> **Observation 2.2**
>
> **this is a check.The exponent written in the relation and the slope taken from the function agree——an implementation departing from the relation would fail it, and the magnitude lies above 11 and below 12.**

Computation 2.3: **life against stress.**

stress	life
0.90	**3.454009**
0.95	1.828417
1.00	1.000000
1.05	0.563267
1.10	**0.325857**
1.20	**0.117073**

> **Observation 2.4**
>
> **this is the core.Ten per cent too much stress leaves 0.325857 of the life, a third——above 0.30 and below 0.35, measured before being bracketed.**

Observation 2.5

twenty per cent comes near a tenth.0.117073——a fifth is a modest error in a load estimate and changes the order of the life.

Observation 2.6

the boundary was checked.Unit stress gives unit life to 12 digits, and the relation is monotone——a confirmation that nothing is broken.

3. Downward It Is Generous

Computation 3.1: **the same relation inverted.**

life wanted	stress required	reduction
10 times	0.822243	17.7757 per cent
100 times	0.676083	32.3917 per cent

Observation 3.2

the inversion is a check.Feeding the stresses back returns exactly ten and a hundred times, within 10^{-9}——confirming separately that they solve the relation.

Observation 3.3

a decade costs 17.7757 per cent.Above 17 and below 18——to gain ten times the life, lower the stress by a little under a fifth. It is the same exponent read in Section 2 as “a tenth costs two thirds”.

Observation 3.4

the reductions do not accelerate.Two decades cost 32.3917 per cent, less than twice the 17.7757 of one (35.5514)——a power law charges the same proportion for each decade, and since proportions multiply rather than add, later decades come cheaper.

Observation 3.5

one exponent, two faces.A tenth more stress drops the life to a third; a fifth less stress multiplies it tenfold ——both are the -11.764706 power. Cruelty and generosity are two sides of one number.

4. The Separator

Observation 4.1

the separator is which quantity is counted in proportions.Count stress in per cent and answer in per cent of life, and the exponent multiplies the cruelty——count life in decades and answer in per cent of stress, and its reciprocal multiplies the generosity. The asymmetry lies in the counting, not in the exponent.

Observation 4.2

an exponent of one erases it.A tenth more costs a tenth; ten times the life takes a tenth of the stress——the view is the same read either way. What makes the asymmetry is the size of the exponent.

Observation 4.3

what varies differs from Paper 558.There $+4$, -3 and -1 lived in one formula and the answer changed with which was moved——here there is one exponent only, but so large that the answer changes with which way it is read. The same power-law leverage, and a different thing varying.

Observation 4.4

one root, under Paper 300.“Cruel upward” and “generous downward” both come from $1/b$——set b to -1 and both vanish. The usual verdict in this body of work is separate roots, and here it is one.

Observation 4.5

this is the seat Paper 262 left.Its scope note states that fatigue and time dependence are not treated, and that only a single application of load is examined——under one loading the limits were buckling, yield and fracture; once cycles enter, the limit itself becomes a function of their number. One material, and the shape of the question changes with how the load is applied.

Observation 4.6

what is not said.Lowering the stress is not called cheap——what an 18 per cent reduction costs to build is not included. What is cheap is the exponent, not the construction.

5. Roll Call and Closing

1. **exponent -11.764706** (Computation 2.1): **recovered without the formula.**
2. **its magnitude between 11 and 12** (Observation 2.2): **an independent comparison.**
3. **1.1 times the stress leaves 0.325857** (Computation 2.3): **the core.**
4. **1.2 times leaves 0.117073** (Computation 2.3): **an order changes.**
5. **unit stress, unit life** (Observation 2.6): **a boundary, to 12 digits.**
6. **a decade costs 17.7757 per cent** (Computation 3.1): **generous.**
7. **feeding it back returns tenfold** (Observation 3.2): **the inversion checks.**
8. **two decades cost 32.3917 per cent** (Computation 3.1): **no acceleration.**
9. **the separator is what is counted in proportions** (Observation 4.1): **the asymmetry is in the counting.**
10. **the seat Paper 262 left** (Observation 4.5): **cycles change the question.**

Closing: **Fatigue life depends steeply on stress, so estimates of stress are urged to be careful.Upward, the dependence is indeed cruel——life goes as the -11.764706 power of stress, an exponent that a numerical log-log slope returns without reading the relation. Ten per cent too much stress leaves 0.325857 of the life, a third: above 0.30 and below**

0.35, measured before being bracketed. Twenty per cent leaves 0.117073, so a fifth — a modest error in a load estimate — changes the order of the life.Yet read the other way, the same exponent is generous——a tenfold life costs a stress reduction of only 17.7757 per cent, and feeding that value back returns exactly tenfold. A second decade costs 32.3917 per cent, less than twice the first (35.5514): a power law charges the same proportion for each decade, and since proportions multiply rather than add, later decades come cheaper. A tenth more stress drops the life to a third; a fifth less multiplies it tenfold — both the same −11.764706 power.The separator is which quantity is counted in proportions——count stress in per cent and answer in per cent of life, and the exponent multiplies the cruelty; count life in decades and answer in per cent of stress, and its reciprocal multiplies the generosity. The asymmetry lies in the counting, not in the exponent. An exponent of one erases it entirely, and the view becomes the same read either way, so what makes the asymmetry is the size. Under Paper 300 the two faces share one root in $1/b$: set b to -1 and both vanish.Placed among the earlier papers——Paper 262 stated in its scope note that fatigue and time dependence are not treated and that only a single application of load is examined, and this paper takes that seat. Under one loading the limits were buckling, yield and fracture; once cycles enter, the limit itself becomes a function of their number. Paper 558 also treated power-law leverage, but there three exponents lived in one formula and the answer changed with which was moved, while here one exponent is so large that the answer changes with which way it is read.To be honest——no design life is provided. Safety factors, scatter and the surface, size and notch factors of a real component were excluded, so the numbers are to be read only as ratios. The exponent $b = -0.085$ is representative of steel and moves with material and treatment, though the conclusion follows from its being large rather than from the value. The existence of an endurance limit was not discussed and only the power-law range examined; mean stress was excluded under fully reversed loading; and the division of life into initiation and growth belongs to Paper 565. Nor is lowering the stress called cheap: what is cheap is the exponent, not the construction.

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