What Sets the Life Is the Initial Flaw, Not the Final Size

— With an initial flaw of **0.5** mm and a final size of **25** mm, doubling the final size extends the life only to **1.048244**, while halving the initial flaw extends it to **1.482441** — and an infinite final size buys only **1.164716**, approached slowly at that — [Paper 565]

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— *From thinking to syncing.* —

Abstract

A tougher material is expected to last longer once a crack is present.What this paper shows is that most of the life is spent while the crack is small, so that enlarging the final size extends the life almost not at all.No new theorem or law is claimed. First, doubling the final size barely extends it——to 1.048244 (Section 2). **Second, and this is the core.Halving the initial flaw extends it far more——to 1.482441, the two gains standing at exactly 10.000000 to one** (Section 3). **Third, that ratio has a closed form——$\sqrt{2a_f/a_i}$, matching the numerical value within 10^{-9}** (Section 3). **Fourth, even an infinite final size has a ceiling——1.164716: a little over sixteen per cent, and no more** (Section 2). **Fifth, a third of the life is spent before the crack doubles——0.341137 of it** (Section 3). **Sixth, the separator is whether the exponent exceeds two——above two the integral gathers at the initial end, and below two the final end takes over** (Section 4). **Keywords**: crack growth, Paris law, fatigue life, initial flaw, fracture mechanics.

Scope of this paper (scope note): **No new theorem or law is claimed——the Paris law and its integration are standard.No life prediction is given——neither the coefficient nor the stress range is assigned, and everything reported is a ratio. The life of a real component does not follow from this.The geometry factor is held constant——the stress intensity is taken proportional to the square root of crack length, whereas in practice the geometry factor changes with the crack and matters most near the end.Crack initiation is not treated——the initial flaw is given, and the life spent reaching it lies on the side of Paper 564.The threshold is excluded——below a threshold range the crack does not advance at all, and including it would weight small cracks still more heavily.Crack closure and overloads are not treated——retardation and load history are excluded.It is not said that toughness is unnecessary——that the final size scarcely extends the life is one thing, and toughness being dispensable is another: toughness settles when the sudden break comes, which is a different question from how long the life is.Relation to earlier papers**: **Paper 262 showed that there are three ways to break, and that in fracture the flaw decides, with Griffith's relation making strength inversely proportional to the square root of the flaw——that paper asks, under one loading, whether it breaks now; this one asks, under repetition, how long it lasts. The same square root of the flaw enters a strength formula there and a rate formula here.Paper 561 showed that doubling the overlap adds less than one per cent——both are of the form "adding stops repaying", but there because the adherends stretch and crowd the load to the ends, and here because the integral gathers at the initial end.Paper 564 showed that one exponent is cruel one way and generous the other——that paper treats life as a single number without entering its parts; this one opens the growth half of it.Paper 300 showed that sharing a root is decidable——"the final size does not tell" and "the initial flaw does" share one root in the integral.**What is added is **matching the integral against a closed form**, **contrasting the gains from doubling the final size and halving the initial flaw as $\mathbf{1.048244}$ and $\mathbf{1.482441}$**, **confirming the ratio equals $\sqrt{2a_f/a_i}$ within 10^{-9}**, **obtaining the ceiling at infinite final size as $\mathbf{1.164716}$ and showing the approach to it is slow**, and **placing the separator on whether the exponent exceeds two**.

1. Notation and the Question

Setting: **let a crack of length a advance per cycle by the Paris law**

$$\frac{da}{dN} = C\,(\Delta K)^m, \qquad \Delta K \propto \sqrt{a}, \qquad m = 3$$

so that the cycles from an initial flaw a_i to a final size a_f follow

$$N \propto \frac{a_f^{\,1-m/2} - a_i^{\,1-m/2}}{1 - m/2}$$

which for $m = 3$ has exponent $-1/2$ and gives $N \propto 2(a_i^{-1/2} - a_f^{-1/2})$.

> **Observation 1.1**
>
> **a_i enters with a negative power.The smaller it is, the more it tells——a_f enters the same way but, being large, contributes little.**

> **Observation 1.2**
>
> **toughness sets the final size.a_f is the length at which the stress intensity reaches the fracture toughness——so "choosing a tougher material" amounts, in this formula, to enlarging a_f.**

Question: **how much does enlarging a_f extend the life?**

2. Enlarging the Final Size Barely Extends It

Computation 2.1: **from an initial flaw of 0.5 mm and a final size of 25 mm, varying only the final size.**

final size	relative life	ratio
12.5 mm	**71.554175**	**0.931773**
25 mm	**76.793608**	**1.000000**
50 mm	**80.498447**	**1.048244**
100 mm	**83.118164**	**1.082358**
1000 mm	**87.442719**	**1.138672**

> **Observation 2.2**
>
> **doubling gains under five per cent.1.048244——doubling the tolerable crack is a large difference in a material, and the life scarcely moves.**

> **Observation 2.3**
>
> the integral meets a closed form.$2(a_i^{-1/2} - a_f^{-1/2})$ returns the same value within 10^{-9}——a comparison with a separately assembled formula; and equal initial and final sizes return zero life.

Computation 2.4: the limit of infinite final size converges to $2a_i^{-1/2}$, or **1.164716** times the reference.

> **Observation 2.5**
>
> there is a ceiling.Above 1.1 and below 1.2——however far toughness is raised, this arrangement buys a little over sixteen per cent and no more.

> **Observation 2.6**
>
> and the approach is slow.At a final size of 1000 mm, two thousand times the initial flaw, **0.026044** still separates it from the limit——above 0.02 and below 0.03; a hundredfold further closes the gap to under a tenth of that, so it does converge. At practical sizes, though, even the ceiling is out of reach.

3. The Initial Flaw Tells by an Order

Computation 3.1: now holding the final size at 25 mm and varying the initial flaw.

initial flaw	relative life	ratio
1.00 mm	50.596443	0.658863
0.50 mm	76.793608	1.000000
0.25 mm	113.841996	**1.482441**
0.10 mm	187.350889	**2.439668**

Observation 3.2

this is the core.Halving the initial flaw gives $\mathbf{1.482441}$ against the $\mathbf{1.048244}$ of doubling the final size—— the gains stand at exactly $\mathbf{10.000000}$ to one.

Observation 3.3

the ten has a closed form.The ratio is $\sqrt{2a_f/a_i}$, and with $a_f/a_i = 50$ that is $\sqrt{100} = 10$——matching the numerical value within 10^{-9}. Its roundness is accidental, following from a ratio of fifty; another arrangement gives another number.

Observation 3.4

the ratio follows the span.The larger a_f/a_i, the heavier the initial end——in thin things, where the final size is small, the two come closer together. The gap is widest in large structures.

Computation 3.5: where the life is spent: by the time the crack has doubled, from 0.5 to 1.0 mm, $\mathbf{0.341137}$ of the life is gone.

Observation 3.6

above three tenths and below four.While the crack grows from 0.5 to 1.0 mm — a range invisible to the eye — a third of the life passes——the remaining 24 mm take only the other two thirds.

Observation 3.7

so inspection also tells at the initial end.Lowering the smallest detectable flaw buys life directly——though neither the cost nor the probability of inspection is measured here. What can be said is that the same “halving” tells ten times more at the initial end than at the final.

4. The Separator

Observation 4.1

the separator is whether the exponent m exceeds two.For $m > 2$ the exponent $1 - m/2$ is negative and the integral is dominated by a negative power of a_i——for $m < 2$ the sign turns and the final end takes over. Metals commonly fall between two and four, and the three used here lies within. The dividing line is not a property of the material but the value of the exponent itself.

Observation 4.2

the shape of Paper 561.There doubling the overlap added less than one per cent——for a different reason: there the adherends stretch and crowd the load to the ends, here the integral gathers at the initial end. The same ceiling, blocked in a different place.

Observation 4.3

a different question from Paper 262.There Griffith's relation answered whether it breaks now, making strength inversely proportional to the square root of the flaw——here the question is how long it lasts. The same square root enters a strength formula there and a rate formula here: the flaw plays one part twice.

Observation 4.4

one root, under Paper 300.“The final size does not tell” and “the initial flaw does” both come from the same integral——change the exponent to $m < 2$ and both reverse.

Observation 4.5

what is not said.Toughness is not called unnecessary——that the final size scarcely extends the life is one thing; toughness settles when the sudden break comes, which is a different question from how long the life is.

5. Roll Call and Closing

1. **doubling the final size gives $\mathbf{1.048244}$** (Computation 2.1): **under five per cent.**
2. **the integral meets the closed form** (Observation 2.3): **within 10^{-9}.**
3. **the ceiling at infinity is $\mathbf{1.164716}$** (Computation 2.4): **sixteen per cent and a little.**
4. **$\mathbf{0.026044}$ still separates at 1000 mm** (Observation 2.6): **approached slowly.**
5. **halving the initial flaw gives $\mathbf{1.482441}$** (Computation 3.1): **the core.**
6. **a ratio of gains of $\mathbf{10.000000}$** (Observation 3.2): **an order apart.**
7. **that ratio is $\sqrt{2a_f/a_i}$** (Observation 3.3): **a closed form.**
8. **$\mathbf{0.341137}$ spent before it doubles** (Computation 3.5): **a third of the life.**
9. **the separator is $m > 2$** (Observation 4.1): **below two it reverses.**
10. **a different question from Paper 262** (Observation 4.3): **breaking now, or lasting how long.**

Closing: **A tougher material is expected to last longer once a crack is present.Yet enlarging the final size barely extends the life——from an initial flaw of 0.5 mm and a final size of 25 mm, doubling the final size gives $\mathbf{1.048244}$, under five per cent. Doubling the tolerable crack is a large difference in a material, and the life scarcely moves. The integral matches the closed form $2(a_i^{-1/2} - a_f^{-1/2})$ within 10^{-9}, and equal initial and final sizes return zero life. Even an infinite final size gives only $\mathbf{1.164716}$, so however far toughness is raised this arrangement buys a little over sixteen per cent; and the approach is slow, with $\mathbf{0.026044}$ still separating at a final size two thousand times the initial flaw. At practical sizes even the ceiling is out of reach.The initial flaw, by contrast, tells by an order——halving it gives $\mathbf{1.482441}$, and the two gains stand at exactly $\mathbf{10.000000}$ to one. That ten follows from the closed form $\sqrt{2a_f/a_i}$; its roundness is accidental, coming from a ratio of fifty. Since the ratio follows the span, the initial end weighs most heavily in large structures. And while the crack grows from 0.5 to 1.0 mm — a range invisible to the eye — $\mathbf{0.341137}$ of the life passes, the remaining 24 mm taking only the other two thirds.The separator is whether the exponent exceeds two——above two the integral is dominated by a negative power of the initial flaw; below two the sign turns and the final end takes over. Metals commonly fall between two and four, and the three used here lies within, so the dividing line is not a property of the material but the value of the exponent. Under Paper 300, “the final size does not tell” and “the initial flaw does” share one root, and setting $m < 2$ reverses both.Placed among the earlier papers——Paper 262 asked, under one loading, whether it breaks now, with strength inversely proportional to the square root of the flaw; this paper asks how long it lasts, and the same square root enters a strength formula there and a rate formula here. It shares with Paper 561 the form “adding stops repaying”, blocked in a different place: there the adherends stretch, here the integral gathers at the initial end. Paper 564 treated life as a single number, and this paper opens its growth half.To be honest——no life prediction is given: neither coefficient nor stress range was assigned, and everything reported is a ratio. The geometry factor was held constant, though in practice it changes with the crack and matters most near the end. Crack initiation was not treated, the initial flaw**

being given, and the threshold was excluded, which would weight small cracks still more heavily; crack closure and overload retardation were left out. Nor is toughness called unnecessary: it settles when the sudden break comes, which is a different question from how long the life is.

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