The Tougher the Material, the Less Its Toughness Can Be Measured

— A valid plane-strain fracture toughness needs a specimen at least $2.5(K_{IC}/\sigma_y)^2$ thick, which is **2.78** mm for a high-strength steel and **3600.00** mm for an austenitic stainless steel, a span of **1296.000000** — the materials most worth measuring are the ones that cannot be — [Paper 566]

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— *From thinking to syncing.* —

Abstract

Fracture toughness is tabulated as a constant of the material.What this paper shows is that it is a constant only above a minimum specimen thickness, and that this thickness grows as the square of the toughness, so the most desirable materials are the hardest to measure.No new theorem or law is claimed. First, the thickness required depends only on K_{IC}/σ_y——double both and it is unchanged, to 12 digits (Section 2). **Second, and this is the core.It spans 1296.000000 across materials——from 2.78 mm for a high-strength steel to 3600.00 mm for an austenitic stainless** (Section 2). **Third, hard and brittle materials are the easy ones——high-strength steel and aluminium alloy both need under 10 mm** (Section 2). **Fourth, tough materials pass a metre——such a specimen cannot be made, and would not mean much if it were** (Section 2). **Fifth, the thickness is a fixed multiple of the plastic zone——$15\pi = 47.123890$ in all five materials, independent of the material** (Section 3). **Sixth, the separator is whether the plastic zone is small against the specimen——if it is not, K no longer represents the field with a single number** (Section 4). **Keywords**: fracture toughness, plane strain, size requirement, plastic zone, fracture toughness testing.

Scope of this paper (scope note): **No new theorem or law is claimed——the size requirement for plane-strain fracture toughness and the plastic-zone estimate are both in the standards.No material is assessed——the five values are representative rather than measurements of a particular grade or treatment, and are set down to show order and magnitude.Elastic-plastic fracture mechanics is not treated——materials that cannot meet the size requirement are handled with the J integral or CTOD; their existence is given as a reason here and nothing more.Test procedure is not discussed——precracking, load recording and the other validity criteria are excluded.Temperature and rate are excluded——the toughness of steel moves greatly with temperature and the requirement relaxes below the transition; only one temperature is examined.The coefficient in the plastic-zone estimate is not asserted——$1/(6\pi)$ is the usual plane-strain form and differs from the plane-stress $1/(2\pi)$, so the $\mathbf{47.123890}$ of Section 3 depends on that choice.The standard is not called too strict——the requirement exists so that K may represent the field with one number; relaxing it does not make the measurement possible but makes it a measurement of something else.Relation to earlier papers**: **Paper 344 showed that strength is a function of volume rather than a constant of the material——there the measured value moved with specimen size. Fracture toughness runs the other way, becoming independent of size above a thickness — except that for some materials that thickness exceeds anything practical. “There is a condition under which it is a constant” and “the condition cannot be met” are different things, and both hold here.Paper 262 showed that there are three ways to break and treated flaw-governed fracture through Griffith’s relation——that paper derives the condition for breaking from surface energy, and this one treats the limits of the instrument that measures it. One fracture, seen once from theory and once from measurement.Paper 565 showed that what sets the life is the initial flaw, not the final size——its final size is precisely the length that this paper’s toughness fixes. That paper says the value scarcely moves the life; this one says the value is hard to measure at all. Together: hard to obtain, and of little effect once obtained.Paper 300 showed that sharing a root is decidable——“the thickness spans 1296” and “the thickness is a fixed multiple of the plastic zone” share one root.**What is added is **setting the required thickness of five materials side by side and measuring the span as $\mathbf{1296.000000}$**, **confirming to 12 digits that only the ratio K/σ_y enters, and recovering it by inversion**, **confirming in all five that the thickness is 15π times the plastic zone**, and **placing the separator on whether the plastic zone is small against the specimen**.

1. Notation and the Question

Setting: **for a plane-strain fracture toughness K_{IC} to mean anything as a constant of the material, the specimen thickness B must satisfy**

$$B \geq 2.5\left(\frac{K_{IC}}{\sigma_y}\right)^2$$

with σ_y the yield stress.Measured on a thinner specimen, the value moves with the thickness.

> **Observation 1.1**
>
> **the requirement enters squared.Twice the toughness needs four times the thickness——and half the yield needs four times as well.**

> **Observation 1.2**
>
> **high toughness with low yield is struck from both sides.That combination is exactly what a tough material is ——so the better the material, the harsher the requirement.**

Question: **how thick, in practice?**

2. A Span of 1296 Across Materials

Computation 2.1: **the required thickness for five representative materials.**

material	K_{IC} (MPa$\sqrt{\text{m}}$)	σ_y (MPa)	thickness required
high-strength steel	50	1500	**2.78** mm
aluminium alloy	25	500	**6.25** mm
titanium alloy	60	900	**11.11** mm
low-alloy steel (pressure vessel)	200	500	**400.00** mm
austenitic stainless steel	300	250	**3600.00** mm

> **Observation 2.2**
>
> **this is the core.From 2.78 mm to 3600.00 mm, a span of 1296.000000——above 1200 and below 1400, measured before being bracketed.**

Observation 2.3

hard and brittle materials are the easy ones.High-strength steel at 2.78 mm and aluminium alloy at 6.25 mm both fit an ordinary specimen——the reluctance to break of a material that breaks easily is easily measured.

Observation 2.4

tough materials pass a metre.Austenitic stainless at 3600.00 mm——a 3.6 m specimen cannot be made, and its behaviour at that thickness would not be the behaviour of the thin article in service. Less unmeasurable than not worth measuring.

Observation 2.5

only the ratio enters.Doubling both K and σ_y leaves the thickness unchanged, to 12 digits——it does not move with a change of units, which makes it a check; and the ratio is recovered by inversion.

Observation 2.6

the directions were checked.Raising toughness at fixed yield increases it, and lowering yield at fixed toughness increases it too——either running the other way would mean the formula is wrong.

3. A Fixed Multiple of the Plastic Zone

Computation 3.1: taking the crack-tip plastic zone as $r_y = \dfrac{1}{6\pi}\left(\dfrac{K_{IC}}{\sigma_y}\right)^2$ and forming the ratio.

material	plastic zone r_y	thickness over zone
high-strength steel	0.0589 mm	$\mathbf{47.123890}$
aluminium alloy	0.1326 mm	$\mathbf{47.123890}$
titanium alloy	0.2358 mm	$\mathbf{47.123890}$
low-alloy steel	8.4883 mm	$\mathbf{47.123890}$
austenitic stainless steel	76.3944 mm	$\mathbf{47.123890}$

Observation 3.2

all five agree.Differing by under 10^{-9}——independent of the material. The requirement states not an absolute length but a number of plastic zones.

Observation 3.3

and that number is $15\pi.2.5 \div \frac{1}{6\pi} = 15\pi = \mathbf{47.123890}$——a comparison with a value fixed entirely outside this computation, above 47 and below 48.

Observation 3.4

so the requirement becomes legible.“At least $2.5(K/\sigma_y)^2$ thick” says the same thing as “the plastic zone is at most a forty-seventh of the specimen”——the meaning is not in the number 2.5 but in the plastic zone being small enough.

Observation 3.5

the zone itself differs by orders.From 0.0589 mm in high-strength steel to 76.3944 mm in austenitic stainless——in the latter, the “small” crack-tip plastic zone is seven centimetres across. That is no longer small.

4. The Separator

Observation 4.1

the separator is whether the plastic zone is small against the specimen.If it is, the crack-tip field is represented by the single number K——if it is not, it is not represented, and K_{IC} ceases to be a constant of the material. The requirement is not a convenience of measurement but the definition of the range in which K holds.

Observation 4.2

so the standard cannot be called too strict.Relaxing it does not make the measurement possible——it makes it a measurement of something else, which is why other quantities, the J integral and CTOD, are used instead. Their existence is given here as a reason and nothing more.

Observation 4.3

the direction is opposite to Paper 344.There strength was a function of volume, the measured value moving with specimen size——here toughness becomes independent of size above a thickness, except that for some materials that thickness exceeds anything practical. “There is a condition under which it is a constant” and “the condition cannot be met” are different, and both hold.

Observation 4.4

with Paper 565, it is unrewarding twice over.There doubling the final size — the length this toughness fixes — extended the life only to 1.048244——here that toughness proves hard to measure at all. Hard to obtain, and of little effect once obtained. It does not follow that toughness is unnecessary: what it settles is when the sudden break comes, which is a different question from how long the life is.

Observation 4.5

a different vantage from Paper 262.There Griffith's relation derived the condition for breaking from surface energy——here the limits of the instrument that measures that condition. One fracture, seen once from theory and once from measurement.

Observation 4.6

one root, under Paper 300.“The thickness spans 1296” and “the thickness is a fixed multiple of the plastic zone” both come from $(K/\sigma_y)^2$——equal ratios give equal thicknesses, and unequal ones open squared.

5. Roll Call and Closing

1. **high-strength steel, 2.78 mm** (Computation 2.1): **an ordinary specimen.**
2. **austenitic stainless, 3600.00 mm** (Computation 2.1): **3.6 metres.**
3. **a span of 1296.000000** (Observation 2.2): **the core, measured then bracketed.**
4. **doubling both changes nothing** (Observation 2.5): **only the ratio, to 12 digits.**
5. **47.123890 plastic zones** (Computation 3.1): **the same in all five.**
6. **which is 15π** (Observation 3.3): **a value fixed outside.**
7. **a plastic zone of 76.3944 mm** (Observation 3.5): **“small” at seven centimetres.**
8. **the separator is a small plastic zone** (Observation 4.1): **the definition of K's range.**
9. **opposite in direction to Paper 344** (Observation 4.3): **a condition that cannot be met.**
10. **unrewarding twice with Paper 565** (Observation 4.4): **hard to get, small in effect.**

Closing: **Fracture toughness is tabulated as a constant of the material.Yet it is a constant only above a minimum specimen thickness——the requirement $B \geq 2.5(K_{IC}/\sigma_y)^2$ enters squared, so twice the toughness needs four times the thickness and half the yield needs four times as well. A tough material is exactly high toughness with low yield, so the better the material the harsher the requirement. In practice the range runs from 2.78 mm for a high-strength steel to 3600.00 mm for an austenitic stainless, a span of 1296.000000 — above 1200 and below 1400, measured before being bracketed. The reluctance to break of a material that breaks easily is easily measured, while a 3.6 m specimen cannot be made and would not behave as the thin article in service. Only the ratio enters, doubling both K and σ_y leaving the thickness unchanged.And in every material that thickness is a fixed multiple of the plastic zone ——47.123890 in all five, differing by under 10^{-9}, which is 15π: a comparison with a value fixed entirely outside this computation. So the requirement becomes legible — “at least $2.5(K/\sigma_y)^2$ thick” says the same as “the plastic**

zone is at most a forty-seventh of the specimen”, and the meaning lies not in the number 2.5 but in the zone being small enough. The zone itself differs by orders, from 0.0589 mm in high-strength steel to 76.3944 mm in austenitic stainless, where the “small” crack-tip zone is seven centimetres across and no longer small at all.The separator is whether the plastic zone is small against the specimen——if it is, the crack-tip field is represented by the single number K; if not, K_{IC} ceases to be a constant of the material. The requirement is not a convenience of measurement but the definition of the range in which K holds. So the standard cannot be called too strict: relaxing it does not make the measurement possible but makes it a measurement of something else, which is why the J integral and CTOD are used instead. Under Paper 300, both facts here share one root in $(K/\sigma_y)^2$.Placed among the earlier papers——Paper 344 had strength as a function of volume, the measured value moving with size; here toughness becomes independent of size above a thickness, except that for some materials that thickness exceeds anything practical. “There is a condition under which it is a constant” and “the condition cannot be met” are different, and both hold. With Paper 565 it is unrewarding twice over: there doubling the final size extended the life only to 1.048244, and here that toughness proves hard to measure at all. It does not follow that toughness is unnecessary, since what it settles is when the sudden break comes. Paper 262 derived the condition for breaking from surface energy, while this paper treats the limits of the instrument that measures it.To be honest——no material is assessed: the five values are representative rather than measurements of a particular grade, set down to show order and magnitude. Elastic-plastic fracture mechanics was not entered, the J integral and CTOD being named only as a reason. Test procedure, precracking and the other validity criteria were excluded, and temperature and rate were left out, though the toughness of steel moves greatly with temperature and the requirement relaxes below the transition. The plastic-zone coefficient $1/(6\pi)$ is the usual plane-strain form and differs from the plane-stress $1/(2\pi)$, so the 47.123890 depends on that choice.

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